

# QCD corrections to Higgs plus jets production with GoSam

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**NLO QCD corrections to the production of Higgs plus two jets at the LHC,**  
*e-Print: arXiv:1301.0493, accepted by Physics Letters B*

[HvD, Greiner, Luisoni, Mastrolia, Mirabella, Ossola, Peraro, von Soden-Fraunhofen, Tramontano, 2013]

# Outline

Motivation

Scattering amplitudes at one-loop

Determining the parametric form of the numerator

Extended rank numerator

$pp \rightarrow H+2j$  with GoSam+Sherpa

$pp \rightarrow H+3j$  with GoSam+Sherpa

Summary

# Motivation

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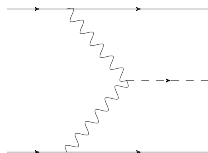
- ▶ Boson discovered by Atlas and CMS → Higgs?

# Motivation

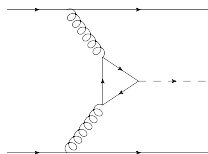
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spin, CP properties, couplings

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- ▶ Need to determine properties: spin, CP properties, couplings



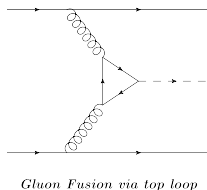
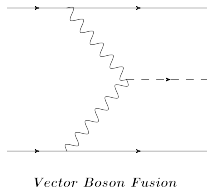
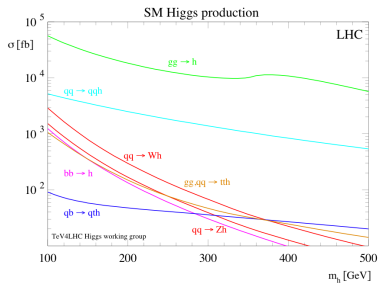
*Vector Boson Fusion*



*Gluon Fusion via top loop*

# Motivation

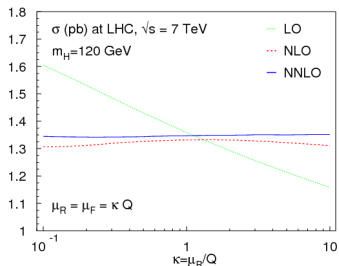
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# Motivation for NLO



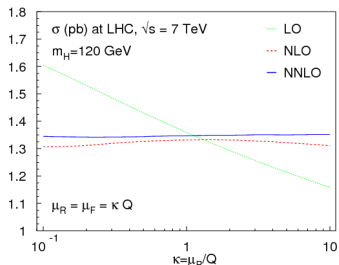
# Motivation for NLO



[Bolzoni, Maltoni, Moch, Zaro, 2011]

- ▶ Leading order too strong dependence on renormalization and factorization scale

# Motivation for NLO



[Bolzoni, Maltoni, Moch, Zaro, 2011]

- ▶ Leading order too strong dependence on renormalization and factorization scale
- ▶ Development of more general framework for NLO automation

# NLO cross section

$$\sigma^{NLO} = \int_{m+1} \left[ d^{(4)}\sigma^R - d^{(4)}\sigma^A \right] + \int_m \left[ d^{(4)}\sigma^B + \int_{loop} d^{(d)}\sigma^V + \int_1 d^{(d)}\sigma^A \right]$$

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↑

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► NLO cross section consists of:

- Leading Order: Born diagram
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- **Real corrections: Radiation**



# NLO cross section

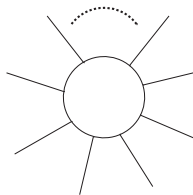
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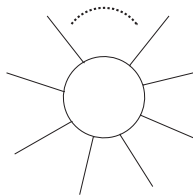
- Leading Order: Born diagram
- Virtual corrections: loop diagrams ← This talk only about this part
- Real corrections: Radiation
- Subtraction terms to regulate infinities



# Scattering amplitudes at one-loop

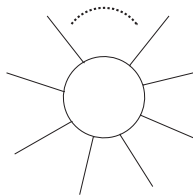


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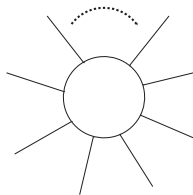
$$\mathcal{M}_n \equiv \int \mathcal{A}_n(\bar{q}) d\bar{q}$$

# Scattering amplitudes at one-loop



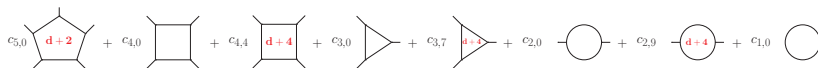
$$\mathcal{M}_n \equiv \int \mathcal{A}_n(\bar{q}) d\bar{q} \equiv \int d^{-2\epsilon}\mu \int d^4q \frac{N(q, \mu^2)}{\bar{D}_0 \dots \bar{D}_{n-1}}$$

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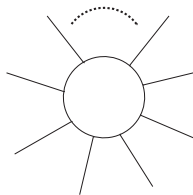


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► Decompose:

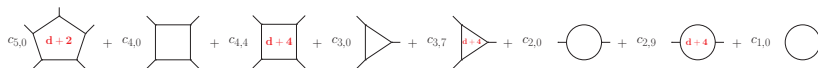


# Scattering amplitudes at one-loop



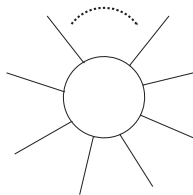
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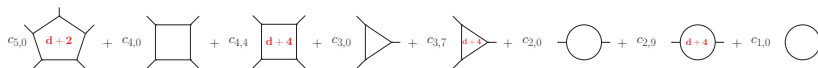
$$\int d^{-2\epsilon} \mu^2 d^4 q \mathcal{A}_n(q) = \int d\bar{q} \frac{c_{5,0} \mu^2}{D_0 D_1 D_2 D_3 D_4} + \int d\bar{q} \frac{c_{4,0} + c_{4,4} \mu^4}{D_0 D_1 D_2 D_3} \\ + \int d\bar{q} \frac{c_{3,0} + c_{3,7} \mu^2}{D_0 D_1 D_2} + \int d\bar{q} \frac{c_{2,0} + c_{2,9} \mu^2}{D_0 D_1} + \int d\bar{q} \frac{c_{1,0}}{D_0}$$

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► Decompose:



$$\begin{aligned} \int d^{-2\epsilon} \mu^2 d^4 q \mathcal{A}_n(q) &= \int d\bar{q} \frac{c_{5,0} \mu^2}{D_0 D_1 D_2 D_3 D_4} + \int d\bar{q} \frac{c_{4,0} + c_{4,4} \mu^4}{D_0 D_1 D_2 D_3} \\ &+ \int d\bar{q} \frac{c_{3,0} + c_{3,7} \mu^2}{D_0 D_1 D_2} + \int d\bar{q} \frac{c_{2,0} + c_{2,9} \mu^2}{D_0 D_1} + \int d\bar{q} \frac{c_{1,0}}{D_0} \end{aligned}$$

► computation of  $\mathcal{M}_n \rightarrow$  computation of **coefficients**

# Integral to Integrand

$$\begin{aligned}
 \int d^{-2\epsilon} \mu^2 d^4 q \mathcal{A}_n(q) = & \int d\bar{q} \frac{c_{5,0} \mu^2}{D_0 D_1 D_2 D_3 D_4} + \int d\bar{q} \frac{c_{4,0} + c_{4,4} \mu^4}{D_0 D_1 D_2 D_3} \\
 & + \int d\bar{q} \frac{c_{3,0} + c_{3,7} \mu^2}{D_0 D_1 D_2} + \int d\bar{q} \frac{c_{2,0} + c_{2,9} \mu^2}{D_0 D_1} + \int d\bar{q} \frac{c_{1,0}}{D_0}
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► integral  $\rightarrow$  integrand:

$$\begin{aligned}
 A_n(q) &= \frac{c_{5,0} \mu^2 + f_{01234}(q, \mu^2)}{D_0 D_1 D_2 D_3 D_4} + \frac{c_{4,0} + c_{4,4} \mu^4 + f_{0123}(q, \mu^2)}{D_0 D_1 D_2 D_3} \\
 &+ \frac{c_{3,0} + c_{3,7} \mu^2 + f_{012}(q, \mu^2)}{D_0 D_1 D_2} + \frac{c_{2,0} + c_{2,9} \mu^2 + f_{01}(q, \mu^2)}{D_0 D_1} + \frac{c_{1,0} + f_0(q, \mu^2)}{D_0}
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 \end{aligned}$$

$$\int d^{-2\epsilon} \mu^2 \int d^4 q \frac{f_{ij\dots}(q, \mu^2)}{D_i D_j \dots} = 0$$

# Determining the parametric form of the numerator

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$$\mathcal{A}_n = \sum_{ijklm} \frac{\Delta_{ijklm}(q, \mu^2)}{D_i D_j D_k D_l D_m} + \sum_{ijkl} \frac{\Delta_{ijkl}(q, \mu^2)}{D_i D_j D_k D_l} + \sum_{ijk} \frac{\Delta_{ijk}(q, \mu^2)}{D_i D_j D_k} + \sum_{ij} \frac{\Delta_{ij}(q, \mu^2)}{D_i D_j} + \sum_i \frac{\Delta_i(q, \mu^2)}{D_i}$$

- ▶ Form residues process independent
- ▶ Values of coefficients process dependent

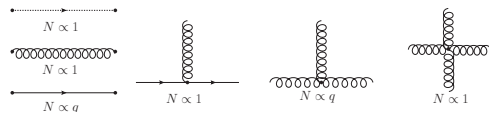
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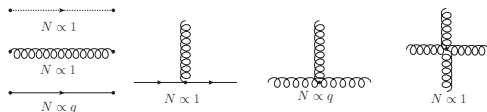
- ▶ Form residues process independent
- ▶ Values of coefficients process dependent
- ▶ Implemented in Samurai

[Ossola, Reiter, Tramontano, Mastrolia, 2010]

# Rankcounting, normal rank

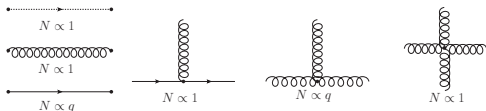


# Rankcounting, normal rank

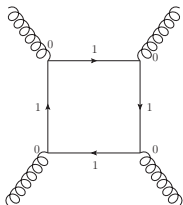


- ▶ Only  $q$  propagators and 3-gluon-vertices contribute one power of  $q$  to numerator

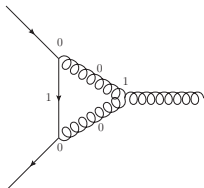
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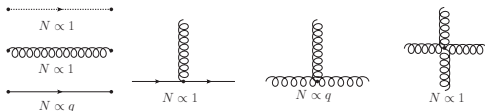
$$r_N = 4$$



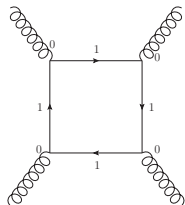
$$r_N = 2$$



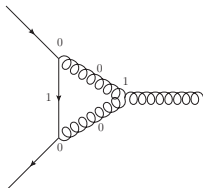
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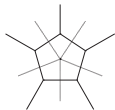
$$r_N = 4$$



$$r_N = 2$$

- ▶  $r_N \leq \#D$

# Integrand decomposition algorithm



$$\Delta_{ijklm}(\vec{q}) = \text{Res}_{ijklm} \left\{ \frac{N(\vec{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}} \right\}$$

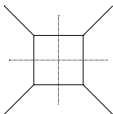
1 coefficient

# Integrand decomposition algorithm



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1 coefficient



$$\Delta_{ijkl\ell}(\bar{q}) = \text{Res}_{ijkl\ell} \left\{ \frac{N(\bar{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}} - \sum_{i << m}^{n-1} \frac{\Delta_{ijklm}(\bar{q})}{\bar{D}_i \bar{D}_j \bar{D}_k \bar{D}_\ell \bar{D}_m} \right\}$$

5 coefficients

# Integrand decomposition algorithm



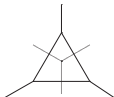
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$$\Delta_{ijkl}(\bar{q}) = \text{Res}_{ijkl} \left\{ \frac{N(\bar{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}} - \sum_{i << m}^{n-1} \frac{\Delta_{ijklm}(\bar{q})}{\bar{D}_i \bar{D}_j \bar{D}_k \bar{D}_\ell \bar{D}_m} \right\}$$

5 coefficients



$$\Delta_{ijk}(\bar{q}) = \text{Res}_{ijk} \left\{ \frac{N(\bar{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}} - \sum_{i << m}^{n-1} \frac{\Delta_{ijklm}(\bar{q})}{\bar{D}_i \bar{D}_j \bar{D}_k \bar{D}_\ell \bar{D}_m} - \sum_{i << \ell}^{n-1} \frac{\Delta_{ijkl}(\bar{q})}{\bar{D}_i \bar{D}_j \bar{D}_k \bar{D}_\ell} \right\}$$

10 coefficients

# Integrand decomposition algorithm



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1 coefficient



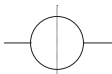
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# Integrand decomposition algorithm



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1 coefficient



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5 coefficients



$$\Delta_{ijk}(\bar{q}) = \text{Res}_{ijk} \left\{ \frac{N(\bar{q})}{D_0 \cdots D_{n-1}} - \sum_{i < < m}^{n-1} \frac{\Delta_{ijklm}(\bar{q})}{D_i D_j D_k D_l D_m} - \sum_{i < < \ell}^{n-1} \frac{\Delta_{ij\ell}(\bar{q})}{D_i D_j D_k D_\ell} \right\}$$

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$$\Delta_i(\bar{q}) = \text{Res}_i \left\{ \frac{N(\bar{q})}{D_0 \cdots D_{n-1}} - \sum_{i < < m}^{n-1} \frac{\Delta_{ijklm}(\bar{q})}{D_i D_j D_k D_l D_m} - \sum_{i < < \ell}^{n-1} \frac{\Delta_{ijk\ell}(\bar{q})}{D_i D_j D_k D_\ell} + \sum_{i < < k}^{n-1} \frac{\Delta_{ijk}(\bar{q})}{D_i D_j D_k} - \sum_{i < j}^{n-1} \frac{\Delta_{ij}(\bar{q})}{D_i D_j} \right\}$$

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1 coefficient



$$\Delta_{ijkl}(\bar{q}) = \text{Res}_{ijkl} \left\{ \frac{N(\bar{q})}{D_0 \cdots D_{n-1}} - \sum_{i < < m}^{n-1} \frac{\Delta_{ijklm}(\bar{q})}{D_i D_j D_k D_\ell D_m} \right\}$$

5 coefficients



$$\Delta_{ijk}(\bar{q}) = \text{Res}_{ijk} \left\{ \frac{N(\bar{q})}{D_0 \cdots D_{n-1}} - \sum_{i < < m}^{n-1} \frac{\Delta_{ijklm}(\bar{q})}{D_i D_j D_k D_\ell D_m} - \sum_{i < < \ell}^{n-1} \frac{\Delta_{ijkl}(\bar{q})}{D_i D_j D_k D_\ell} \right\}$$

10 coefficients



$$\Delta_{ij}(\bar{q}) = \text{Res}_{ij} \left\{ \frac{N(\bar{q})}{D_0 \cdots D_{n-1}} - \sum_{i < < m}^{n-1} \frac{\Delta_{ijklm}(\bar{q})}{D_i D_j D_k D_\ell D_m} - \sum_{i < < \ell}^{n-1} \frac{\Delta_{ijkl}(\bar{q})}{D_i D_j D_k D_\ell} - \sum_{i < < k}^{n-1} \frac{\Delta_{ijk}(\bar{q})}{D_i D_j D_k} \right\}$$

10 coefficients

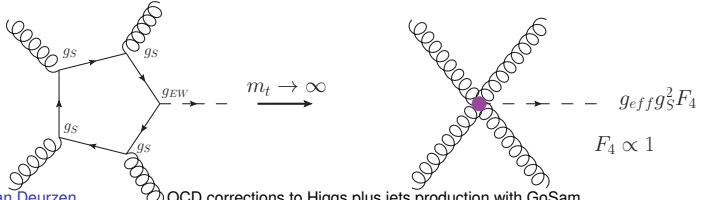
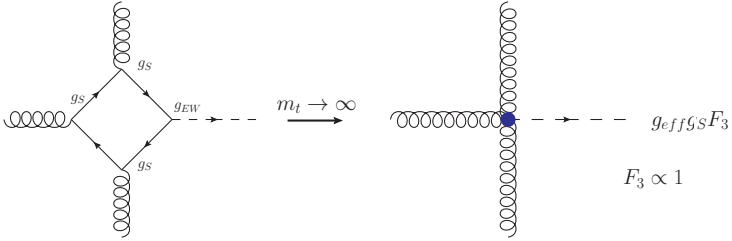
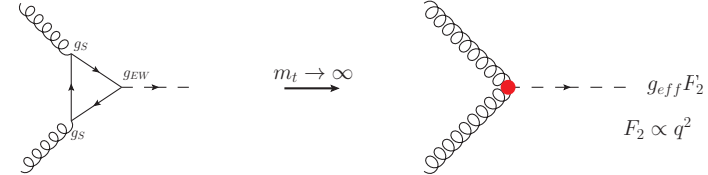


$$\Delta_i(\bar{q}) = \text{Res}_i \left\{ \frac{N(\bar{q})}{D_0 \cdots D_{n-1}} - \sum_{i < < m}^{n-1} \frac{\Delta_{ijklm}(\bar{q})}{D_i D_j D_k D_\ell D_m} - \sum_{i < < \ell}^{n-1} \frac{\Delta_{ijkl}(\bar{q})}{D_i D_j D_k D_\ell} + \sum_{i < < k}^{n-1} \frac{\Delta_{ijk}(\bar{q})}{D_i D_j D_k} - \sum_{i < j}^{n-1} \frac{\Delta_{ij}(\bar{q})}{D_i D_j} \right\}$$

5 coefficients

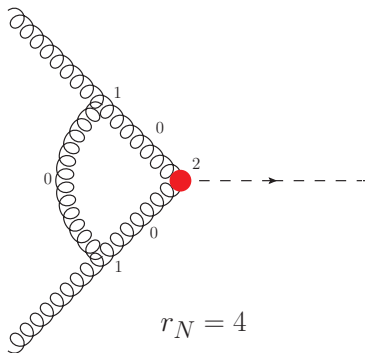
$$\text{Hexagon: } \binom{6}{5} \cdot 1 + \binom{6}{4} \cdot 5 + \binom{6}{3} \cdot 10 + \binom{6}{2} \cdot 10 + \binom{6}{1} \cdot 5 = 386 \text{ coefficients}$$

# Effective Vertices

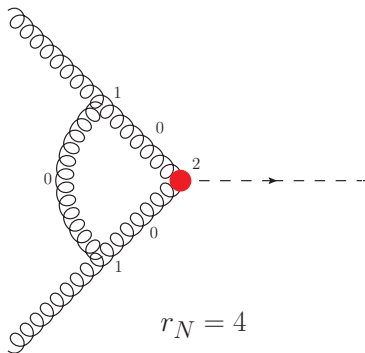




# Rankcounting, higher rank



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- ▶ One effective vertex:  $r_N \leq \#D + 1$

# Extended rank Integrand decomposition algorithm



$$\Delta_{ijklm}(\bar{q}) = \text{Res}_{ijklm} \left\{ \frac{N(\bar{q})}{D_0 \cdots D_{n-1}} \right\}$$

1 → 1 coefficient



$$\Delta_{ijkl}(\bar{q}) = \text{Res}_{ijkl} \left\{ \frac{N(\bar{q})}{D_0 \cdots D_{n-1}} - \sum_{i < < m}^{n-1} \frac{\Delta_{ijklm}(\bar{q})}{D_i D_j D_k D_l D_m} \right\}$$

5 → 6 coefficients



$$\Delta_{ijk}(\bar{q}) = \text{Res}_{ijk} \left\{ \frac{N(\bar{q})}{D_0 \cdots D_{n-1}} - \sum_{i < < m}^{n-1} \frac{\Delta_{ijklm}(\bar{q})}{D_i D_j D_k D_l D_m} - \sum_{i < < l}^{n-1} \frac{\Delta_{ijkl}(\bar{q})}{D_i D_j D_k D_l} \right\}$$

10 → 15 coefficients



$$\Delta_{ij}(\bar{q}) = \text{Res}_{ij} \left\{ \frac{N(\bar{q})}{D_0 \cdots D_{n-1}} - \sum_{i < < m}^{n-1} \frac{\Delta_{ijklm}(\bar{q})}{D_i D_j D_k D_l D_m} - \sum_{i < < l}^{n-1} \frac{\Delta_{ijkl}(\bar{q})}{D_i D_j D_k D_l} - \sum_{i < < k}^{n-1} \frac{\Delta_{ijk}(\bar{q})}{D_i D_j D_k} \right\}$$

10 → 20 coefficients



$$\Delta_i(\bar{q}) = \text{Res}_i \left\{ \frac{N(\bar{q})}{D_0 \cdots D_{n-1}} - \sum_{i < < m}^{n-1} \frac{\Delta_{ijklm}(\bar{q})}{D_i D_j D_k D_l D_m} - \sum_{i < < l}^{n-1} \frac{\Delta_{ijkl}(\bar{q})}{D_i D_j D_k D_l} + \sum_{i < < k}^{n-1} \frac{\Delta_{ijk}(\bar{q})}{D_i D_j D_k} - \sum_{i < j}^{n-1} \frac{\Delta_{ij}(\bar{q})}{D_i D_j} \right\}$$

5 → 15 coefficients

[Mastrolia, Mirabella, Peraro, 2012]

Hexagon:  $\binom{6}{5} \cdot 1 + \binom{6}{4} \cdot 6 + \binom{6}{3} \cdot 15 + \binom{6}{2} \cdot 20 + \binom{6}{1} \cdot 15 = (386 \rightarrow) 786$  coefficients

► Samurai → XSamurai

# Discrete Fourier Transformation (DFT)

- ▶  $\Delta(q, \mu^2)$  multivariate polynomial in  $q$  and  $\mu^2$

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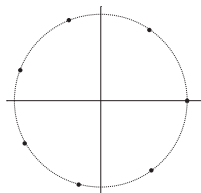
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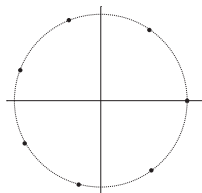
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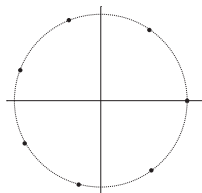
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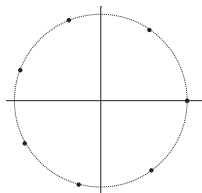
$$P(x) = c_0 + c_1x + c_2x^2 + \dots + c_nx^n$$

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$$c_l = \frac{\rho^{-l}}{n+1} \sum_{k=0}^n P_k \exp \left[ 2\pi i \frac{k}{n+1} l \right]$$



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  - ▶ Use DFT twice,  $\Delta(\mu^2, x_3, C/x_3)$  and  $\Delta(\mu^2 C/x_4, x_4)$  solutions  $\propto \frac{1}{1-C}$ , problem if  $C = 1$



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  - ▶ Use DFT: solutions  $\propto \frac{1}{C}$ , problem if  $C = 0$
  - ▶ Use DFT twice,  $\Delta(\mu^2, x_3, C/x_3)$  and  $\Delta(\mu^2 C/x_4, x_4)$   
solutions  $\propto \frac{1}{1-C}$ , problem if  $C = 1$
  - ▶ Branching:  
if  $(C=0)$ : Use  $\Delta(\mu^2, x_3, C/x_3)$  and  $\Delta(\mu^2 C/x_4, x_4)$   
else: Use  $\Delta(\mu^2, x_3, C/x_3)$

# Sampling problems

- ▶ At double cut:  $\Delta(\mu^2, x_1, x_3, x_4)$  with  $x_3 x_4 = F(x_1) = Ax_1^2 + Bx_1 + C$  lot of branchings:

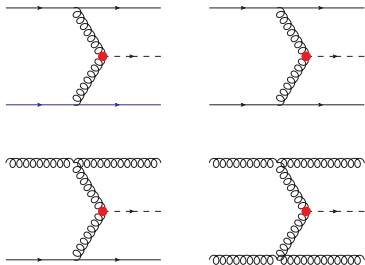
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- ▶ At single cut:  $\Delta(\mu^2, x_1, x_2, x_3, x_4)$  with  $x_3x_4 - x_1x_2 = G$  similar to the triple cut

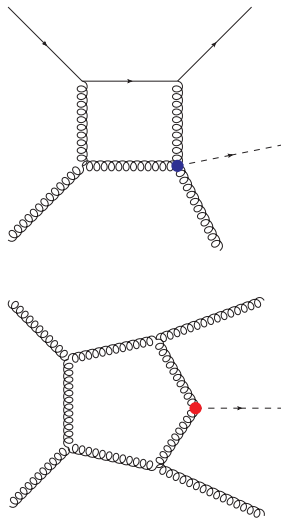
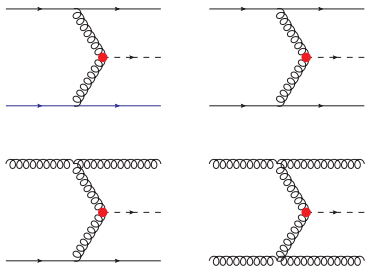
# $pp \rightarrow H+2j$ with GoSam+Sherpa



Number of diagrams:

$ud \rightarrow Hud$	1 tree	32 NLO
$uu \rightarrow Huu$	2 tree	64 NLO
$ug \rightarrow Hug$	8 tree	179 NLO
$gg \rightarrow Hgg$	26 tree	651 NLO
Total	37 tree	926 NLO

# pp → H+2j with GoSam+Sherpa

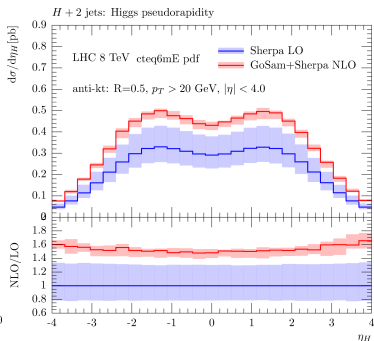
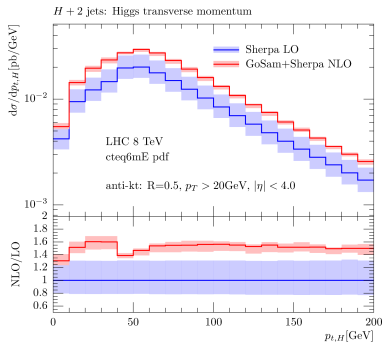


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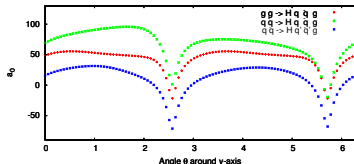
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# Results $pp \rightarrow H + 2j$ with GoSam+Sherpa

- ▶ Interface GoSam + Sherpa
- ▶ Pole cancellation
- ▶ Agreement with MCFM(v6.4) and R. K. Ellis, W. Giele, and G. Zanderighi



# Results $pp \rightarrow H+3j$ with GoSam+Sherpa



$ud \rightarrow Hudg$	12 tree	467 NLO
$uu \rightarrow Huug$	24 tree	868 NLO
$ug \rightarrow Hugg$	74 tree	2519 NLO
$gg \rightarrow Hggg$	230 tree	9325 NLO
<b>Total</b>	<b>340 tree</b>	<b>13179 NLO</b>

$$\frac{2\Re\{\mathcal{M}^{\text{tree-level}}*\mathcal{M}^{\text{one-loop}}\}}{(4\pi\alpha_s)|\mathcal{M}^{\text{tree-level}}|^2} \equiv \frac{a_{-2}}{\epsilon^2} + \frac{a_{-1}}{\epsilon} + a_0$$

$gg \rightarrow Hq\bar{q}g$		
$b_0$	$0.6309159660038877 \cdot 10^{-4}$	
$a_0$	48.68424097859422	
$a_{-1}$	-36.08277727147958	-36.08277728199094
$a_{-2}$	-11.66666666667209	
$q\bar{q} \rightarrow Hq\bar{q}g$		
$b_0$	$0.3609139855530763 \cdot 10^{-4}$	
$a_0$	69.32351140490162	
$a_{-1}$	-29.98862932963380	-29.98862932963629
$a_{-2}$	-8.333333333333339	
$q\bar{q} \rightarrow Hq'\bar{q}'g$		
$b_0$	$0.2687990772405433 \cdot 10^{-5}$	
$a_0$	15.79262767177915	
$a_{-1}$	-32.35320587070861	-32.35320587073038
$a_{-2}$	-8.333333333333398	



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# Backup slides

# table H+2j

$$\frac{2\Re\{\mathcal{M}^{\text{tree-level*}}\mathcal{M}^{\text{one-loop}}\}}{(4\pi\alpha_s)|\mathcal{M}^{\text{tree-level}}|^2} \equiv \frac{a_{-2}}{\epsilon^2} + \frac{a_{-1}}{\epsilon} + a_0$$

$gg \rightarrow Hgg$		
$c_0$	$0.1507218951429643 \cdot 10^{-3}$	
$a_0$	59.8657965614009	
$a_{-1}$	-26.4694115468536	-26.46941154671207
$a_{-2}$	-12.000000000000001	-12.000000000000000
$gg \rightarrow Hq\bar{q}$		
$c_0$	$0.5677813961826772 \cdot 10^{-6}$	
$a_0$	66.6635142370683	
$a_{-1}$	-16.5816633315627	-16.58166333155405
$a_{-2}$	-8.666666666666669	-8.666666666666668
$q\bar{q} \rightarrow Hq\bar{q}$		
$c_0$	$0.1099527895267439 \cdot 10^{-5}$	
$a_0$	88.2959834057198	
$a_{-1}$	-10.9673755313443	-10.96737553134440
$a_{-2}$	-5.333333333333332	-5.333333333333334
$q\bar{q} \rightarrow Hq'q'$		
$c_0$	$0.1011096724203529 \cdot 10^{-6}$	
$a_0$	33.9521626734153	
$a_{-1}$	-13.8649292834138	-13.86492928341388
$a_{-2}$	-5.333333333333334	-5.333333333333334

# Extended rank residues

$$\Delta_{ijklm}(q, \mu^2) = c_{5,0}^{(ijklm)} \mu^2,$$

$$\Delta_{ijkl}(q, \mu^2) = \Delta_{ijkl}^R(q, \mu^2) + c_{4,0}^{(ijkl)} + c_{4,2}^{(ijkl)} \mu^2 + c_{4,4}^{(ijkl)} \mu^4,$$

$$\Delta_{ijk}(q, \mu^2) = \Delta_{ijk}^R(q, \mu^2) + c_{3,0}^{(ijk)} + c_{3,7}^{(ijk)} \mu^2,$$

$$\Delta_{ij}(q, \mu^2) = \Delta_{ij}^R(q, \mu^2) + c_{2,0}^{(ij)} + c_{2,9}^{(ij)} \mu^2,$$

$$\begin{aligned} \Delta_i(q, \mu^2) &= c_{1,0}^{(i)} + c_{1,1}^{(i)}((q + p_i) \cdot e_1) + c_{1,2}^{(i)}((q + p_i) \cdot e_2) \\ &\quad + c_{1,3}^{(i)}((q + p_i) \cdot e_3) + c_{1,4}^{(i)}((q + p_i) \cdot e_4). \end{aligned}$$

$$\Delta_{ijkl}^R(q, \mu^2) = \left( c_{4,1}^{(ijkl)} + c_{4,3}^{(ijkl)} \mu^2 \right) (q + p_i) \cdot v_{\perp},$$

$$\begin{aligned} \Delta_{ijk}^R(q, \mu^2) &= \left( c_{3,1}^{(ijk)} + c_{3,8}^{(ijk)} \mu^2 \right) (q + p_i) \cdot e_3 + \left( c_{3,4}^{(ijk)} + c_{3,9}^{(ijk)} \mu^2 \right) (q + p_i) \cdot e_4 \\ &\quad + c_{3,2}^{(ijk)} ((q + p_i) \cdot e_3)^2 + c_{3,5}^{(ijk)} ((q + p_i) \cdot e_4)^2 \\ &\quad + c_{3,3}^{(ijk)} ((q + p_i) \cdot e_3)^3 + c_{3,6}^{(ijk)} ((q + p_i) \cdot e_4)^3, \end{aligned}$$

$$\begin{aligned} \Delta_{ij}^R(q, \mu^2) &= c_{2,1}^{(ij)} (q + p_i) \cdot e_2 + c_{2,2}^{(ij)} ((q + p_i) \cdot e_2)^2 \\ &\quad + c_{2,3}^{(ij)} (q + p_i) \cdot e_3 + c_{2,4}^{(ij)} ((q + p_i) \cdot e_3)^2 \\ &\quad + c_{2,5}^{(ij)} (q + p_i) \cdot e_4 + c_{2,6}^{(ij)} ((q + p_i) \cdot e_4)^2 \\ &\quad + c_{2,7}^{(ij)} ((q + p_i) \cdot e_2) ((q + p_i) \cdot e_3) + c_{2,8}^{(ij)} ((q + p_i) \cdot e_2) ((q + p_i) \cdot e_4). \end{aligned}$$

# Extended rank residues

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$$\Delta_{ij}(q, \mu^2) = \Delta_{ij}^R(q, \mu^2) + c_{2,0}^{(ij)} + c_{2,9}^{(ij)} \mu^2,$$

$$\Delta_i(q, \mu^2) = c_{1,0}^{(i)} + c_{1,1}^{(i)}((q + p_i) \cdot e_1) + c_{1,2}^{(i)}((q + p_i) \cdot e_2) + c_{1,3}^{(i)}((q + p_i) \cdot e_3) + c_{1,4}^{(i)}((q + p_i) \cdot e_4).$$

$$\Delta_{ijkl}^R(q, \mu^2) = (c_{4,1}^{(ijkl)} + c_{4,3}^{(ijkl)} \mu^2)(q + p_i) \cdot v_{\perp},$$

$$\Delta_{ijk}^R(q, \mu^2) = (c_{3,1}^{(ijk)} + c_{3,8}^{(ijk)} \mu^2)(q + p_i) \cdot e_3 + (c_{3,4}^{(ijk)} + c_{3,9}^{(ijk)} \mu^2)(q + p_i) \cdot e_4 + c_{3,2}^{(ijk)}((q + p_i) \cdot e_3)^2 + c_{3,5}^{(ijk)}((q + p_i) \cdot e_4)^2 + c_{3,3}^{(ijk)}((q + p_i) \cdot e_3)^3 + c_{3,6}^{(ijk)}((q + p_i) \cdot e_4)^3,$$

$$\Delta_{ij}^R(q, \mu^2) = c_{2,1}^{(ij)}(q + p_i) \cdot e_2 + c_{2,2}^{(ij)}((q + p_i) \cdot e_2)^2 + c_{2,3}^{(ij)}(q + p_i) \cdot e_3 + c_{2,4}^{(ij)}((q + p_i) \cdot e_3)^2 + c_{2,5}^{(ij)}(q + p_i) \cdot e_4 + c_{2,6}^{(ij)}((q + p_i) \cdot e_4)^2 + c_{2,7}^{(ij)}((q + p_i) \cdot e_2)((q + p_i) \cdot e_3) + c_{2,8}^{(ij)}((q + p_i) \cdot e_2)((q + p_i) \cdot e_4).$$

$$\Lambda_{ijklm}(q, \mu^2) = \Delta_{ijklm}(q, \mu^2),$$

$$\Lambda_{ijkl}(q, \mu^2) = \Delta_{ijkl}(q, \mu^2) + c_{4,5}^{(ijkl)} \mu^4 (q + p_i) \cdot v_{\perp},$$

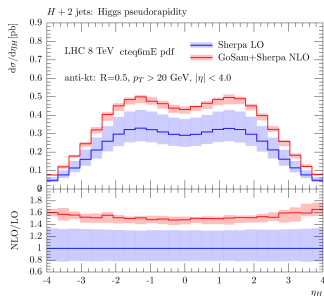
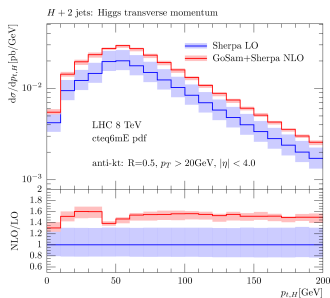
$$\Lambda_{ijk}(q, \mu^2) = \Delta_{ijk}(q, \mu^2) + c_{3,14}^{(ijk)} \mu^4 + c_{3,10}^{(ijk)} \mu^2 ((q + p_i) \cdot e_3)^2 + c_{3,11}^{(ijk)} \mu^2 ((q + p_i) \cdot e_4)^2 + c_{3,12}^{(ijk)} ((q + p_i) \cdot e_3)^4 + c_{3,13}^{(ijk)} ((q + p_i) \cdot e_4)^4,$$

$$\Lambda_{ij}(q, \mu^2) = \Delta_{ij}(q, \mu^2) + \mu^2 (c_{2,10}^{(ij)}(q + p_i) \cdot e_2 + c_{2,11}^{(ij)}(q + p_i) \cdot e_3 + c_{2,12}^{(ij)}(q + p_i) \cdot e_4) + c_{2,13}^{(ij)}((q + p_i) \cdot e_2)^3 + c_{2,14}^{(ij)}((q + p_i) \cdot e_3)^3 + c_{2,15}^{(ij)}((q + p_i) \cdot e_4)^3 + c_{2,16}^{(ij)}((q + p_i) \cdot e_2)^2((q + p_i) \cdot e_3) + c_{2,17}^{(ij)}((q + p_i) \cdot e_2)^2((q + p_i) \cdot e_4) + c_{2,18}^{(ij)}((q + p_i) \cdot e_2)((q + p_i) \cdot e_3)^2 + c_{2,19}^{(ij)}((q + p_i) \cdot e_2)((q + p_i) \cdot e_4)^2,$$

$$\Lambda_i(q, \mu^2) = \Delta_i(q, \mu^2) + c_{1,5}^{(i)}((q + p_i) \cdot e_1)^2 + c_{1,6}^{(i)}((q + p_i) \cdot e_2)^2 + c_{1,7}^{(i)}((q + p_i) \cdot e_3)^2 + c_{1,8}^{(i)}((q + p_i) \cdot e_4)^2 + c_{1,10}^{(i)}((q + p_i) \cdot e_1)((q + p_i) \cdot e_3) + c_{1,11}^{(i)}((q + p_i) \cdot e_1)((q + p_i) \cdot e_4) + c_{1,12}^{(i)}((q + p_i) \cdot e_2)((q + p_i) \cdot e_3) + c_{1,13}^{(i)}((q + p_i) \cdot e_2)((q + p_i) \cdot e_4) + c_{1,14}^{(i)} \mu^2 + c_{1,15}^{(i)}((q + p_i) \cdot e_3)((q + p_i) \cdot e_4),$$



# Results $pp \rightarrow H + 2j$ with GoSam+Sherpa



LHC 8 TeV  
PDF: cteq6mE  
anti-kt:  
 $R = 0.5$   
 $p_T > 20 \text{ GeV}$   
 $|\eta| < 4.0$   
 $M_H = 125 \text{ GeV}$   
 $\mu_R = \mu_F = M_H$