QCD corrections to Higgs plus jets production with GoSam

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NLO QCD corrections to the production of Higgs plus two jets at the LHC,
e-Print: arXiv:1301.0493, accepted by Physics Letters B

[HvD, Greiner, Luisoni, Mastrolia, Mirabella, Ossola, Peraro, von Soden-Fraunhofen, Tramontano, 2013]

Outline

Motivation

Scattering amplitudes at one-loop

Determining the parametric form of the numerator

Extended rank numerator

Higgs plus two jets

Higgs plus three jets

Summary

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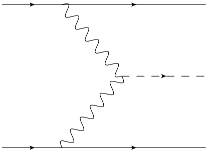
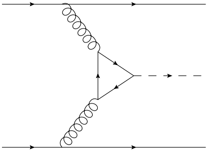


Vector Boson Fusion



Gluon Fusion via top loop

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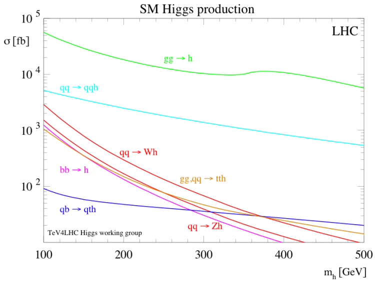

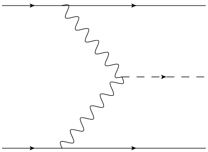

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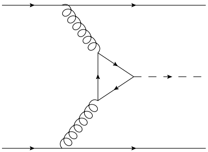

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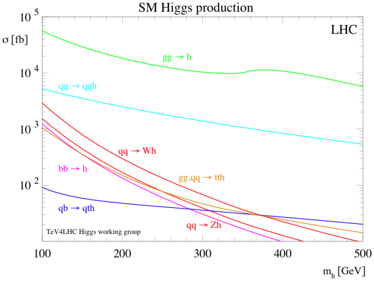

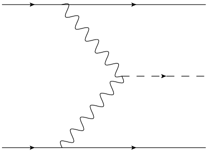

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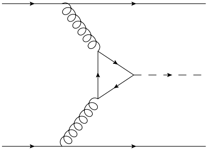

Gluon Fusion via top loop

- Leading order too strong dependence on renormalization and factorization scale

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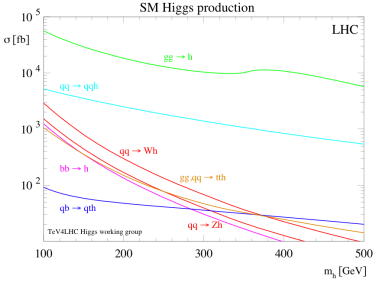

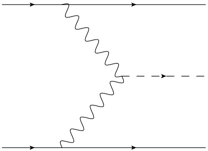

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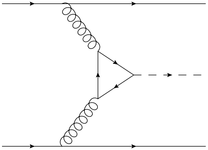

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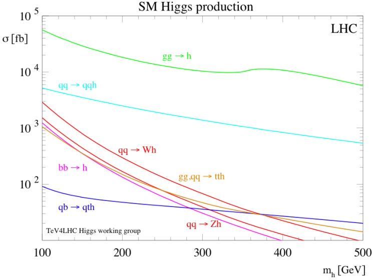

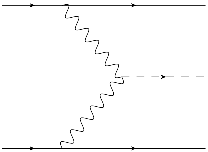

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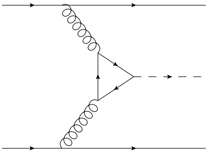

Gluon Fusion via top loop

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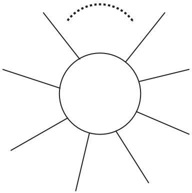
$$\sigma^{NLO} = \int_{m+1} \left[d^{(4)}\sigma^R - d^{(4)}\sigma^A \right] + \int_m \left[d^{(4)}\sigma^B + \int_{loop} d^{(d)}\sigma^V + \int_1 d^{(d)}\sigma^A \right]$$

Scattering amplitudes at one-loop



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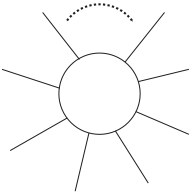

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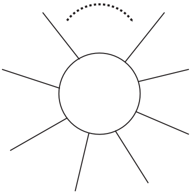

$$\mathcal{M}_n \equiv \int \mathcal{A}_n(\bar{q})\, d\bar{q} \equiv \int d^{-2\epsilon}\mu \int d^4q \frac{N(q,\mu^2)}{\bar{D}_0 \ldots \bar{D}_{n-1}}$$

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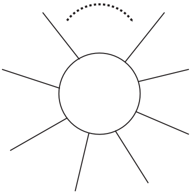

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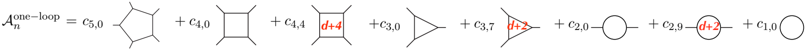

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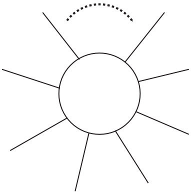

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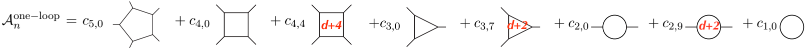

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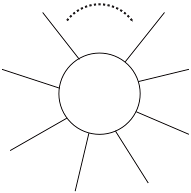

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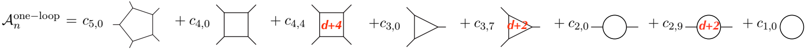

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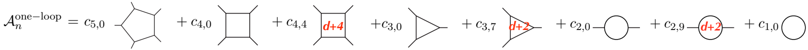
- computation of $\mathcal{M}_n \rightarrow$ computation of coefficients

Integral to Integrand



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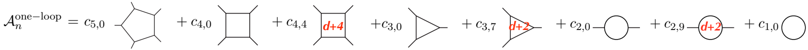

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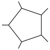
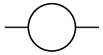
- integral $\to$ integrand:

Integral to Integrand

$\mathcal{A}_n^{\text{one-loop}} = c_{5,0}$  $+ c_{4,0}$  $+ c_{4,4}$ 

$+ c_{3,0}$  $+ c_{3,7}$ 

$+ c_{2,0}$  $+ c_{2,9}$ 

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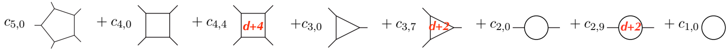
$$A_n(q) = \frac{c_{5,0} + f_{01234}(q,\mu^2)}{D_0 D_1 D_2 D_3 D_4} + \frac{c_{4,0} + c_{4,4}\mu^4 + f_{0123}(q,\mu^2)}{D_0 D_1 D_2 D_3} + \frac{c_{3,0} + c_{3,7}\mu^2 + f_{012}(q,\mu^2)}{D_0 D_1 D_2} + \frac{c_{2,0} + c_{2,9}\mu^2 + f_{01}(q,\mu^2)}{D_0 D_1} + \frac{c_{1,0} + f_0(q,\mu^2)}{D_0}$$

Integral to Integrand

$\mathcal{A}_n^{\text{one-loop}} =$



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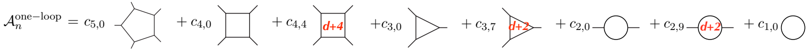

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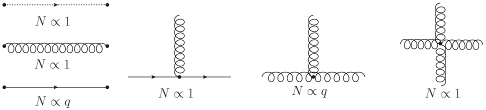
$$\int d^{-2\epsilon}\mu^2 \int d^4 q \quad \frac{f_{ij\ldots}(q,\mu^2)}{D_i D_j \ldots} = 0$$

$$\mathcal{A}_n = \sum_{ijkl} \frac{\Delta_{ijkl}(q,\mu^2)}{D_i D_j D_k D_l} + \sum_{ijk} \frac{\Delta_{ijk}(q,\mu^2)}{D_i D_j D_k} + \sum_{ij} \frac{\Delta_{ij}(q,\mu^2)}{D_i D_j} + \sum_{i} \frac{\Delta_{i}(q,\mu^2)}{D_i}$$

Rankcounting, normal rank



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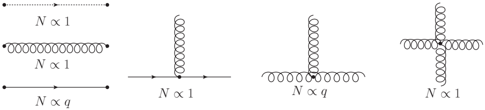

- Only q propagators and 3-gluon-vertices contribute one power of q to numerator

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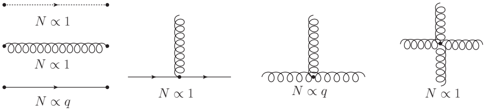

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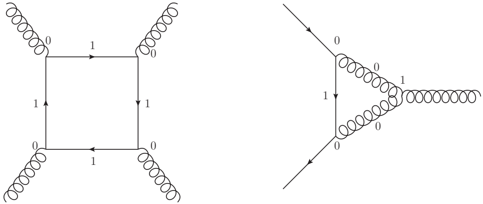
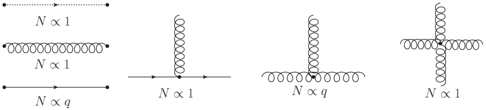


$r_N = 4$ $r_N = 2$

Rankcounting, normal rank



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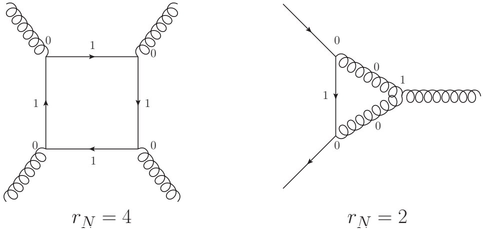
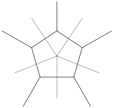


- $r_N \leq \#D$

Integrand decomposition algorithm



$$\Delta_{ijk\ell m}(\bar{q}) = \mathrm{Res}_{ijk\ell m}\left\{\frac{N(\bar{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}}\right\}$$

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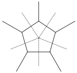

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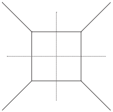

$$\Delta_{ijk\ell}(\bar{q}) = \mathrm{Res}_{ijk\ell}\left\{\frac{N(\bar{q})}{\bar{D}_0\cdots\bar{D}_{n-1}} - \sum_{i<<m}^{n-1}\frac{\Delta_{ijk\ell m}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell\bar{D}_m}\right\}$$

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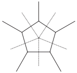

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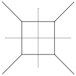
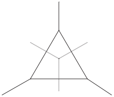
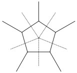


$$\Delta_{ijk\ell}(\bar{q}) = \mathrm{Res}_{ijk\ell}\left\{\frac{N(\bar{q})}{\bar{D}_0\cdots\bar{D}_{n-1}} - \sum_{i<<m}^{n-1}\frac{\Delta_{ijk\ell m}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell\bar{D}_m}\right\}$$

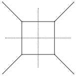


$$\Delta_{ijk}(\bar{q}) = \mathrm{Res}_{ijk}\left\{\frac{N(\bar{q})}{\bar{D}_0\cdots\bar{D}_{n-1}} - \sum_{i<<m}^{n-1}\frac{\Delta_{ijk\ell m}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell\bar{D}_m} - \sum_{i<<\ell}^{n-1}\frac{\Delta_{ijk\ell}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell}\right\}$$

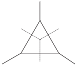
Integrand decomposition algorithm



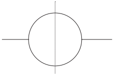
$$\Delta_{ijk\ell m}(\bar{q}) = \mathrm{Res}_{ijk\ell m}\left\{\frac{N(\bar{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}}\right\}$$



$$\Delta_{ijk\ell}(\bar{q}) = \mathrm{Res}_{ijk\ell}\left\{\frac{N(\bar{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}} - \sum_{i<<m}^{n-1} \frac{\Delta_{ijk\ell m}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell\bar{D}_m}\right\}$$

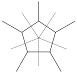


$$\Delta_{ijk}(\bar{q}) = \mathrm{Res}_{ijk}\left\{\frac{N(\bar{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}} - \sum_{i<<m}^{n-1} \frac{\Delta_{ijk\ell m}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell\bar{D}_m} - \sum_{i<<\ell}^{n-1} \frac{\Delta_{ijk\ell}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell}\right\}$$

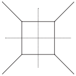


$$\Delta_{ij}(\bar{q}) = \mathrm{Res}_{ij}\left\{\frac{N(\bar{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}} - \sum_{i<<m}^{n-1} \frac{\Delta_{ijk\ell m}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell\bar{D}_m} - \sum_{i<<\ell}^{n-1} \frac{\Delta_{ijk\ell}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell} - \sum_{i<<k}^{n-1} \frac{\Delta_{ijk}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k}\right\}$$

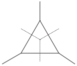
Integrand decomposition algorithm



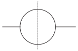
$$\Delta_{ijk\ell m}(\bar{q}) = \mathrm{Res}_{ijk\ell m}\left\{ \frac{N(\bar{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}} \right\}$$



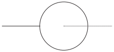
$$\Delta_{ijk\ell}(\bar{q}) = \mathrm{Res}_{ijk\ell}\left\{ \frac{N(\bar{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}} - \sum_{i<<m}^{n-1} \frac{\Delta_{ijk\ell m}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell\bar{D}_m} \right\}$$



$$\Delta_{ijk}(\bar{q}) = \mathrm{Res}_{ijk}\left\{ \frac{N(\bar{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}} - \sum_{i<<m}^{n-1} \frac{\Delta_{ijk\ell m}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell\bar{D}_m} - \sum_{i<<\ell}^{n-1} \frac{\Delta_{ijk\ell}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell} \right\}$$



$$\Delta_{ij}(\bar{q}) = \mathrm{Res}_{ij}\left\{ \frac{N(\bar{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}} - \sum_{i<<m}^{n-1} \frac{\Delta_{ijk\ell m}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell\bar{D}_m} - \sum_{i<<\ell}^{n-1} \frac{\Delta_{ijk\ell}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell} - \sum_{i<<k}^{n-1} \frac{\Delta_{ijk}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k} \right\}$$



$$\Delta_{i}(\bar{q}) = \mathrm{Res}_{i}\left\{ \frac{N(\bar{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}} - \sum_{i<<m}^{n-1} \frac{\Delta_{ijk\ell m}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell\bar{D}_m} - \sum_{i<<\ell}^{n-1} \frac{\Delta_{ijk\ell}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell} + \right.$$
$$\left. - \sum_{i<<k}^{n-1} \frac{\Delta_{ijk}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k} - \sum_{i<j}^{n-1} \frac{\Delta_{ij}(\bar{q})}{\bar{D}_i\bar{D}_j} \right\}$$

Residues

$$\Delta_{ijk\ell m}(q,\mu^2) = c_{5,0}^{(ijk\ell m)}\ \mu^2\ ,$$

$$\Delta_{ijk\ell}(q,\mu^2) = \Delta^R_{ijk\ell}(q,\mu^2) + c_{4,0}^{(ijk\ell)} + c_{4,2}^{(ijk\ell)}\mu^2 + c_{4,4}^{(ijk\ell)}\mu^4\ ,$$

$$\Delta_{ijk}(q,\mu^2) = \Delta^R_{ijk}(q,\mu^2) + c_{3,0}^{(ijk)} + c_{3,7}^{(ijk)}\mu^2\ ,$$

$$\Delta_{ij}(q,\mu^2) = \Delta^R_{ij}(q,\mu^2) + c_{2,0}^{(ij)} + c_{2,9}^{(ij)}\mu^2\ ,$$

$$\begin{aligned}\Delta_{i}(q,\mu^2) &= c_{1,0}^{(i)} + c_{1,1}^{(i)}((q+p_i)\cdot e_1) + c_{1,2}^{(i)}((q+p_i)\cdot e_2)\\ &+ c_{1,3}^{(i)}((q+p_i)\cdot e_3) + c_{1,4}^{(i)}((q+p_i)\cdot e_4)\ .\end{aligned}$$

$$\Delta^R_{ijk\ell}(q,\mu^2) = \left(c_{4,1}^{(ijk\ell)} + c_{4,3}^{(ijk\ell)}\ \mu^2\right)(q+p_i)\cdot v_\perp\ ,$$

$$\begin{aligned}\Delta^R_{ijk}(q,\mu^2) &= \left(c_{3,1}^{(ijk)} + c_{3,8}^{(ijk)}\mu^2\right)(q+p_i)\cdot e_3 + \left(c_{3,4}^{(ijk)} + c_{3,9}^{(ijk)}\mu^2\right)(q+p_i)\cdot e_4\\ &+ c_{3,2}^{(ijk)}((q+p_i)\cdot e_3)^2 + c_{3,5}^{(ijk)}((q+p_i)\cdot e_4)^2\\ &+ c_{3,3}^{(ijk)}((q+p_i)\cdot e_3)^3 + c_{3,6}^{(ijk)}((q+p_i)\cdot e_4)^3\ ,\end{aligned}$$

$$\begin{aligned}\Delta^R_{ij}(q,\mu^2) &= c_{2,1}^{(ij)}(q+p_i)\cdot e_2 + c_{2,2}^{(ij)}((q+p_i)\cdot e_2)^2\\ &+ c_{2,3}^{(ij)}(q+p_i)\cdot e_3 + c_{2,4}^{(ij)}((q+p_i)\cdot e_3)^2\\ &+ c_{2,5}^{(ij)}(q+p_i)\cdot e_4 + c_{2,6}^{(ij)}((q+p_i)\cdot e_4)^2\\ &+ c_{2,7}^{(ij)}((q+p_i)\cdot e_2)((q+p_i)\cdot e_3) + c_{2,8}^{(ij)}((q+p_i)\cdot e_2)((q+p_i)\cdot e_4)\,.\end{aligned}$$

Residues

$$\begin{aligned}
\Delta_{ijk\ell m}(q,\mu^2) &= c_{5,0}^{(ijk\ell m)}\ \mu^2\ , \\
\Delta_{ijk\ell}(q,\mu^2) &= \Delta^R_{ijk\ell}(q,\mu^2) + c_{4,0}^{(ijk\ell)} + c_{4,2}^{(ijk\ell)}\mu^2 + c_{4,4}^{(ijk\ell)}\mu^4\ , \\
\Delta_{ijk}(q,\mu^2) &= \Delta^R_{ijk}(q,\mu^2) + c_{3,0}^{(ijk)} + c_{3,7}^{(ijk)}\mu^2\ , \\
\Delta_{ij}(q,\mu^2) &= \Delta^R_{ij}(q,\mu^2) + c_{2,0}^{(ij)} + c_{2,9}^{(ij)}\mu^2\ , \\
\Delta_{i}(q,\mu^2) &= c_{1,0}^{(i)} + c_{1,1}^{(i)}((q+p_i)\cdot e_1) + c_{1,2}^{(i)}((q+p_i)\cdot e_2) \\
&\quad + c_{1,3}^{(i)}((q+p_i)\cdot e_3) + c_{1,4}^{(i)}((q+p_i)\cdot e_4)\ .
\end{aligned}$$

$$\begin{aligned}
\Delta^R_{ijk\ell}(q,\mu^2) &= \left(c_{4,1}^{(ijk\ell)} + c_{4,3}^{(ijk\ell)}\ \mu^2\right)(q+p_i)\cdot v_\perp\ , \\
\Delta^R_{ijk}(q,\mu^2) &= \left(c_{3,1}^{(ijk)} + c_{3,8}^{(ijk)}\mu^2\right)(q+p_i)\cdot e_3 + \left(c_{3,4}^{(ijk)} + c_{3,9}^{(ijk)}\mu^2\right)(q+p_i)\cdot e_4 \\
&\quad + c_{3,2}^{(ijk)}((q+p_i)\cdot e_3)^2 + c_{3,5}^{(ijk)}((q+p_i)\cdot e_4)^2 \\
&\quad + c_{3,3}^{(ijk)}((q+p_i)\cdot e_3)^3 + c_{3,6}^{(ijk)}((q+p_i)\cdot e_4)^3\ , \\
\Delta^R_{ij}(q,\mu^2) &= c_{2,1}^{(ij)}(q+p_i)\cdot e_2 + c_{2,2}^{(ij)}((q+p_i)\cdot e_2)^2 \\
&\quad + c_{2,3}^{(ij)}(q+p_i)\cdot e_3 + c_{2,4}^{(ij)}((q+p_i)\cdot e_3)^2 \\
&\quad + c_{2,5}^{(ij)}(q+p_i)\cdot e_4 + c_{2,6}^{(ij)}((q+p_i)\cdot e_4)^2 \\
&\quad + c_{2,7}^{(ij)}((q+p_i)\cdot e_2)((q+p_i)\cdot e_3) + c_{2,8}^{(ij)}((q+p_i)\cdot e_2)((q+p_i)\cdot e_4)\,.
\end{aligned}$$

- coefficients:
  - 5ple cut: 1 coefficient
  - 4ple cut: 5 coefficients
  - 3ple cut: 10 coefficients
  - 2ple cut: 10 coefficients
  - 1ple cut: 5 coefficients

Residues

$$
\begin{aligned}
\Delta_{ijk\ell m}(q,\mu^2) &= c_{5,0}^{(ijk\ell m)}\ \mu^2\ , \\
\Delta_{ijk\ell}(q,\mu^2) &= \Delta^R_{ijk\ell}(q,\mu^2) + c_{4,0}^{(ijk\ell)} + c_{4,2}^{(ijk\ell)}\mu^2 + c_{4,4}^{(ijk\ell)}\mu^4\ , \\
\Delta_{ijk}(q,\mu^2) &= \Delta^R_{ijk}(q,\mu^2) + c_{3,0}^{(ijk)} + c_{3,7}^{(ijk)}\mu^2\ , \\
\Delta_{ij}(q,\mu^2) &= \Delta^R_{ij}(q,\mu^2) + c_{2,0}^{(ij)} + c_{2,9}^{(ij)}\mu^2\ , \\
\Delta_{i}(q,\mu^2) &= c_{1,0}^{(i)} + c_{1,1}^{(i)}((q+p_i)\cdot e_1) + c_{1,2}^{(i)}((q+p_i)\cdot e_2) \\
&\quad + c_{1,3}^{(i)}((q+p_i)\cdot e_3) + c_{1,4}^{(i)}((q+p_i)\cdot e_4)\ .
\end{aligned}
$$

$$
\begin{aligned}
\Delta^R_{ijk\ell}(q,\mu^2) &= \left(c_{4,1}^{(ijk\ell)} + c_{4,3}^{(ijk\ell)}\ \mu^2\right)(q+p_i)\cdot v_\perp\ , \\
\Delta^R_{ijk}(q,\mu^2) &= \left(c_{3,1}^{(ijk)} + c_{3,8}^{(ijk)}\mu^2\right)(q+p_i)\cdot e_3 + \left(c_{3,4}^{(ijk)} + c_{3,9}^{(ijk)}\mu^2\right)(q+p_i)\cdot e_4 \\
&\quad + c_{3,2}^{(ijk)}((q+p_i)\cdot e_3)^2 + c_{3,5}^{(ijk)}((q+p_i)\cdot e_4)^2 \\
&\quad + c_{3,3}^{(ijk)}((q+p_i)\cdot e_3)^3 + c_{3,6}^{(ijk)}((q+p_i)\cdot e_4)^3\ , \\
\Delta^R_{ij}(q,\mu^2) &= c_{2,1}^{(ij)}(q+p_i)\cdot e_2 + c_{2,2}^{(ij)}((q+p_i)\cdot e_2)^2 \\
&\quad + c_{2,3}^{(ij)}(q+p_i)\cdot e_3 + c_{2,4}^{(ij)}((q+p_i)\cdot e_3)^2 \\
&\quad + c_{2,5}^{(ij)}(q+p_i)\cdot e_4 + c_{2,6}^{(ij)}((q+p_i)\cdot e_4)^2 \\
&\quad + c_{2,7}^{(ij)}((q+p_i)\cdot e_2)((q+p_i)\cdot e_3) + c_{2,8}^{(ij)}((q+p_i)\cdot e_2)((q+p_i)\cdot e_4)\,.
\end{aligned}
$$

- coefficients:
  - 5ple cut: 1 coefficient
  - 4ple cut: 5 coefficients
  - 3ple cut: 10 coefficients
  - 2ple cut: 10 coefficients
  - 1ple cut: 5 coefficients
- form residues process independent
- values of coefficients process dependent

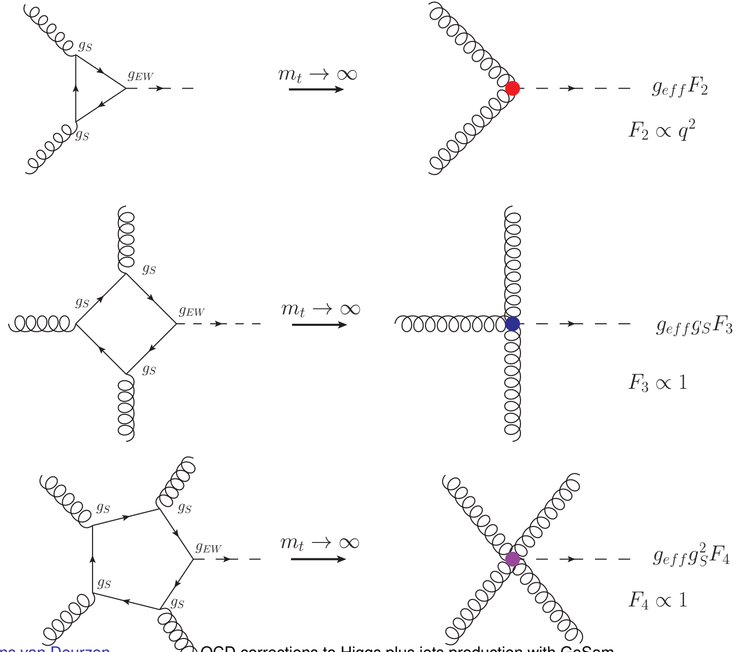
Residues

$$
\begin{aligned}
\Delta_{ijk\ell m}(q,\mu^2) &= c_{5,0}^{(ijk\ell m)}\ \mu^2 \ , \\
\Delta_{ijk\ell}(q,\mu^2) &= \Delta^R_{ijk\ell}(q,\mu^2) + c_{4,0}^{(ijk\ell)} + c_{4,2}^{(ijk\ell)}\mu^2 + c_{4,4}^{(ijk\ell)}\mu^4 \ , \\
\Delta_{ijk}(q,\mu^2) &= \Delta^R_{ijk}(q,\mu^2) + c_{3,0}^{(ijk)} + c_{3,7}^{(ijk)}\mu^2 \ , \\
\Delta_{ij}(q,\mu^2) &= \Delta^R_{ij}(q,\mu^2) + c_{2,0}^{(ij)} + c_{2,9}^{(ij)}\mu^2 \ , \\
\Delta_{i}(q,\mu^2) &= c_{1,0}^{(i)} + c_{1,1}^{(i)}((q+p_i)\cdot e_1) + c_{1,2}^{(i)}((q+p_i)\cdot e_2) \\
&\quad + c_{1,3}^{(i)}((q+p_i)\cdot e_3) + c_{1,4}^{(i)}((q+p_i)\cdot e_4) \ .
\end{aligned}
$$

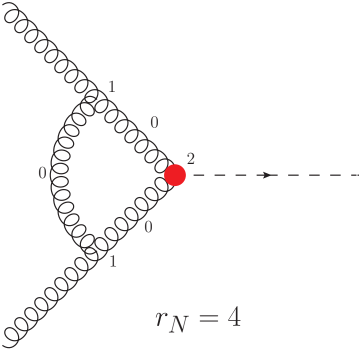
$$
\begin{aligned}
\Delta^R_{ijk\ell}(q,\mu^2) &= \left(c_{4,1}^{(ijk\ell)} + c_{4,3}^{(ijk\ell)}\ \mu^2\right)(q+p_i)\cdot v_\perp \ , \\
\Delta^R_{ijk}(q,\mu^2) &= \left(c_{3,1}^{(ijk)} + c_{3,8}^{(ijk)}\mu^2\right)(q+p_i)\cdot e_3 + \left(c_{3,4}^{(ijk)} + c_{3,9}^{(ijk)}\mu^2\right)(q+p_i)\cdot e_4 \\
&\quad + c_{3,2}^{(ijk)}((q+p_i)\cdot e_3)^2 + c_{3,5}^{(ijk)}((q+p_i)\cdot e_4)^2 \\
&\quad + c_{3,3}^{(ijk)}((q+p_i)\cdot e_3)^3 + c_{3,6}^{(ijk)}((q+p_i)\cdot e_4)^3 \ , \\
\Delta^R_{ij}(q,\mu^2) &= c_{2,1}^{(ij)}(q+p_i)\cdot e_2 + c_{2,2}^{(ij)}((q+p_i)\cdot e_2)^2 \\
&\quad + c_{2,3}^{(ij)}(q+p_i)\cdot e_3 + c_{2,4}^{(ij)}((q+p_i)\cdot e_3)^2 \\
&\quad + c_{2,5}^{(ij)}(q+p_i)\cdot e_4 + c_{2,6}^{(ij)}((q+p_i)\cdot e_4)^2 \\
&\quad + c_{2,7}^{(ij)}((q+p_i)\cdot e_2)((q+p_i)\cdot e_3) + c_{2,8}^{(ij)}((q+p_i)\cdot e_2)((q+p_i)\cdot e_4) \, .
\end{aligned}
$$

- coefficients:
    - 5ple cut: 1 coefficient
    - 4ple cut: 5 coefficients
    - 3ple cut: 10 coefficients
    - 2ple cut: 10 coefficients
    - 1ple cut: 5 coefficients
- form residues process independent
- values of coefficients process dependent
- Implemented in Samurai
  [Ossola, Reiter, Tramontano, Mastrolia, 2010]

Effective Vertices

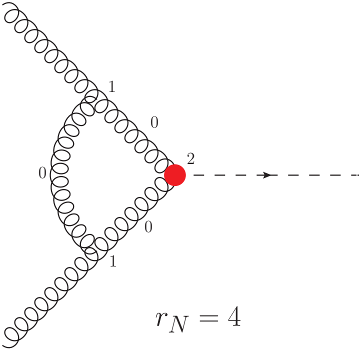


Rankcounting, higher rank



$r_N = 4$

Rankcounting, higher rank



- One effective vertex: $r_N \leq \#D + 1$

Extended rank residues

$$
\begin{aligned}
\Delta_{ijk\ell m}(q,\mu^2) &= c_{5,0}^{(ijk\ell m)}\ \mu^2 \ , \\
\Delta_{ijk\ell}(q,\mu^2) &= \Delta^R_{ijk\ell}(q,\mu^2) + c_{4,0}^{(ijk\ell)} + c_{4,2}^{(ijk\ell)}\mu^2 + c_{4,4}^{(ijk\ell)}\mu^4 \ , \\
\Delta_{ijk}(q,\mu^2) &= \Delta^R_{ijk}(q,\mu^2) + c_{3,0}^{(ijk)} + c_{3,7}^{(ijk)}\mu^2 \ , \\
\Delta_{ij}(q,\mu^2) &= \Delta^R_{ij}(q,\mu^2) + c_{2,0}^{(ij)} + c_{2,9}^{(ij)}\mu^2 \ , \\
\Delta_{i}(q,\mu^2) &= c_{1,0}^{(i)} + c_{1,1}^{(i)}((q+p_i)\cdot e_1) + c_{1,2}^{(i)}((q+p_i)\cdot e_2) \\
&\quad + c_{1,3}^{(i)}((q+p_i)\cdot e_3) + c_{1,4}^{(i)}((q+p_i)\cdot e_4) \ .
\end{aligned}
$$

$$
\begin{aligned}
\Delta^R_{ijk\ell}(q,\mu^2) &= \left(c_{4,1}^{(ijk\ell)} + c_{4,3}^{(ijk\ell)}\ \mu^2\right)(q+p_i)\cdot v_\perp \ , \\
\Delta^R_{ijk}(q,\mu^2) &= \left(c_{3,1}^{(ijk)} + c_{3,8}^{(ijk)}\mu^2\right)(q+p_i)\cdot e_3 + \left(c_{3,4}^{(ijk)} + c_{3,9}^{(ijk)}\mu^2\right)(q+p_i)\cdot e_4 \\
&\quad + c_{3,2}^{(ijk)}((q+p_i)\cdot e_3)^2 + c_{3,5}^{(ijk)}((q+p_i)\cdot e_4)^2 \\
&\quad + c_{3,3}^{(ijk)}((q+p_i)\cdot e_3)^3 + c_{3,6}^{(ijk)}((q+p_i)\cdot e_4)^3 \ , \\
\Delta^R_{ij}(q,\mu^2) &= c_{2,1}^{(ij)}(q+p_i)\cdot e_2 + c_{2,2}^{(ij)}((q+p_i)\cdot e_2)^2 \\
&\quad + c_{2,3}^{(ij)}(q+p_i)\cdot e_3 + c_{2,4}^{(ij)}((q+p_i)\cdot e_3)^2 \\
&\quad + c_{2,5}^{(ij)}(q+p_i)\cdot e_4 + c_{2,6}^{(ij)}((q+p_i)\cdot e_4)^2 \\
&\quad + c_{2,7}^{(ij)}((q+p_i)\cdot e_2)((q+p_i)\cdot e_3) + c_{2,8}^{(ij)}((q+p_i)\cdot e_2)((q+p_i)\cdot e_4) .
\end{aligned}
$$

Extended rank residues

$$
\begin{aligned}
\Delta_{ijk\ell m}(q,\mu^2) &= c_{5,0}^{(ijk\ell m)}\ \mu^2 \ , \\
\Delta_{ijk\ell}(q,\mu^2) &= \Delta^R_{ijk\ell}(q,\mu^2) + c_{4,0}^{(ijk\ell)} + c_{4,2}^{(ijk\ell)}\mu^2 + c_{4,4}^{(ijk\ell)}\mu^4 \ , \\
\Delta_{ijk}(q,\mu^2) &= \Delta^R_{ijk}(q,\mu^2) + c_{3,0}^{(ijk)} + c_{3,7}^{(ijk)}\mu^2 \ , \\
\Delta_{ij}(q,\mu^2) &= \Delta^R_{ij}(q,\mu^2) + c_{2,0}^{(ij)} + c_{2,9}^{(ij)}\mu^2 \ , \\
\Delta_{i}(q,\mu^2) &= c_{1,0}^{(i)} + c_{1,1}^{(i)}((q+p_i)\cdot e_1) + c_{1,2}^{(i)}((q+p_i)\cdot e_2) \\
&\quad + c_{1,3}^{(i)}((q+p_i)\cdot e_3) + c_{1,4}^{(i)}((q+p_i)\cdot e_4) \ .
\end{aligned}
$$

$$
\begin{aligned}
\Delta^R_{ijk\ell}(q,\mu^2) &= \left(c_{4,1}^{(ijk\ell)} + c_{4,3}^{(ijk\ell)}\ \mu^2\right)(q+p_i)\cdot v_\perp \ , \\
\Delta^R_{ijk}(q,\mu^2) &= \left(c_{3,1}^{(ijk)} + c_{3,8}^{(ijk)}\mu^2\right)(q+p_i)\cdot e_3 + \left(c_{3,4}^{(ijk)} + c_{3,9}^{(ijk)}\mu^2\right)(q+p_i)\cdot e_4 \\
&\quad + c_{3,2}^{(ijk)}((q+p_i)\cdot e_3)^2 + c_{3,5}^{(ijk)}((q+p_i)\cdot e_4)^2 \\
&\quad + c_{3,3}^{(ijk)}((q+p_i)\cdot e_3)^3 + c_{3,6}^{(ijk)}((q+p_i)\cdot e_4)^3 \ , \\
\Delta^R_{ij}(q,\mu^2) &= c_{2,1}^{(ij)}(q+p_i)\cdot e_2 + c_{2,2}^{(ij)}((q+p_i)\cdot e_2)^2 \\
&\quad + c_{2,3}^{(ij)}(q+p_i)\cdot e_3 + c_{2,4}^{(ij)}((q+p_i)\cdot e_3)^2 \\
&\quad + c_{2,5}^{(ij)}(q+p_i)\cdot e_4 + c_{2,6}^{(ij)}((q+p_i)\cdot e_4)^2 \\
&\quad + c_{2,7}^{(ij)}((q+p_i)\cdot e_2)((q+p_i)\cdot e_3) + c_{2,8}^{(ij)}((q+p_i)\cdot e_2)((q+p_i)\cdot e_4) .
\end{aligned}
$$

$$
\begin{aligned}
\Lambda_{ijk\ell m}(q,\mu^2) &= \Delta_{ijk\ell m}(q,\mu^2) \ , \\
\Lambda_{ijk\ell}(q,\mu^2) &= \Delta_{ijk\ell}(q,\mu^2) + c_{4,5}^{(ijk\ell)}\ \mu^4\ (q+p_i)\cdot v_\perp \ , \\
\Lambda_{ijk}(q,\mu^2) &= \Delta_{ijk}(q,\mu^2) + c_{3,14}^{(ijk)}\ \mu^4 + c_{3,10}^{(ijk)}\ \mu^2\ ((q+p_i)\cdot e_3)^2 \\
&\quad + c_{3,11}^{(ijk)}\ \mu^2\ ((q+p_i)\cdot e_4)^2 + c_{3,12}^{(ijk)}((q+p_i)\cdot e_3)^4 \\
&\quad + c_{3,13}^{(ijk)}((q+p_i)\cdot e_4)^4 \ , \\
\Lambda_{ij}(q,\mu^2) &= \Delta_{ij}(q,\mu^2) + \mu^2\Big(c_{2,10}^{(ij)}\ (q+p_i)\cdot e_2 + c_{2,11}^{(ij)}\ (q+p_i)\cdot e_3 \\
&\quad + c_{2,12}^{(ij)}(q+p_i)\cdot e_4\Big) + c_{2,13}^{(ij)}\ ((q+p_i)\cdot e_2)^3 + c_{2,14}^{(ij)}((q+p_i)\cdot e_3)^3 \\
&\quad + c_{2,15}^{(ij)}((q+p_i)\cdot e_4)^3 + c_{2,16}^{(ij)}((q+p_i)\cdot e_2)^2((q+p_i)\cdot e_3) \\
&\quad + c_{2,17}^{(ij)}((q+p_i)\cdot e_2)^2((q+p_i)\cdot e_4) \\
&\quad + c_{2,18}^{(ij)}((q+p_i)\cdot e_2)((q+p_i)\cdot e_3)^2 \\
&\quad + c_{2,19}^{(ij)}((q+p_i)\cdot e_2)((q+p_i)\cdot e_4)^2 \ , \\
\Lambda_{i}(q,\mu^2) &= \Delta_i(q,\mu^2) + c_{1,5}^{(i)}((q+p_i)\cdot e_1)^2 + c_{1,6}^{(i)}((q+p_i)\cdot e_2)^2 \\
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&\quad + c_{1,14}^{(i)}\ \mu^2 + c_{1,15}^{(i)}((q+p_i)\cdot e_3)((q+p_i)\cdot e_4) \ ,
\end{aligned}
$$

[Mastrolia, Mirabella, Peraro, 2012]

Extended rank residues

$$
\begin{aligned}
\Delta_{ijk\ell m}(q,\mu^2) &= c_{5,0}^{(ijk\ell m)}\ \mu^2\ , \\
\Delta_{ijk\ell}(q,\mu^2) &= \Delta_{ijk\ell}^R(q,\mu^2) + c_{4,0}^{(ijk\ell)} + c_{4,2}^{(ijk\ell)}\mu^2 + c_{4,4}^{(ijk\ell)}\mu^4\ , \\
\Delta_{ijk}(q,\mu^2) &= \Delta_{ijk}^R(q,\mu^2) + c_{3,0}^{(ijk)} + c_{3,7}^{(ijk)}\mu^2\ , \\
\Delta_{ij}(q,\mu^2) &= \Delta_{ij}^R(q,\mu^2) + c_{2,0}^{(ij)} + c_{2,9}^{(ij)}\mu^2\ , \\
\Delta_{i}(q,\mu^2) &= c_{1,0}^{(i)} + c_{1,1}^{(i)}((q+p_i)\cdot e_1) + c_{1,2}^{(i)}((q+p_i)\cdot e_2) \\
&\quad + c_{1,3}^{(i)}((q+p_i)\cdot e_3) + c_{1,4}^{(i)}((q+p_i)\cdot e_4)\ .
\end{aligned}
$$

$$
\begin{aligned}
\Delta_{ijk\ell}^R(q,\mu^2) &= \left(c_{4,1}^{(ijk\ell)} + c_{4,3}^{(ijk\ell)}\ \mu^2\right)(q+p_i)\cdot v_\perp\ , \\
\Delta_{ijk}^R(q,\mu^2) &= \left(c_{3,1}^{(ijk)} + c_{3,8}^{(ijk)}\mu^2\right)(q+p_i)\cdot e_3 + \left(c_{3,4}^{(ijk)} + c_{3,9}^{(ijk)}\mu^2\right)(q+p_i)\cdot e_4 \\
&\quad + c_{3,2}^{(ijk)}((q+p_i)\cdot e_3)^2 + c_{3,5}^{(ijk)}((q+p_i)\cdot e_4)^2 \\
&\quad + c_{3,3}^{(ijk)}((q+p_i)\cdot e_3)^3 + c_{3,6}^{(ijk)}((q+p_i)\cdot e_4)^3\ , \\
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\end{aligned}
$$

- coefficients:
  - 5ple cut: 1→1 coefficient
  - 4ple cut: 5→6 coefficients
  - 3ple cut: 10→15 coefficients
  - 2ple cut: 10→20 coefficients
  - 1ple cut: 5→15 coefficients
- Samurai → XSamurai

Discrete Fourier Transformation (DFT)

- $\Delta(q, \mu^2)$ multivariate polynomial in q and μ^2

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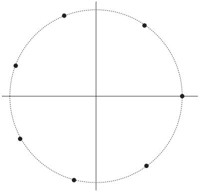
$$x_k = \rho \exp\left[-2\pi i \frac{k}{n+1}\right]$$



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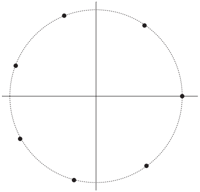

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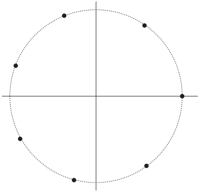

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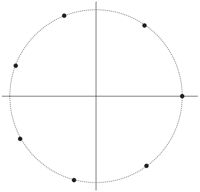
$$c_l = \frac{\rho^{-l}}{n+1} \sum_{k=0}^{n} P_k \exp\left[2\pi i \frac{k}{n+1} l\right]$$



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  - Use DFT twice, $\Delta(x3, C/x3)$ and $\Delta(C/x_4, x_4)$
    solutions $\propto \frac{1}{1-C}$, problem if $C = 1$
  - Branching:
    if(C=0): Use $\Delta(x_3, C/x_3)$ and $\Delta(C/x_4, x_4)$
    else: Use $\Delta(x_3, C/x_3)$

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  - F=0 has one non-zero solution
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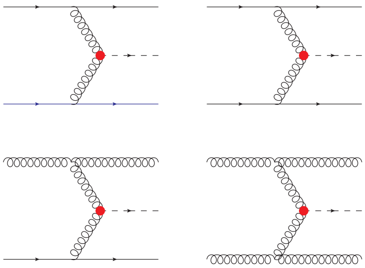
  - ... think carefully
- At single cut: $\Delta(x_1, x_2, x_3, x_4)$ with $x_3 x_4 - x_1 x_2 = G$
  similar to the triple cut

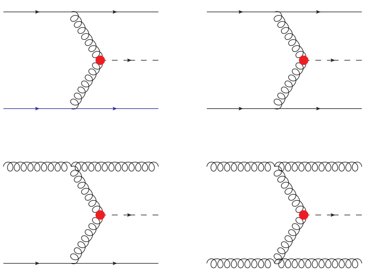
Higgs plus two jets

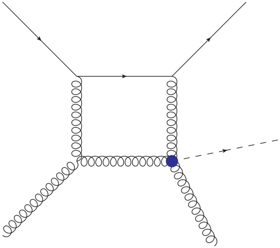


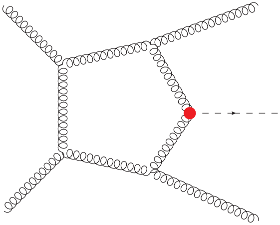
Number of diagrams:

$ud \to Hud$	1 tree	32 NLO
$uu \to Huu$	2 tree	64 NLO
$ug \to Hug$	8 tree	179 NLO
$gg \to Hgg$	26 tree	651 NLO
Total	37 tree	926 NLO

Higgs plus two jets







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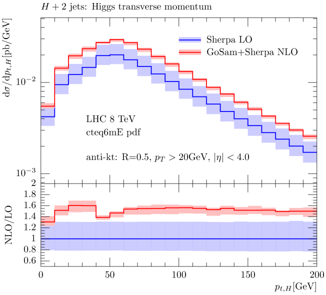
Results Higgs plus two jets

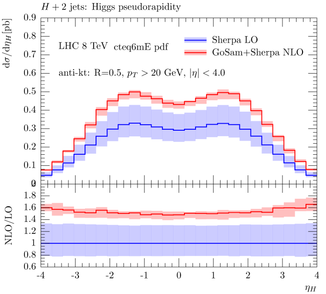
- Interface GoSam + Sherpa (talk Gionata Luisoni)
- Pole cancellation
- Agreement with MCFM(v6.4) and R. K. Ellis, W. Giele, and G. Zanderighi

$$\frac{2\mathfrak{Re}\left\{\mathcal{M}^{\text{tree-level}*}\mathcal{M}^{\text{one-loop}}\right\}}{(4\pi\alpha_s)\left|\mathcal{M}^{\text{tree-level}}\right|^2} \equiv \frac{a_{-2}}{\epsilon^2} + \frac{a_{-1}}{\epsilon} + a_0$$

	$gg \to Hgg$	
c_0	$0.1507218951429643 \cdot 10^{-3}$	
a_0	59.8657965614009	
a_{-1}	−26.4694115468536	−26.46941154671207
a_{-2}	−12.00000000000001	−12.00000000000000
	$gg \to Hq\bar{q}$	
c_0	$0.5677813961826772 \cdot 10^{-6}$	
a_0	66.6635142370683	
a_{-1}	−16.5816633315627	−16.58166333155405
a_{-2}	−8.66666666666669	−8.666666666666668
	$q\bar{q} \to Hq\bar{q}$	
c_0	$0.1099527895267439 \cdot 10^{-5}$	
a_0	88.2959834057198	
a_{-1}	−10.9673755313443	−10.96737553134440
a_{-2}	−5.33333333333332	−5.333333333333334
	$q\bar{q} \to Hq'\bar{q}'$	
c_0	$0.1011096724203529 \cdot 10^{-6}$	
a_0	33.9521626734153	
a_{-1}	−13.8649292834138	−13.86492928341388
a_{-2}	−5.33333333333334	−5.333333333333334

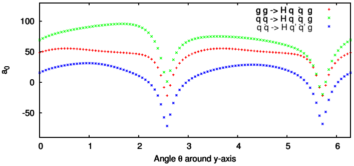
Results Higgs plus two jets





LHC 8 TeV
PDF: cteq6mE
anti-kt:
$R = 0.5$
$p_T > 20$ GeV
$|\eta| < 4.0$
$M_H = 125$ GeV
$\mu_R = \mu_F = M_H$

Results Higgs plus three jets



$ud \to Hudg$	12 tree	467 NLO
$uu \to Huug$	24 tree	868 NLO
$ug \to Hugg$	74 tree	2519 NLO
$gg \to Hggg$	230 tree	9325 NLO
Total	340 tree	13179 NLO

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	$gg \to Hq\bar{q}g$	
b_0	$0.6309159660038877 \cdot 10^{-4}$	
a_0	48.68424097859422	
a_{-1}	−36.08277727147958	−36.08277728199094
a_{-2}	−11.66666666667209	−11.66666666666667
	$q\bar{q} \to Hq\bar{q}g$	
b_0	$0.3609139855530763 \cdot 10^{-4}$	
a_0	69.32351140490162	
a_{-1}	−29.98862932963380	−29.98862932963629
a_{-2}	−8.333333333333339	−8.333333333333334
	$q\bar{q} \to Hq'\bar{q}'g$	
b_0	$0.2687990772405433 \cdot 10^{-5}$	
a_0	15.79262767177915	
a_{-1}	−32.35320587070861	−32.35320587073038
a_{-2}	−8.333333333333398	−8.333333333333332

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- Higgs plus three jets in production