The Non-Abelian Born-Infeld Lagrangian in Open Superstring Theory

Georg Puhlfürst

12.04.2013

Outline

- Born-Infeld theory in electrodynamics
- Born-Infeld theory in string theory
- S-matrix approach

(linear) electrodynamics

Maxwell equations:

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$
$$\nabla \cdot \vec{B} = 0$$
$$\nabla \cdot \vec{E} = \rho$$
$$\nabla \times \vec{B} = \frac{\vec{j}}{c} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

Lagrangian:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

non-linear electrodynamics

Maxwell equations:

$$\begin{aligned}\nabla \times \vec{E} &= -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}\\ \nabla \cdot \vec{B} &= 0\\ \nabla \cdot \vec{D} &= \rho\\ \nabla \times \vec{H} &= \frac{\vec{j}}{c} + \frac{1}{c}\frac{\partial \vec{D}}{\partial t}\end{aligned}$$

In general non-linear relations:

$$\vec{D} = \vec{D}(\vec{E}, \vec{B}), \qquad \vec{H} = \vec{H}(\vec{E}, \vec{B})$$

e.g. linear dielectrics: $\vec{D} = \epsilon \vec{E}$

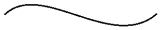
e.g. the Born-Infeld Lagrangian:

$$\mathcal{L} = -b^2 \sqrt{-\det\left(\eta_{\mu\nu} + \frac{1}{b} F_{\mu\nu}\right)} + b^2$$

The concept of D-branes as a generalization of boundary conditions

Dp-branes are extended objects with p spatial dimensions.

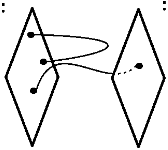
- Neumann b.c.: $\partial_\sigma X^\mu|_{\text{endpoints}} = 0$



- Dirichlet b.c.: $\partial_\tau X^\mu|_{\text{endpoints}} = 0$

- bounded to lines

- bounded to planes



⋮

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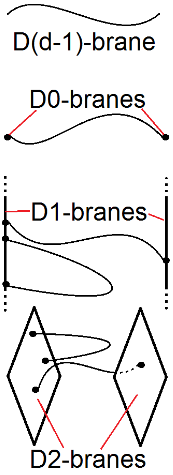
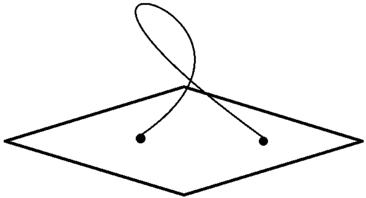


Abelian Born-Infeld Lagrangian in string theory



A Dp-brane has a $U(1)$ gauge field living on its world volume. The effective low energy theory for this gauge field is the Born-Infeld theory (in the case of constant F_{mn}): [Tseytlin et al., 1985]

$$\mathcal{L} = -T_p \sqrt{-\det\left(\eta_{mn} + 2\pi\alpha' F_{mn}\right)}$$

Compare with the Born-Infeld Lagrangian from non-linear electrodynamics:

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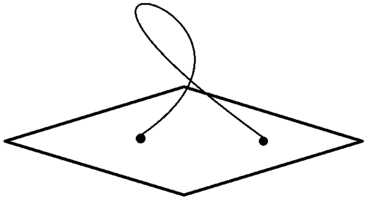

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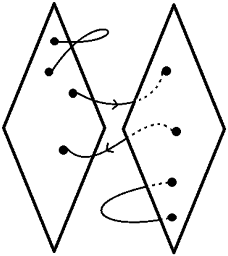

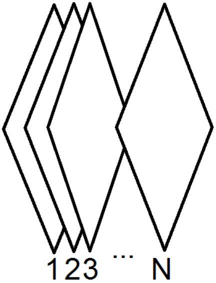

D-branes	2	N
sectors	4	N^2
gauge group for $d \to 0$	$U(2)$	$U(N)$

N coincident D-branes carry $U(N)$ gauge fields.

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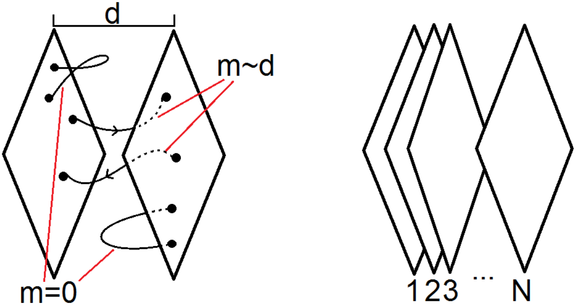

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String amplitudes

Latest result for the scattering of N open strings at tree-level:
[1106.2645,1106.2646]

$$A(1,...,N) = \sum_{\sigma \in S_{N-3}} A_{YM}(1, 2_\sigma, ..., (N-2)_\sigma, N-1, N) F^{\sigma}_{(1,...,N)}(\alpha')$$

- A_{YM} - color ordered Yang-Mills subamplitudes
- $F^{\sigma}(\alpha')$ - generalized Euler integrals

For example N=4:

$$A(1,2,3,4) = A_{YM}(1,2,3,4)\left(1 - \frac{\pi^2}{6} su\alpha'^2 + su(s+u)\zeta(3)\alpha'^3 + \mathcal{O}(\alpha'^4)\right)$$

S-matrix approach

$$\text{Amplitudes} \xleftarrow{\text{S-matrix}} \mathcal{L}$$

- → general ansatz for $\mathcal{L}$ with arbitrary coefficients
- → construct Feynman rules for this $\mathcal{L}$ to calculate corresponding amplitude
- → compare the result with the known string amplitude to fix the coeficients

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Use this procedure for every order in α' starting with the lowest one, e.g. $N = 4$:

$$A(1,2,3,4) = A_{YM}(1,2,3,4)\left(1 - \frac{\pi^2}{6} su\alpha'^2 + \ldots\right)$$
$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1\alpha' + \mathcal{L}_2\alpha'^2 + \ldots$$

$\Rightarrow \mathcal{L}_0 = \mathcal{L}_{YM}; \quad \mathcal{L}_1 = 0; \ldots$

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State of the art

α'^0	α'^2	α'^3	α'^4
Witten (9610043)	Tseytlin (9701125)	Koerber, Sevrin (0108169); Medina et al. (0208121); ...	Koerber, Sevrin (0208044); ...

Results for all orders in α':

- Tseytlin (9701125): symmetrized trace proposal
- Chandia, Medina (0310015): 4-field terms (e.g. $F_{\mu_1\mu_2}F^{\mu_2\mu_3}F_{\mu_3\mu_4}F^{\mu_4\mu_1}$ at $\mathcal{O}(\alpha'^2)$ or $D_{\mu_5}F_{\mu_1\mu_2}D^{\mu_5}F^{\mu_2\mu_3}F_{\mu_3\mu_4}F^{\mu_4\mu_1}$ at $\mathcal{O}(\alpha'^3)$)
- Barreiro, Medina (0503182): 5-field terms

Const. $F_{\mu\nu}$ in abelian case $\Rightarrow$ no terms with derivatives, since $\partial F = 0$
No similar limit in the non-abelian case $\Rightarrow$ due to the relation $[D_\mu, D_\nu]\, F_{\rho\lambda} = [F_{\mu\nu}, F_{\rho\lambda}]$ it makes no sense to set $DF = 0$.

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Conclusion

- A non-abelian version of the Born-Infeld Lagrangian can be used to understand the dynamics of D-branes.
- This would be valuable for the study of string theory models which use various D-brane configurations (and compactified dimensions).
- An important purpose of such models, is to explain the difference between the predicted number of spacetime dimensions in string theory (d=10) and the number we perseive (d=4).