Black Holes as Bose Einstein Condensates of Gravitons

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Microscopic Picture of Non-Relativistic Classicalons

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The usual Black Hole picture

Foundations

- BH descricption within GR: $r_g = 2G_N M$, J, Q
  $\rightarrow$ no hair theorem
- Hawking's semiclassical treatment
  ($M_{Pl} \rightarrow \infty$, $L_{Pl} \rightarrow 0$, r_g finite, $\hbar$ finite)
  $\rightarrow$Hawking radiation $T \propto \frac{1}{r_g}$
  $\rightarrow$Bekenstein entropy $S \propto A$

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Consequences

- Information paradox
- negative heat capacity...

Dvali / Gomez: the above treatment is not appropriate; quantum effects cannot be neglected

Quantum N-Portrait of Black Holes

- graviton interaction strength momentum dependent: $\alpha_{gr} = \frac{\hbar G_N}{\lambda^2}$
- BEC: universal coupling of N gravitons with $\lambda \sim r_g$
  $\rightarrow \alpha_{gr} = \frac{1}{\boldsymbol{N}}$, $\lambda = \sqrt{\boldsymbol{N}} L_P$, $M = \sqrt{\boldsymbol{N}} \frac{\hbar}{L_P}$ (no-hair)
- at quantum phase transition (leakiness) $\rightarrow$depletion
  evaporation rate $\frac{dM}{dt} = -\frac{T^2}{\hbar}$ $\quad (T := \frac{\hbar}{\sqrt{N} L_P})$
  half life-time $\tau = \frac{\hbar^2}{T^3 G_N}$

...**IF** $N \rightarrow \infty$, $L_P \rightarrow 0$, r_g finite, $\hbar$ finite (semiclassical limit) leading corrections of order $\mathcal{O}(\frac{1}{N})$ **NOT** $\mathcal{O}(\exp(-N))$

$\frac{1}{N}$-corrections become important at the phase transition

$$\Downarrow$$

- $\frac{1}{N}$-corrections account for the information paradox
- Black Holes carry $\frac{1}{N}$-hair

Toy Model I: 1D Bose-Einstein condensate with attractive interactions (Flassig, Pritzel, Wintergerst)

$$\hat{H} = \frac{\hbar^2}{2m}\int_0^V \mathrm{d}x\,(\partial_x\hat{\psi}^\dagger)\,(\partial_x\hat{\psi}) - \frac{U}{2}\int_0^V \mathrm{d}x\,\hat{\psi}^\dagger\hat{\psi}^\dagger\hat{\psi}\hat{\psi}$$

ground state:
$U < U_c$ homogeneous condensate
$U > U_c$ inhomogeneous condensate / soliton

Bogoliubov approximation: $\hat{\psi} = \psi_0 + \delta\hat{\psi}$

Energy spectrum of the Bogoliubov excitations:

$$\varepsilon(k) = \left(\frac{\hbar^2\delta k^2}{2m}\right)\left[\left(\frac{V}{2\pi}\right)^2\delta k^2 - \frac{U}{U_c}\right]^{1/2} \quad \left(\delta k = n\frac{2\pi}{V}\right)$$

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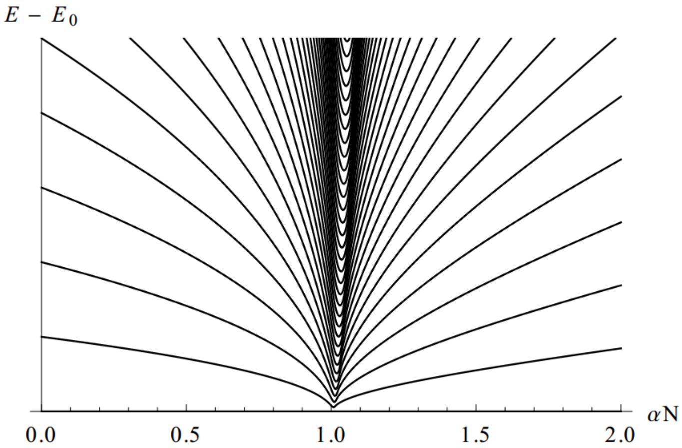
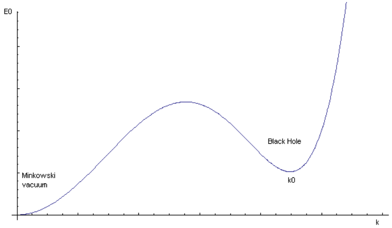


Toy Model II: 1D Bose-Einstein condensate with derivatively coupled interaction terms

$$\hat{H} = \frac{\hbar^2}{2m}\int_0^V \mathrm{d}x : (\partial_x \hat{\psi}^\dagger)(\partial_x \hat{\psi}) :$$

$$+\lambda \int_0^V \mathrm{d}x : ((\partial_x \hat{\psi}^\dagger)(\partial_x \hat{\psi}))^2 : +\kappa \int_0^V \mathrm{d}x : ((\partial_x \hat{\psi}^\dagger)(\partial_x \hat{\psi}))^3 :$$



Toy Model II: 1D Bose-Einstein condensate with derivatively coupled interaction terms

Bogoliubov approximation: $\hat{\psi} = \psi_0 + \delta\hat{\psi}$

Energy spectrum of quasi Bogoliubov excitations:

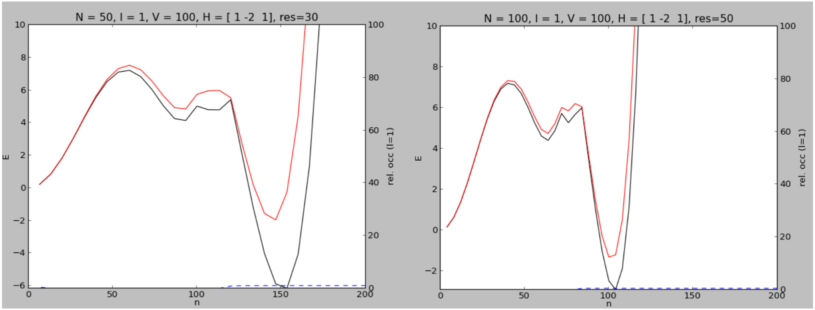
$$\varepsilon(\delta k) = \begin{cases} 8P_0 k_0 \delta k & \text{for } \delta k > 0 \\ 0 & \text{for } \delta k < 0 \end{cases}$$

Number of depleted real particles $v^2(\delta k) = \frac{1}{2}\left(\frac{k_0^2 + \delta k^2}{2k_0|\delta k|} - 1\right)$
high for $\delta k \gg k_0$

$\rightarrow$ Breakdown of Bogoliubov approximation / point of quantum phase transition

Toy Model II: 1D Bose-Einstein condensate with derivatively coupled interaction terms

Energy gap closing



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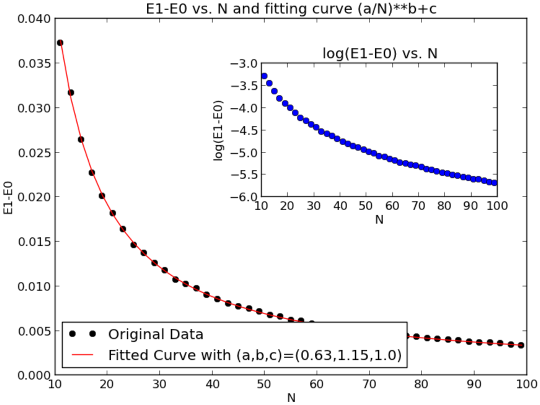

Toy Model II: 1D Bose-Einstein condensate with derivatively coupled interaction terms

Future work:

- numerical analysis for a modified Hamiltonian (e.g. adding $\int_0^V \mathrm{d}x : \left(\left(\partial_x^2 \psi^\dagger\right)\left(\partial_x^2 \psi\right)\right)^2 :$)
- higher spin models $\rightarrow$spin 2
- relativistic models