Non-geometry in string theory

based on 1211.6437 in collaboration with

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Basic idea

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Innocent idea:

1. String theory as a field theory with fields $X^\mu(\tau, \sigma)$
2. Canonical quantisation: $[X^\mu, \Pi^\nu] = \mathrm{i}$

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3. T-duality to a non-geometric situation: $X^\mu \to Z^\mu$

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2. Canonical quantisation: $[X^\mu, \Pi^\nu] = \mathrm{i}$
3. T-duality to a non-geometric situation: $X^\mu \to Z^\mu$
4. What are the commutators $[Z^\mu, Z^\nu]$?

String theory as a field theory

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- Define three-dimensional torus as "**target space**"

$$X^\mu(\tau, \sigma + 2\pi) = X^\mu(\tau, \sigma) + 2\pi N^\mu$$

String theory as a field theory 2

- **Choose** simple metric and "*H*-flux"

$$G = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \; B = \begin{pmatrix} 0 & HX^3 & 0 \\ -HX^3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

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- Highly non-trivial **equations of motion**

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- Our **ansatz**: dilute flux approximation

$$X^\mu(\tau,\sigma) = X_0^\mu(\tau,\sigma) + H X_H^\mu(\tau,\sigma) + \mathcal{O}(H^2)$$

String theory as a field theory 3

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- $\mathcal{O}(H^0)$: **free string**

$$X_0^\mu(\tau,\sigma) = x^\mu + p^\mu\tau + N^\mu\sigma + \frac{i}{2}\sum_{n\neq 0}\frac{1}{n}\left(\tilde{\alpha}_n^\mu e^{-in\sigma_+} + \alpha_n^\mu e^{-in\sigma_-}\right)$$

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- $\mathcal{O}(H^1)$: much more involved

$$\begin{aligned} X_H^\mu(\tau,\sigma) = &\; x_H^\mu + p_H^\mu\,\tau + \frac{\mathrm{i}}{2}\sum_{n\neq 0}\frac{1}{n}\left(\tilde{\gamma}_n^\mu e^{-\mathrm{i}n\sigma_+} + \gamma_n^\mu e^{-\mathrm{i}n\sigma_-}\right) \\ & -\epsilon^\mu{}_{\nu\rho}\; p^\rho N^\nu\;\frac{\tau^2}{2} \\ & -\epsilon^\mu{}_{\nu\rho}\;\frac{1}{2}\;\tau\;\left(N^\nu X_0^\rho|_\Sigma - p^\nu \tilde{X}_0^\rho|_\Sigma\right) \\ & -\epsilon^\mu{}_{\nu\rho}\;\frac{1}{4}\tilde{X}_0^\nu|_\Sigma X_0^\rho|_\Sigma \end{aligned}$$

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- Impose **canonical** equal-"time" commutation relations

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with momentum

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- Again, proceed **order by order**

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\end{aligned}$$

- Plug in mode expansion and **read off**

$$\begin{aligned}
&[\tilde{\alpha}_m^\mu, \tilde{\alpha}_n^\nu] = [\alpha_m^\mu, \alpha_n^\nu] = m\ \delta_{m,-n}\ \eta^{\mu\nu} \\
&[x^\mu, p^\nu] = \tfrac{\mathrm{i}}{2}\eta^{\mu\nu}
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- Instead, **two terms**

$$\begin{aligned} 0 &= [X^\mu(\tau,\sigma), X^\nu(\tau,\sigma')]\big|_H \\ &= H\,[X^\mu_0(\tau,\sigma), X^\nu_H(\tau,\sigma')] + H\,[X^\mu_H(\tau,\sigma), X^\nu_0(\tau,\sigma')] \end{aligned}$$

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- Finally, **read off** some - but by far not all - commutators

$$\begin{aligned} [\gamma^1, \alpha^2] - [\gamma^2, \alpha^1] &\neq 0 \\ [\gamma^1, p^2] - [p^2_H, \alpha^1] &\neq 0 \\ [\gamma^1, N^2] &= 0 \end{aligned}$$

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- Found compatible commutators for combinations of expansion coefficients
- No exhaustive list of such commutators $\rightarrow$ eventually, not possible to judge whether quantisation is fully consistent
- Difficulties come from solving order by order and cannot be overcome in that procedure, even in principle

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- Here: metric and B-field only depend on $X^3 \rightarrow$ **2 isometries**

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$$G = f\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{f} \end{pmatrix}, \quad B = f\begin{pmatrix} 0 & -HZ^3 & 0 \\ HZ^3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

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- Unknown **integration constants** (operators)

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- Had to **choose** particular commutators for the unknown integration constants

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