Hydrodynamics from gravity

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Recipe

How to get hydrodynamics from gravity:

- Preparation time: 25 min.
- Difficulty: medium.

Ingredients:

- Usual stuff: maximal spin 2, supersymmetry...
- AdS/CFT
- Einstein gravity theory conveniently modified.

AdS/CFT in a nutshell

Gravity + Susy = Supergravity 11d

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Take IIB.
Here have D3-branes.

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find geometric structure of SU(N) group

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Bottomline

We combined gravity with susy and ended up with a supersymmetric system in 4d with SU(N) symmetry.

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AdS/CFT conjecture

$$\begin{array}{c} \mathcal{N} = 4\ SU(N)\ \text{SYM} \\ \text{Conformal field theory} \end{array} = \begin{array}{c} \text{Type IIB superstring theory} \\ \text{on } AdS_5 \times S^5 \end{array}$$

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Matching of parameters

Which point of view is mathematically tractable depends on whether the gauge theory is strongly or weakly coupled:

GRAVITY THEORY	GAUGE THEORY
☺ easy	☹ hard
☹ hard	☺ easy

AdS/CFT in a nutshell

Take limit to make life simpler:

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Easy and cool version of AdS/CFT

Einstein gravity		$\mathcal{N} = 4$ $SU(N)$ SYM
(consistent truncation)	$=$	$N \to \infty$, strongly coupled
AdS 5d		∂AdS 4d

AdS/CFT in a nutshell

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Einstein gravity (consistent truncation) AdS 5d	$=$	$\mathcal{N}=4$ $SU(N)$ SYM $N \to \infty$, strongly coupled ∂AdS 4d

AdS/CFT dictionary

GRAVITY	GAUGE
A_μ	J^μ
$g_{\mu\nu}$	$T^{\mu\nu}$

AdS/CFT in a nutshell

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What about temperature?Easy generalization!

Thermal AdS/CFT

$$AdS \text{ Blackbrane 5d} \quad \leftrightarrow \quad \mathcal{N} = 4 \; SU(N) \text{ SYM } T > 0$$

$$r_h \propto T$$

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Message to take

To describe a thermal strongly coupled planar **field theory** in 4d just need

$$\mathcal{L}_{bulk} = \sqrt{-g}\left[\frac{1}{2\kappa}(R - \Lambda)\right] \qquad \text{in 5d}$$

Field content: $g_{\mu\nu}$ AdS Schwarzschild metric.

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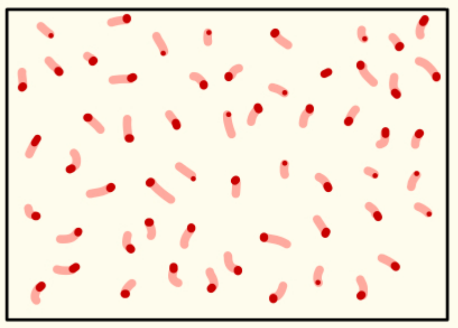

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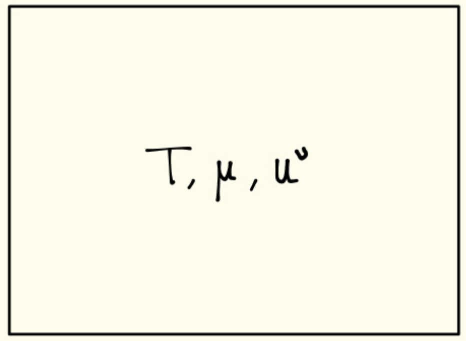

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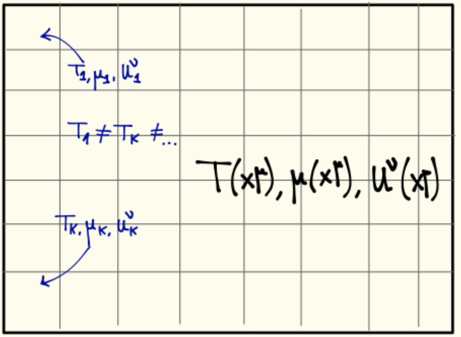

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When is hydrodynamics valid?

Length scale of interacting system given by L_{MFP}

Hydrodynamics applies when $\Delta T \gg L_{MFP}$

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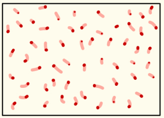
Hydrodynamic variables



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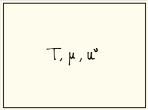

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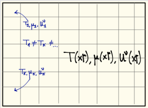

Noether: cont. symmetries
$\Rightarrow$ conserved currents

Effective variables:
T,u^ν, μ,. . .

Slight deviations from equilibrium:
$T(x)$,$u^\nu(x)$, $\mu(x)$,. . .

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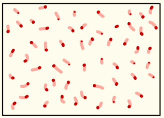

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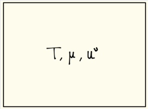

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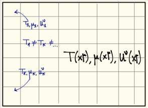

Slight deviations from equilibrium:
$T(x)$,$u^\nu(x)$, $\mu(x)$,...

Conservation equations

$$\partial_\mu T^{\mu\nu}(x) = 0 \qquad \partial_\mu J^\mu(x) = 0$$

Hydrodynamics

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Constitutive relations

Equilibrium:

$$T^{\mu\nu} = \rho u^\mu u^\nu + P \underbrace{\left(\eta^{\mu\nu} + u^\mu u^\nu\right)}_{\Delta^{\mu\nu}}$$

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Hydrodynamics:

$$T^{\mu\nu}(x) = \rho(x) u^\mu(x) u^\nu(x) + P(x)\Delta^{\mu\nu}(x) + \Pi^{\mu\nu}(x)$$

$\Pi^{\mu\nu}(x)$ represents contributions of gradients of $T(x), u^\nu(x)$

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- Use symmetry of the system.
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$$\Delta^{\mu\alpha}\Delta^{\nu\beta}\left(\partial_\alpha u_\beta + \partial_\beta u_\alpha - \frac{2}{d}\eta_{\alpha\beta}\partial_\sigma u^\sigma\right) \qquad \Delta^{\mu\nu}\partial_\sigma u^\sigma$$

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1st order dissipative hydrodynamics

$$T^{\mu\nu} = T_0^{\mu\nu}(x) + \eta\Delta^{\mu\alpha}\Delta^{\nu\beta}\left(\partial_\alpha u_\beta + \partial_\beta u_\alpha - \frac{2}{d}\eta_{\alpha\beta}\partial_\sigma u^\sigma\right) - \zeta\Delta^{\mu\nu}\partial_\sigma u^\sigma$$

Fluid/gravity

We've seen

AdS/CFT duality

AdS Schwarzschild metric to describe thermal dual field theory.

$$g_{\mu\nu} \leftrightarrow T^{\mu\nu}$$

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Drop global equilibrium and assume local equilibrium.

$$T^{\mu\nu} \rightarrow T^{\mu\nu}(x)$$

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$$g_{\mu\nu}(r, T, u^{\nu}) \longrightarrow g_{\mu\nu}(r, T(x), u^{\nu}(x))??$$

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☹ But then Einstein's equations are no longer fulfilled.

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Systematic procedure to solve Einstein's equations order by order in derivatives.

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Promote our parameters

$$T = T^{eq}(x_0) + x^\sigma \partial_\sigma T(x_0) + \mathcal{O}(\partial^2) \qquad u^\nu = u^{\nu eq}(x_0) + x^\sigma \partial_\sigma u^\nu(x_0) + \mathcal{O}(\partial^2)$$

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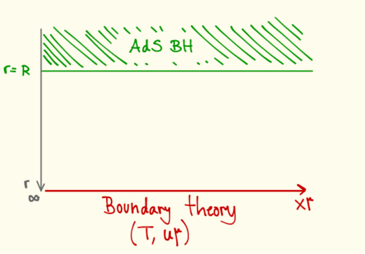

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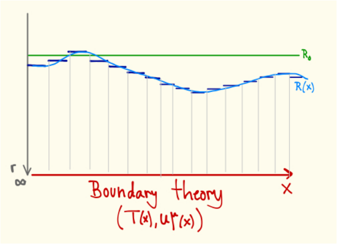

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It is all about tuning the correcting functions $G_{\mu\nu}(r)$, $\partial_\sigma T(x_0)$ and $\partial_\sigma u^\nu(x_0)$ so that Einstein's equations are solved again at each order.

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Einstein's equations are second order equations:

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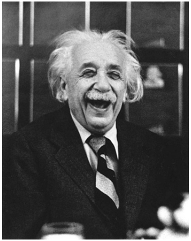
$$PDE \longrightarrow ODE$$



Fluid/gravity

Fluid/gravity machinery

Algorithmically solve for the correcting functions by requiring that Einstein's equations be satisfied order by order.

1. Take known solution to the equations. (Global equilibrium).
2. Introduce dependence on x^μ.
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One finds that the correcting functions must obey the eoms of fluid dynamics

$$\partial_\mu T^{\mu\nu} = 0$$

Fluid/gravity

Following the previous instructions for our system one finds:

$$\Pi^{\mu\nu} = \eta \Delta^{\mu\alpha} \Delta^{\nu\beta} \left(\partial_\alpha u_\beta + \partial_\beta u_\alpha - \frac{2}{d} \eta_{\alpha\beta} \partial_\sigma u^\sigma \right) - \zeta \Delta^{\mu\nu} \partial_\sigma u^\sigma$$

whereby

$$\eta = \frac{1}{2\kappa} (\pi T)^3 \qquad \zeta = 0$$

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Bottomline

Fluid dynamics can be derived from gravity using the AdS/CFT duality.

Uses of fluid/gravity

Fluid gravity has proven useful in some interesting ways:

1. Einstein's equations, when considered in a convenient limit (small amplitudes, non-relativistic velocities) can be shown to reduce to the Navier-Stokes equations:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \Lambda g_{\mu\nu} = 0 \Longrightarrow \dot{\vec{v}} + \vec{v}\cdot\vec{\nabla}\vec{v} = -\vec{\nabla}P + \nu\nabla^2\vec{v}$$

2. Deriving the fluid dynamics from Einstein-Maxwell-Chern-Simons gravity, anomalies were found which account for additional transport coefficients that had not been foreseen before. So additional transport coefficients were predicted for field theories with U(1) triangular anomaly. This could be experimentally tested.

Outlook

Our work

Try to apply this to p-wave superfluids.

Potential research lines:

- New approach to membrane paradigm
- New approach to turbulence
- Look for universality relations among transport coefficients.
- Link to thermalization.

Thank you for your attention

Have a lovely weekend!