Vacuum Persistence in Massive Gravity

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Motivation to massive gravity

Why massive gravity?

A possible solution to the cosmological constant problem.

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- What is the cosmological constant problem?

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- What is the cosmological constant problem?
- What is the vacuum persistence?

The cosmological constant problem

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- The Einstein equations from general relativity

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu} \quad (1)$$
$$\Lambda \leq 10^{-48} GeV^4 \quad (2)$$

by the observation.

The cosmological constant problem

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by the observation.

- The vacuum energy density from quantum field theory

$$\Lambda \sim E_{\text{UV}}^4 \geq 10^2 GeV^4 \text{ at EW scale} \quad (3)$$

Discrepancy is at least of order of 56!!

Massive gravity in action

- The gravitational potential

$$V(r) \sim \frac{1}{r} e^{-mr} \tag{4}$$

Explains the smallness of the observed value of the vacuum energy density

Massive gravity in a bigger set-up

- View massive gravity as an effective description of some underlying theory.
- The underlying theory does not necessarily violate 4-dimensional general covariance (e.g. DGP-model).

Vacuum persistence amplitude (VPA)

- Vacuum to vacuum transition, given there exist external sources exchanging a particle

$$\langle 0|0\rangle_J \sim \exp\left\{ \int J(x)G(x,y)J(y) \right\} \tag{5}$$

J is an external source. For any sensible quantum theory the mod of this expression is constrained to lie between zero and one.

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- Massless gravity from the Einstein-Hilbert action

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- Linearize the action in flat background

$$\begin{aligned} g_{\mu\nu}(x) &= \eta_{\mu\nu} + h_{\mu\nu}(x), \text{ for } |h_{\mu\nu}| << 1 \quad (7) \\ S &= -\frac{1}{2} \int d^4x \; h^{\mu\nu} \epsilon_{\mu\nu,\rho\sigma} h^{\rho\sigma} \quad (8) \end{aligned}$$

after canonical normalization.

1. Massive gravity in flat background - Mass term

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- Include a generic mass term into the action

$$S = -\frac{1}{2}\int d^4x \left[h^{\mu\nu}\epsilon_{\mu\nu,\rho\sigma}h^{\rho\sigma} + m^2(\alpha h_{\mu\nu}h^{\mu\nu} + \beta h^2)\right] \tag{9}$$

where α and β are arbitrary real coefficients, and have to be tuned for stability of the theory.

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- Two ways of tuning coefficients:

1. decompose $h_{\mu\nu}$ into tensor, vector and scalar components
2. Stueckelberg formalism

- Fierz-Pauli form of the mass term

$$\mathcal{L}_m = -\frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2) \tag{10}$$

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- VPA of the massive spin-2 in flat background

$$\langle 0|0\rangle_T = \exp\left\{\frac{i}{2}\int d^4x \int d^4x'\, T^{\mu\nu}(x)\, G_{\mu\nu,\rho\sigma}(x,x')\, T^{\rho\sigma}(x')\right\} \quad (11)$$

1. Massive gravity in flat background - VPA (2)

- VPA in momentum space

$$\langle 0|0\rangle_T = \exp\left\{ \frac{i}{2} \int \frac{d^4k}{(2\pi)^4} \frac{|T^{\mu\nu}|^2 - \frac{1}{3}|T|^2}{k^2 + m^2 - i\epsilon} \right\} \tag{12}$$

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- Q: How do we evaluate this integral?
- A: The easiest one is by specifying the external source

$$T^{\mu\nu}(x) = M\delta^\mu_0\delta^\nu_0\delta^{(3)}(\mathbf{x}) \quad (13)$$

1. Massive gravity in flat background - VPA (3)

- Taking the Wick rotation, the amplitude simplified

$$\langle 0|0\rangle_T = \exp\Big\{-\frac{1}{3}\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2+m^2}|T(k)|^2\Big\} \tag{14}$$

k^2 has an Euclidean signature, so the integrand is strictly positive.

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- Massive gravity in flat spacetime is a healthy theory at the tree level!!

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- A vacuum w.r.t. an "a-particle" state is not necessarily a vacuum w.r.t. a "b-particle" state.

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- A non-minimally coupled free massive scalar field theory in flat FRW universe

$$S = -\frac{1}{2}\int d\eta d^3\mathbf{x}\left[\partial^\mu\phi\partial_\mu\phi + \left(m^2a^2 - (1-6\xi)\frac{a''}{a}\right)\phi^2\right] \qquad (15)$$

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- Effective mass term changes w.r.t. a (conformal) time.

2. Adiabatic vacuum (1)

- EOM for a harmonic oscillator

$$\phi_k'' + \left(k^2 + m^2 a^2 - (1 - 6\xi)\frac{a''}{a}\right)\phi_k = 0 \tag{16}$$

Difficult to choose a basis. One can assume the slow expansion of the universe, and the solution is of a WKB type.

- We solve up to a second adiabatic order and its corresponding vacuum is a "second adiabatic vacuum".

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- In de Sitter spacetime, this relation translates into the comparison between size of the normal neighborhood and the curvature scale

$$y^2 << \Lambda \tag{18}$$

3. Massive gravity in de Sitter background - Linearization

- Linearized EH (after canonical normalization)

$$\begin{aligned}\mathcal{L}_{EH} &= \nabla^\sigma h^{\alpha\rho}\nabla_\alpha h_{\rho\sigma} - \frac{1}{2}\nabla^\sigma h^{\alpha\rho}\nabla_\sigma h_{\alpha\rho} + \frac{1}{2}(\nabla^\rho h - 2\nabla^\alpha h^\rho_\alpha)\nabla_\rho h \\ &- \frac{1}{2}\left(h^{\alpha\beta}h_{\alpha\beta} - \frac{1}{2}h^2\right)(\mathcal{R} - 2\Lambda) + \left(2h^{\alpha\sigma}h^\beta_\sigma - hh^{\alpha\beta}\right)\mathcal{R}_{\alpha\beta} \quad (19)\end{aligned}$$

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- Non-minimal coupling

$$\mathcal{L}_\xi = -\frac{1}{2}\xi\mathcal{R}(h_{\mu\nu}h^{\mu\nu} - h^2) \quad (20)$$

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$$\mathcal{L}_\xi = -\frac{1}{2}\xi\mathcal{R}(h_{\mu\nu}h^{\mu\nu} - h^2) \quad (20)$$

- Mass

$$\mathcal{L}_m = -\frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2) \quad (21)$$

3. Massive gravity in de Sitter background - VPA

- Source term

$$\mathcal{L}_T = h_{\mu\nu} T^{\mu\nu} \tag{22}$$

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$$\begin{aligned} S &= \int d^4x' \sqrt{-g'} \Big[\mathcal{L}_{EH} + \mathcal{L}_m + \mathcal{L}_\xi + \mathcal{L}_T \Big] \\ &= \int d^4x' \sqrt{-g'} \left[-\frac{1}{2} h^{\mu\nu} \mathcal{K}_{\mu\nu,\rho\sigma} h^{\rho\sigma} + h_{\mu\nu} T^{\mu\nu} \right] \end{aligned} \tag{23}$$

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- VPA

$$\langle 0|0 \rangle_T = \exp\left\{ \frac{i}{2} \int d^4x' \sqrt{-g'} \Big[T^{\mu\nu}(x') (\mathcal{K}^{-1} T)_{\mu\nu}(x') \Big] \right\} \tag{24}$$

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- EOM after using all constraints

$$\left(-\Box^c + \mu^2 + \frac{2\Lambda}{3}\right) h_{\alpha\rho} = T_{\alpha\rho} + \frac{1}{3\mu^2 - 2\Lambda}\left[\nabla_\rho \nabla_\alpha T - \mu^2 g_{\alpha\rho} T + \frac{\Lambda}{3} g_{\alpha\rho} T\right] \tag{27}$$

expand both sides up to second adiabatic order!

3. Massive gravity in de Sitter background - Vacuum persistence

- VPA in momentum space

$$\begin{aligned}\langle 0|0\rangle_T &= \exp\Big\{-\frac{1}{2}\int_{|k|>>\Lambda^{1/2}}\frac{d^4k}{(2\pi)^4}\Big(T^{\alpha\rho}[G_0+G_1+G_2]^{\tau\sigma}_{\alpha\rho}(\Pi T_0)_{\tau\sigma} \\ &\quad +T^{\alpha\rho}[G_0+G_1]^{\tau\sigma}_{\alpha\rho}(\Pi T_1)_{\tau\sigma}+T^{\alpha\rho}(G_0)^{\tau\sigma}_{\alpha\rho}(\Pi T_2)_{\tau\sigma}\Big)\Big\} \\ &= \exp\Big\{-\frac{1}{2}\int_{|k|>>\Lambda^{1/2}}\frac{d^4k}{(2\pi)^4}F(\Delta_F,\Lambda)|T(k)|^2\Big\} \qquad (28)\end{aligned}$$

$T^{\mu\nu}$ is a source localized in space.

3. Massive gravity in de Sitter background - Positivity

- Positivity of $F(\Delta_F, \Lambda)$: Given the coefficient of the leading order in polynomial positive, one has to check the function f is bounded below by a positive value

$$
\begin{aligned}
f(x) \;=\;& (x+M)^4 + \frac{\Lambda}{18M-11\Lambda}\left(22M - \frac{62\Lambda}{3}\right)(x+M)^3 \\
& - \frac{16\Lambda^2}{9(18M-11\Lambda)} x^2 (x+M) \\
& + \frac{\Lambda}{18M-11\Lambda}\left(\frac{104\Lambda}{9} - 16M\right) x (x+M)^2 \\
& + \ldots
\end{aligned}
\tag{29}
$$

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- Unitarity preserved as long as either one of two conditions are fulfilled

$$m^2 > \left(\frac{2}{3} - 4\xi\right)\Lambda \text{ or } m^2 < \left(\frac{11}{18} - 4\xi\right)\Lambda \qquad (30)$$

for $\xi \sim 1$. Second condition ruled out by the UV cutoff

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- A sensible constraint to preserve the unitarity

$$m^2 > \left(\frac{2}{3} - 4\xi\right)\Lambda \sim -\Lambda \tag{31}$$

Cosmological constant is positive in dS, so this bound is trivial.

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- Ours - non-minimal coupling $\xi \neq 0$
  Modifies the constraint on the deformation parameter, and implies only trivial mass bound for the graviton.

Discussion - Anti de Sitter spacetime

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- AdS - another maximally symmetric spacetime

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- Mass bound

$$m^2 > \left(4\xi - \frac{11}{18}\right)\Lambda \tag{33}$$

but trivial when the coupling is properly adjusted

Discussion - FRW

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- Slowly expanding FRW ($\dot{H} << H^2$)

$$\mathcal{R}(t)y^2 << 1 \tag{34}$$

and obtain the mass bound

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and obtain the mass bound

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- Slightly running mass bound, but a constant in short time span. Trivial mass bound again.

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