Theory Seminar@MPP, München
21. May 2013

Beyond Heisenberg's uncertainty principle:

Error-disturbance uncertainty relation studied in neutron's successive spin measurements

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I. Introduction: neutron optical experiments

II. Uncertainty relation for error-disturbance

V. Summary

The neutron

Particle | Feels four-forces | **Wave**

$m = 1.674928\,(1) \times 10^{-27}$ kg

$s = \frac{1}{2}\hbar$

$\mu = -\,9.6491783(18) \times 10^{-27}$ J/T

$\tau = 887(2)$ s

R = 0.7 fm

$\alpha = 12.0\,(2.5) \times 10^{-4}$ fm^3

u - d - d - quark structure

CONNECTION

de Broglie

$$\lambda_B = \frac{h}{m.v}$$

Schrödinger

$$H\psi(\bar{r},t) = i\hbar \frac{\delta\psi(\bar{r},t)}{\delta\ t}$$

&

boundary conditions

$$\lambda_c = \frac{h}{m.c} = 1.319695\,(20) \times 10^{-15}\ \text{m}$$

For thermal neutrons
= 2 Å, 2000 m/s, 20meV

$$\lambda_B = \frac{h}{m.v} = 1.8 \times 10^{-10}\ \text{m}$$

$$\Delta_c = \frac{1}{2\delta k} \cong 10^{-8}\ \text{m}$$

$$\Delta_p = v.\Delta t \cong 10^{-2}\ \text{m}$$

$$\Delta_d = v.\tau = 1.942(5) \times 10^{6}\ \text{m}$$

$$0 \le \chi \le 2\pi\ (4\pi)$$

m ... mass, s ... spin, μ ... magnetic moment, τ ... β-decay lifetime, R ... (magnetic) confinement radius, α ... electric polarizability; all other measured quantities like electric charge, magnetic monopole and electric dipole moment are compatible with zero

-μB ________________

μB ________________ two level system

λ_c ... Compton wavelength, λ_B ... deBroglie wavelength, Δ_c ... coherence length, Δ_p ... packet length, Δ_d ... decay length, δk.... momentum width, Δt ... chopper opening time, v ... group velocity, χ ... ... phase.

Neutrons in quantum mechanics

Particle and wave properties

$$p = mv = h/\lambda$$

(L. De Broglie)

Schroedinger equation

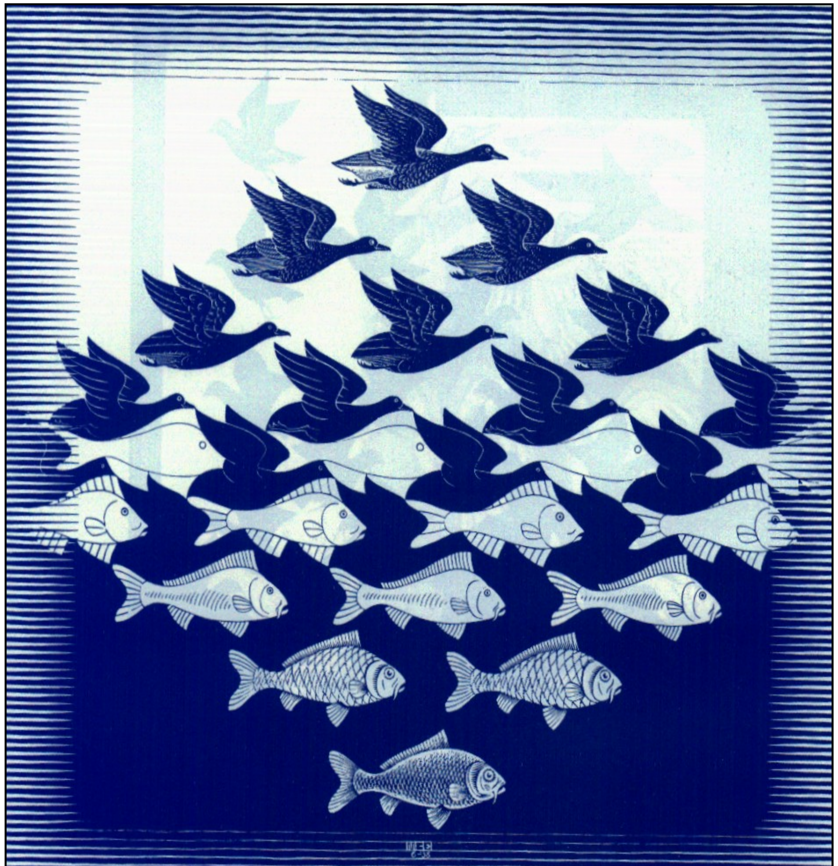
$$i\hbar \frac{\partial \Psi(\vec{r},t)}{\partial t} = H\Psi(\vec{r},t)$$

(E. Schrödinger)

Uncertainity

$$\Delta x \, \Delta p \geq h/4\pi$$

(W. Heisenberg)



M.C.Escher, 1938

Neutron interferometry

Neutrons

$m = 1.67 \times 10^{-27}$ kg

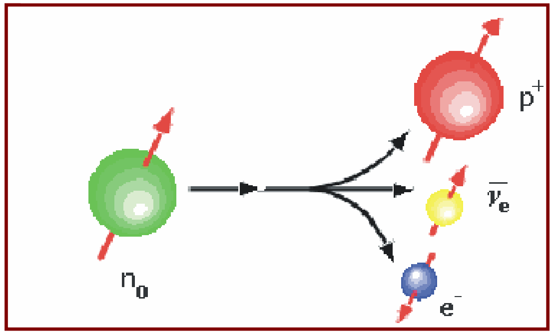
$s = \frac{1}{2}\hbar$

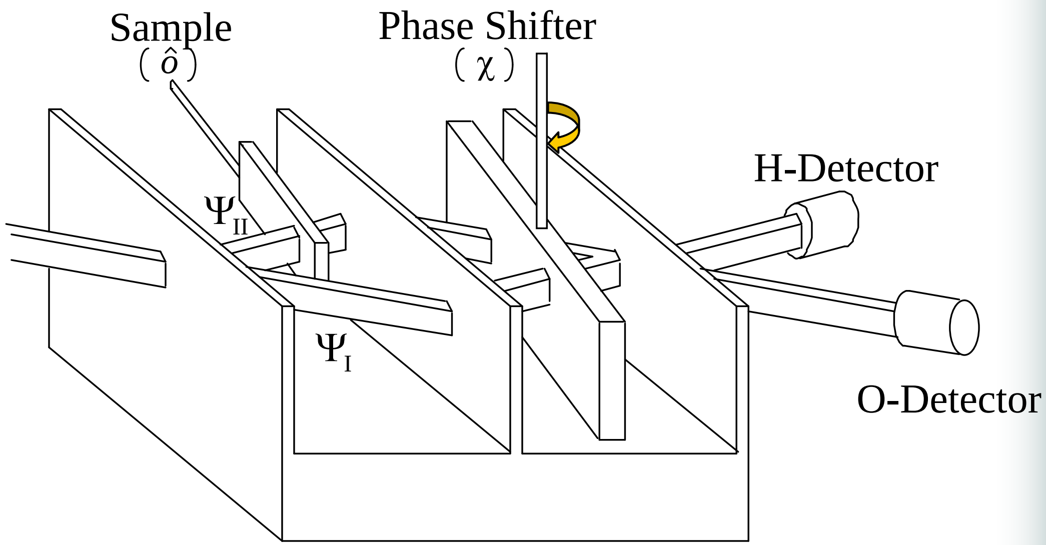
$\mu = -9.66 \times 10^{-27}$ J/T

$\tau = 887$ s

$R = 0.7$ fm

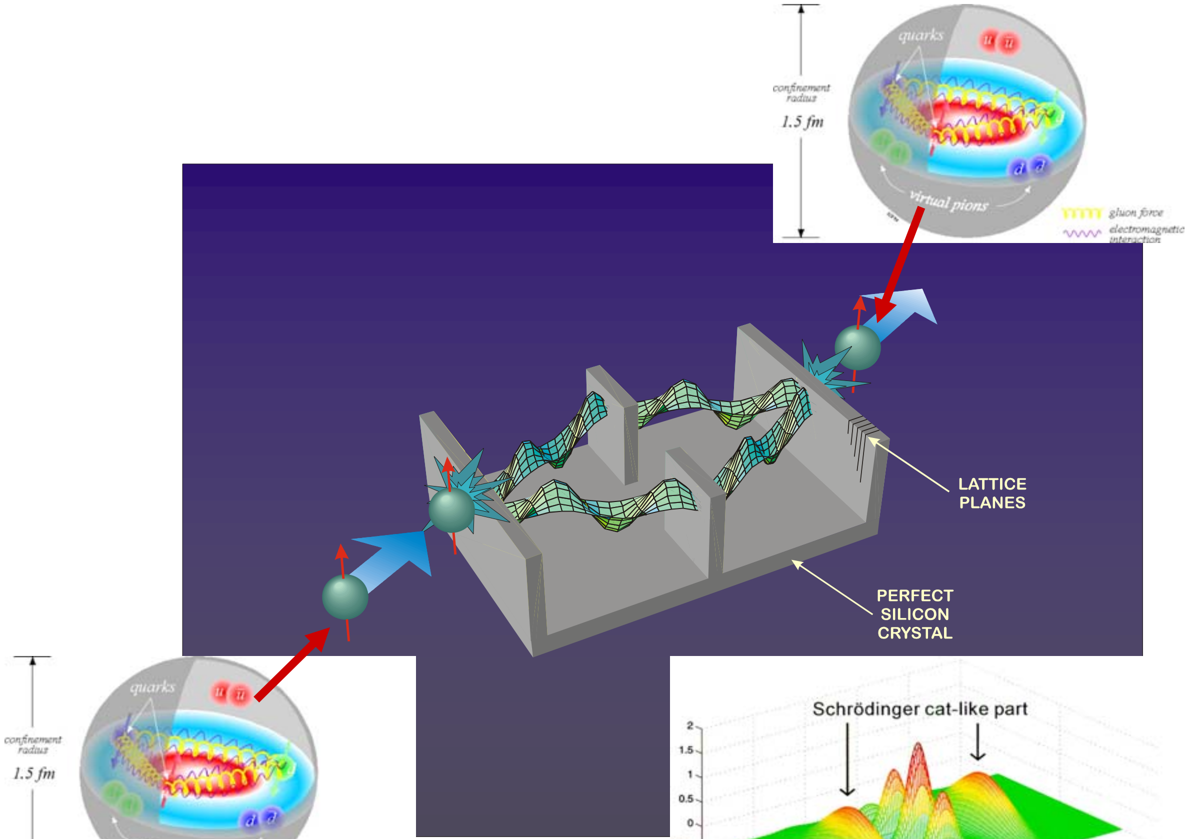
u–d–d quark structure

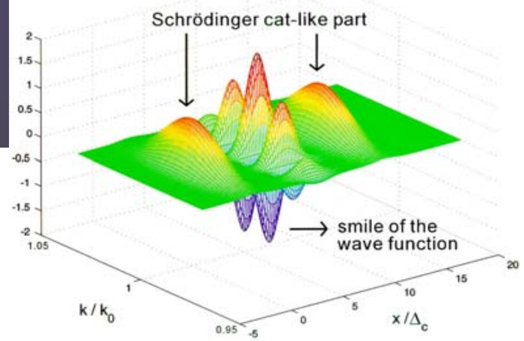




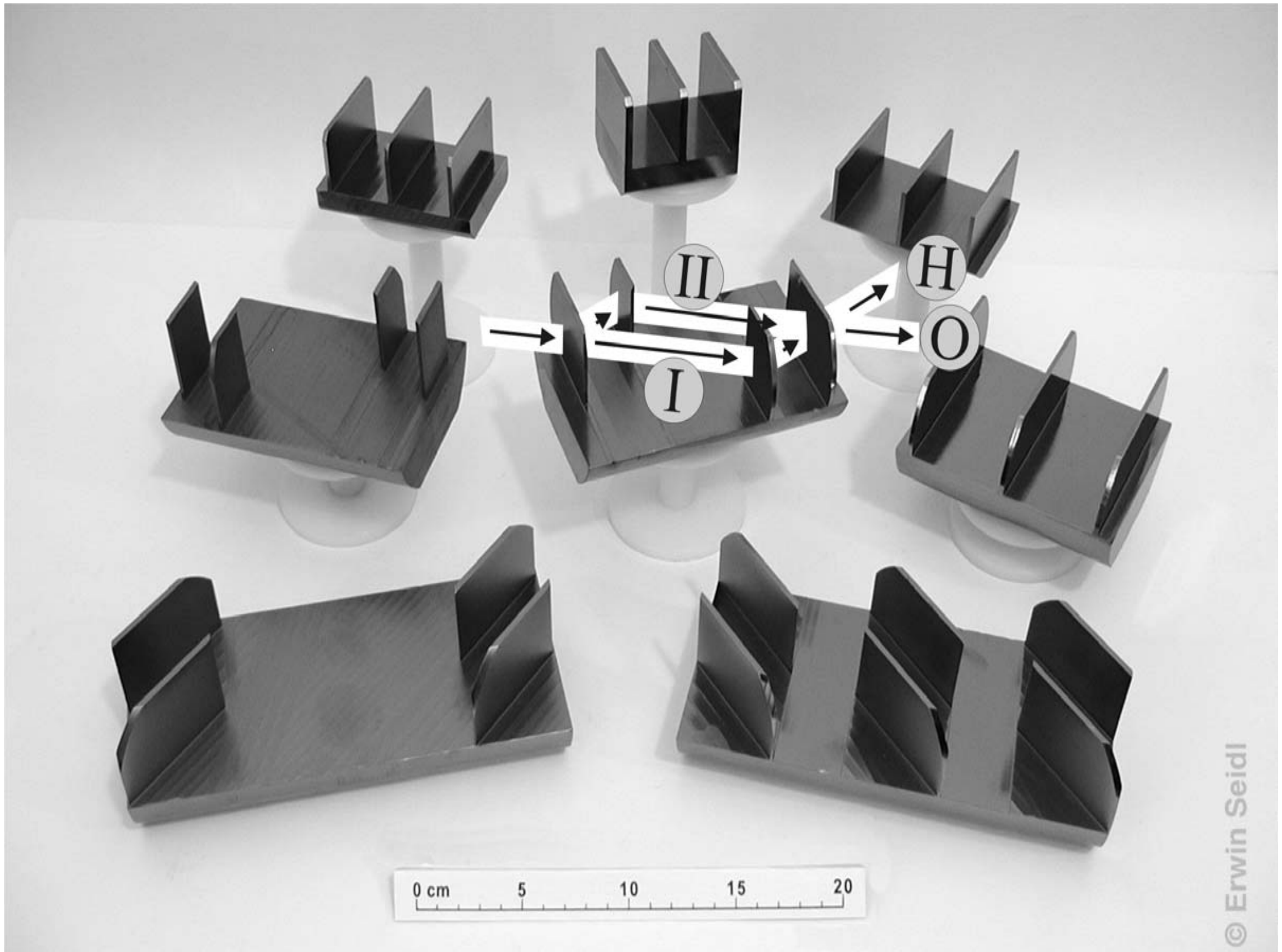
$$I = |\Psi_I + e^{i\chi} \cdot \hat{o} \cdot \Psi_{II}|^2$$

Neutron interferometer



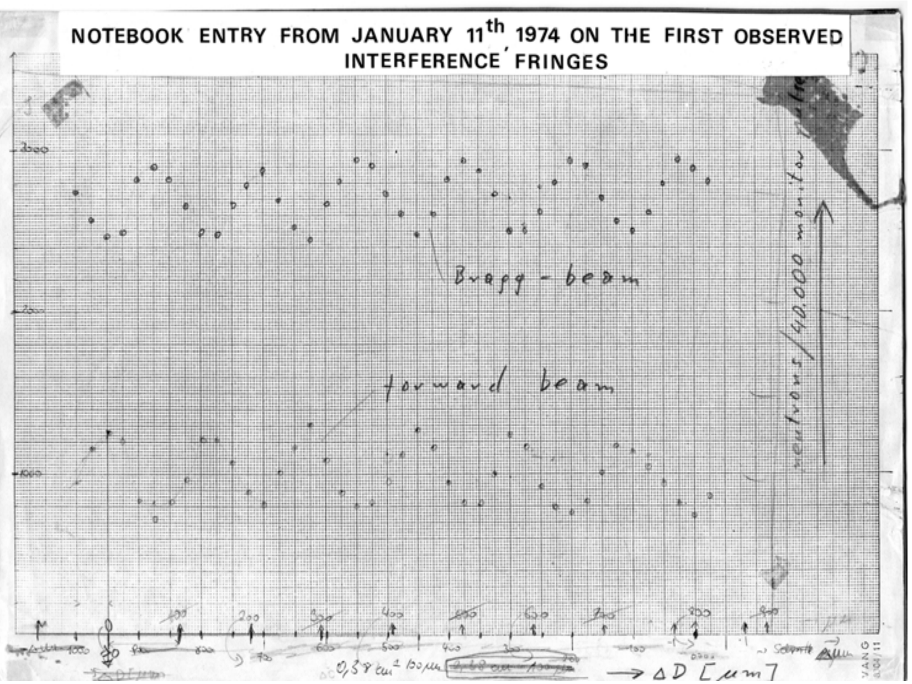


Neutron interferometer family

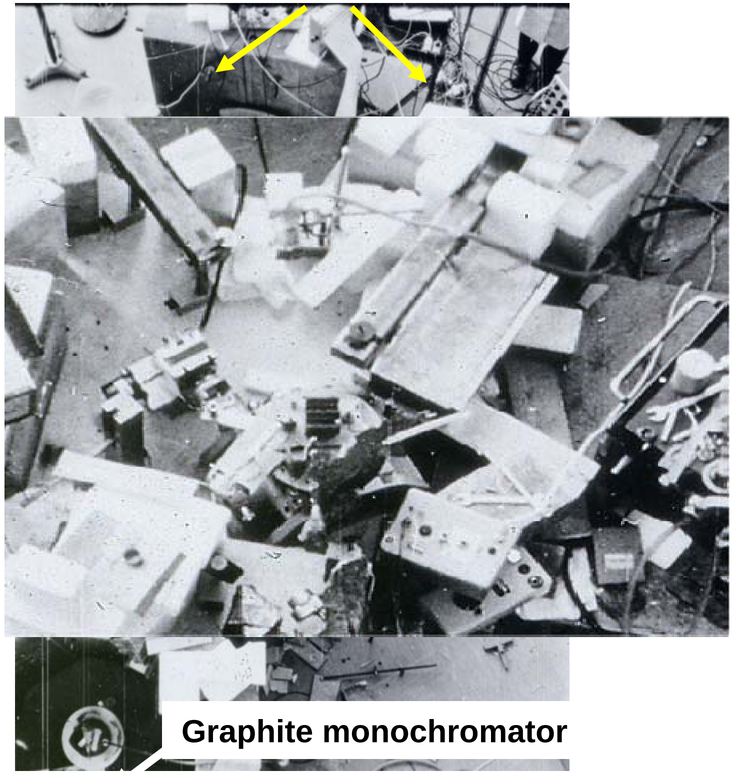


First results

TRIGA reactor Vienna 1974



detectors



Graphite monochromator

H.Rauch, W.Treimer, U.Bonse, Phys.Lett.A47(1974)369

Neutron interferometer experiment(1)

4π-symmetry of spinor wavefunction

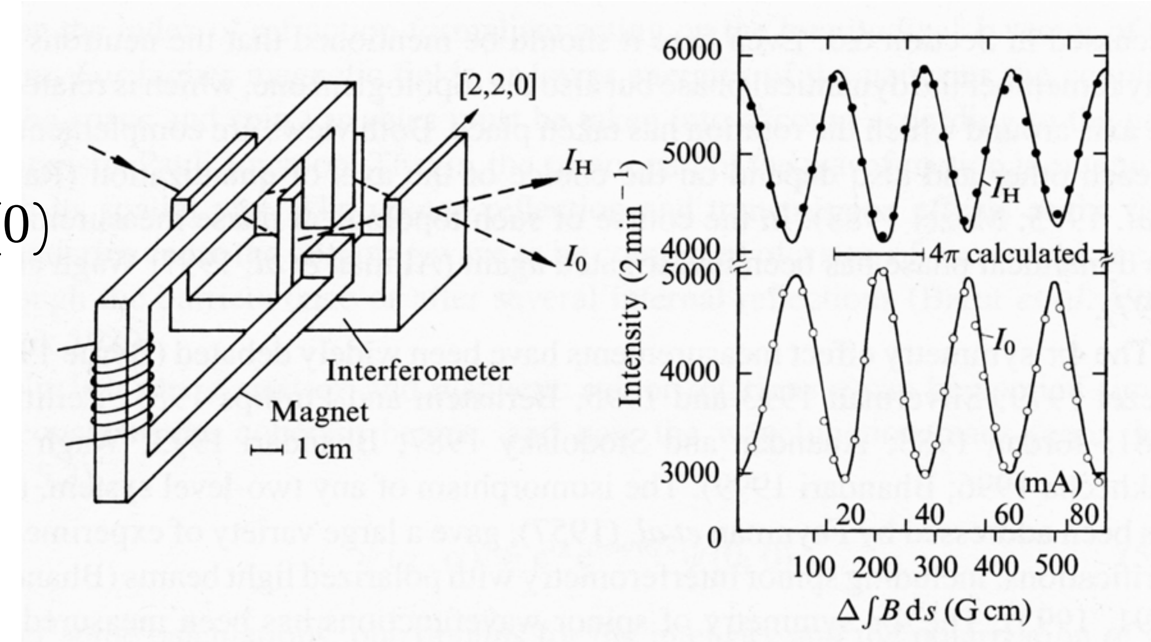
Hamiltonian

$$\hat{H} = -\mu\cdot\mathbf{B} = -\mu\sigma\cdot\mathbf{B}$$

Wave function

$$\Psi(t) = \exp[-{}^{iHt}/_{\hbar}]\cdot\Psi(0)$$
$$= \exp[-{}^{i\mu\cdot\mathbf{B}t}/_{\hbar}]\cdot\Psi(0)$$
$$= \exp[-{}^{i\sigma\cdot\alpha}/_{2}]\cdot\Psi(0)$$

Where $\alpha = \frac{2\mu}{\hbar}\int B\,dt$



H. Rauch et al., PL A**54** (1975) 425.

Neutron interferometer experiment(2)

Gravitationally induced quantum phase

Energy of neutron

gravitational potential

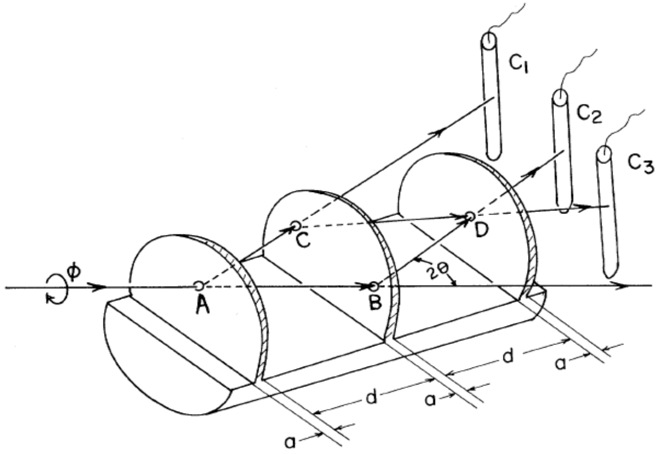
$$E_0 = \frac{\hbar^2 k_0^2}{2m} = \frac{\hbar^2 k^2}{2m} + mgH(\alpha)$$

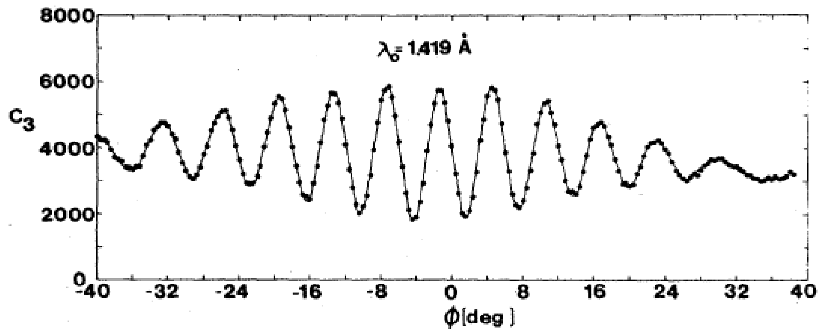
and

$$\Delta k = (k - k_0) \cong -\frac{m^2 gH}{\hbar^2 k_0} \sin\alpha$$

Phase shift

$$\Delta\Phi_{COW} = -2\pi\lambda \frac{m^2}{\hbar^2} g A_0 \sin\alpha$$



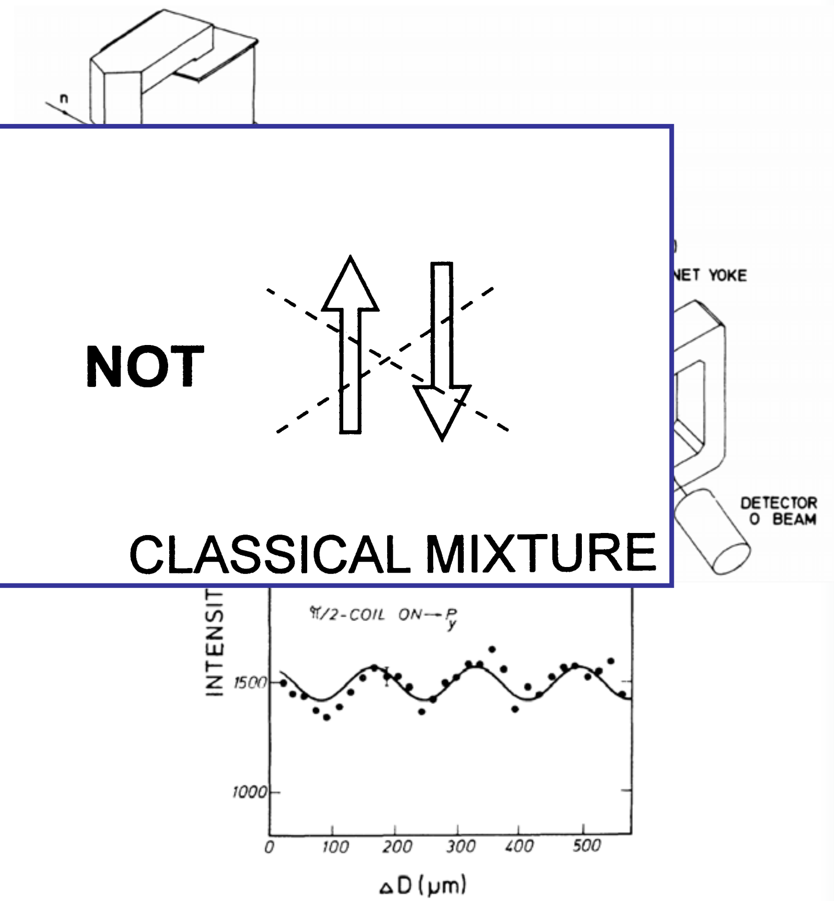


R. Colella et al., PRL **34** (1975) 1472; J.L. Staudenmann et al., PR A21 (1980) 1419.

Neutron interferometer experiment (3)

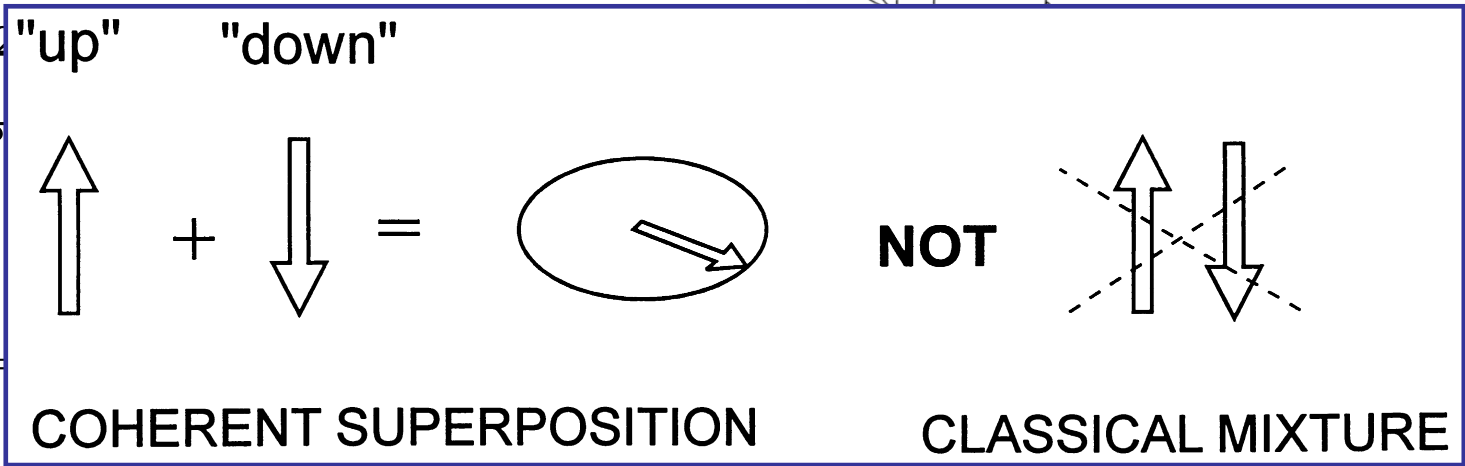
Superposition of spinor wavefunctions

§ General framework of spin-$\frac{1}{2}$ system



(i) 2

σ



(ii)

$|s_{\pm x}\rangle =$

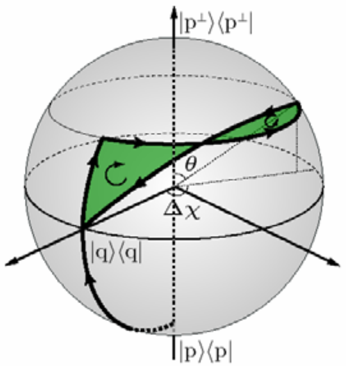
➔➔ **Superposition**

$$\frac{1}{\sqrt{2}}(|s_{+z}\rangle + e^{i\phi}|s_{-z}\rangle) = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ e^{i\phi} \end{bmatrix}$$

$$|s_{+x}\rangle \rightarrow |s_{+y}\rangle \rightarrow |s_{-x}\rangle \rightarrow |s_{-y}\rangle$$

J. Summhammer et al., PL A**27** (1983) 2532.

Geometric/Topological phases



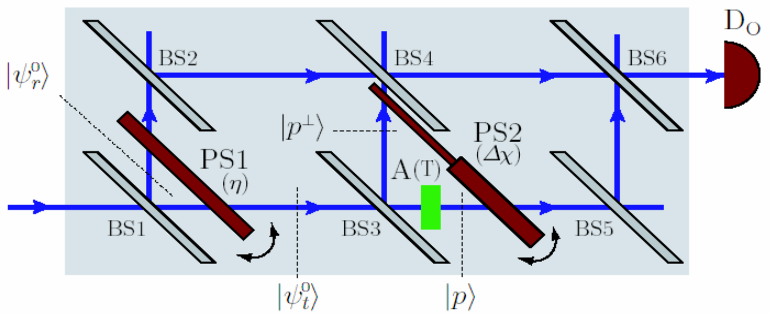
Berry Phase (adiabatic & cyclic evolution)

[Berry; Proc.R.S.Lond. A 392, 45 (1984)]

$$|\psi(t)\rangle = e^{-i\phi_d} e^{i\phi_g} |n(R(t))\rangle$$

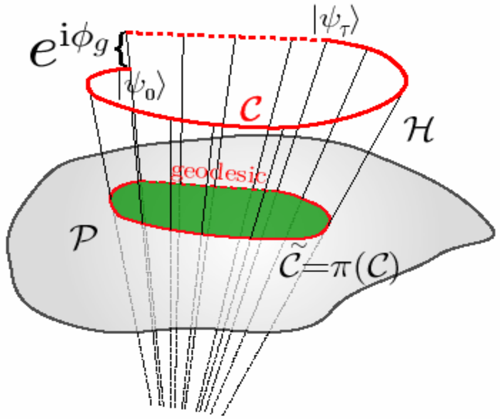
$$\phi_d(t) = \frac{1}{\hbar}\int_0^t dt' E_n(t')$$

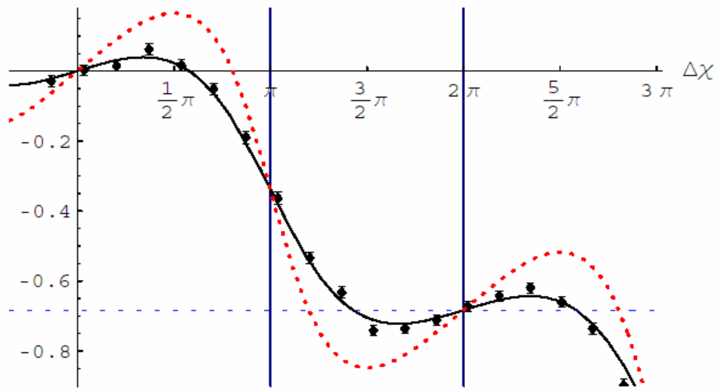
$$\phi_g = -\frac{1}{2}\,\mathbf{\Omega}$$ (for 2-level systems)



Non-adiabatic & non-cyclic evolution

[Samuel & Bhandari, PRL 60, 2339 (1988)]





S. Filipp, et al., PRA72 (2005) 021602

Two-particle vs. two-space entanglement

2-Particle Bell-State

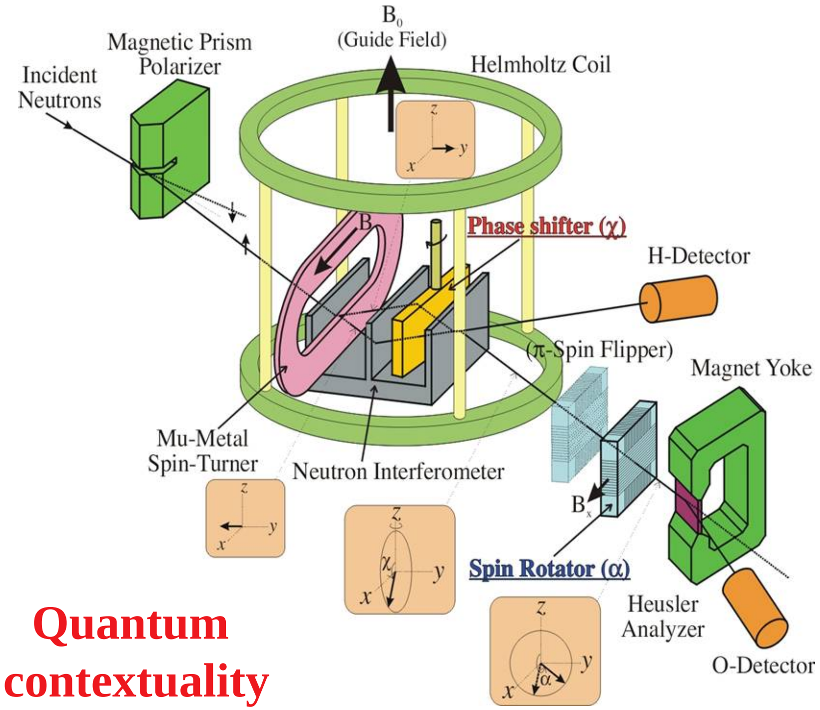
$$|\Psi\rangle = \frac{1}{\sqrt{2}}\{|\uparrow\rangle_{\mathrm{I}}\otimes|\downarrow\rangle_{\mathrm{II}} + |\downarrow\rangle_{\mathrm{I}}\otimes|\uparrow\rangle_{\mathrm{II}}\}$$

I, II represent 2-Particles

2-Space Bell-State

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\{|\uparrow\rangle_{\mathrm{s}}\otimes|\mathrm{I}\rangle_{\mathrm{p}} + |\downarrow\rangle_{\mathrm{s}}\otimes|\mathrm{II}\rangle_{\mathrm{p}}\}$$

s, p represent 2-Spaces, e.g., spin & path



Quantum contextuality

Violation of Bell-like inequality

$$S' \equiv E'(\alpha_1,\chi_1) + E'(\alpha_1,\chi_2) - E'(\alpha_2,\chi_1) + E'(\alpha_2,\chi_2)$$

$$= \mathbf{2.051} \pm \mathbf{0.019} \quad > 2$$

Hasegawa et al., Nature2003, NJP2011

Kochen-Specker-like contradiction 1

$$E_x \cdot E_y = 0.407 \xleftrightarrow{63\%} E' \equiv \langle \hat{X}_1 \hat{Y}_2 \cdot \hat{Y}_1 \hat{X}_2 \rangle = -0.861$$

Hasegawa et al., PRL2006/2009

Tri-partite entanglement (GHZ-state)

$$|\Psi_{\mathrm{Neutron}}\rangle = \{|\Psi_{\mathrm{I}}\rangle \otimes |\uparrow\rangle \otimes |\Psi(E_0)\rangle + (e^{i\chi}|\Psi_{\mathrm{II}}\rangle) \otimes (e^{i\alpha}|\downarrow\rangle) \otimes (e^{i\gamma}|\Psi(E_0+\hbar\omega_r)\rangle)\}$$

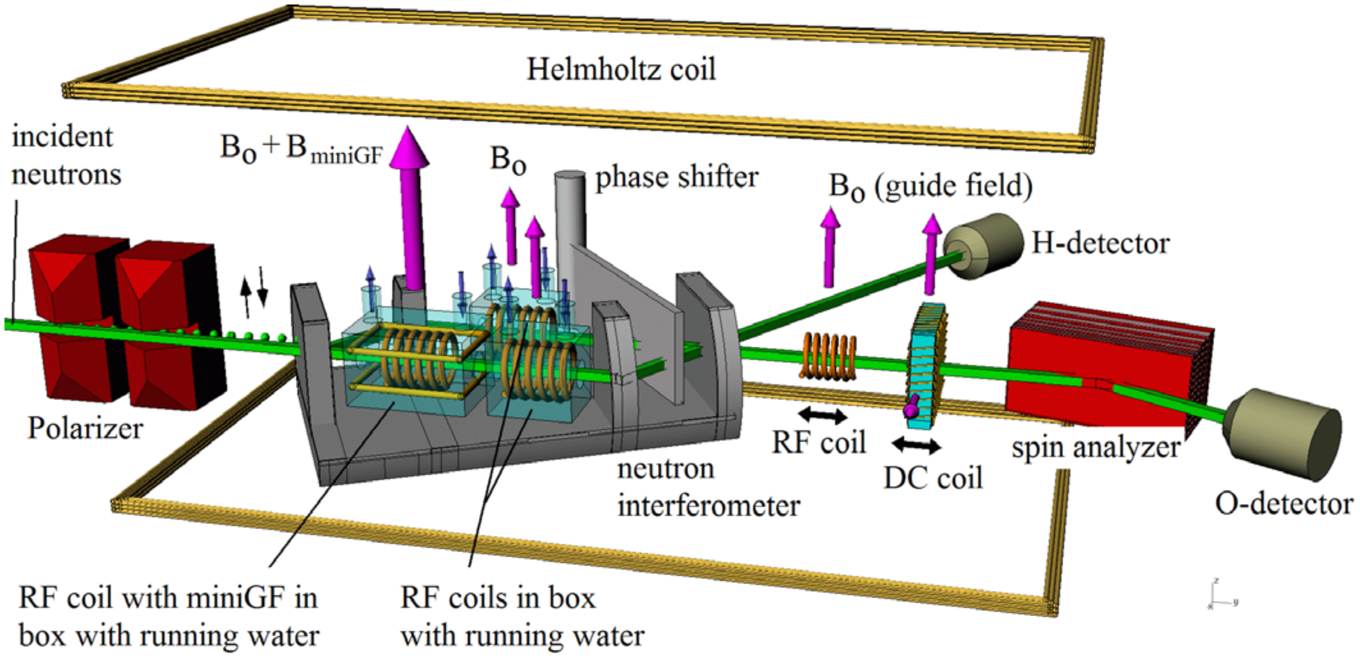
$$M_{Measured} = 2.558 \pm 0.004 \quad > 2$$

Hasegawa et al., PRA2010

W- and GHZ- states in a single neutron system

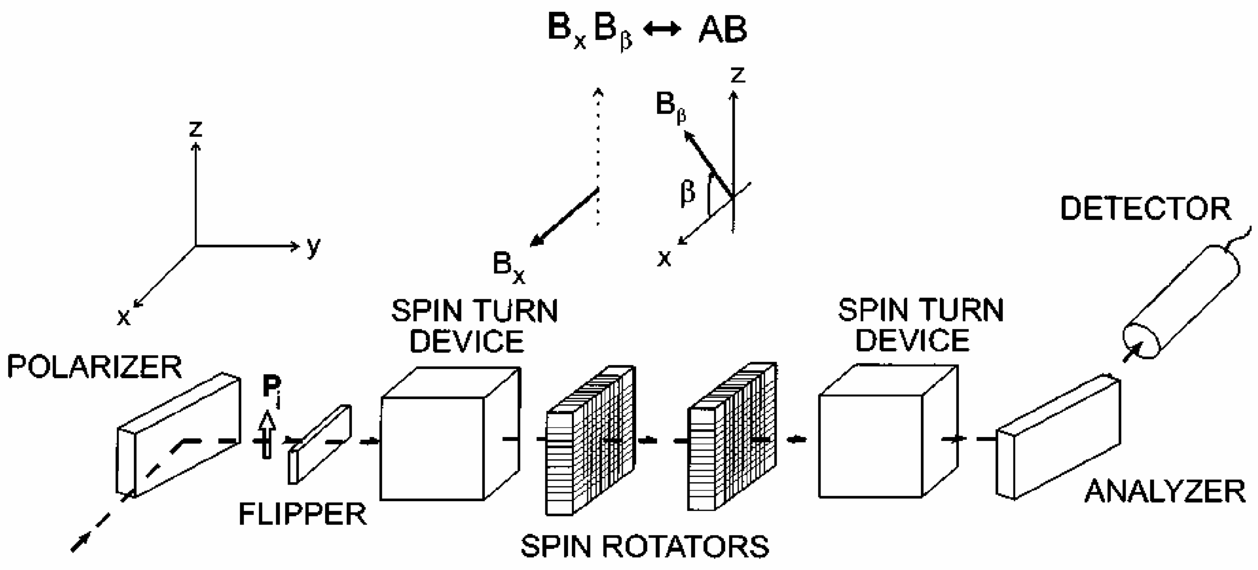
W-state: $$|\Psi\rangle_{\mathrm{W}} = \frac{1}{\sqrt{3}} \cdot |\mathrm{II} \downarrow \hbar\omega\rangle + \frac{1}{\sqrt{3}} \cdot |\mathrm{I} \uparrow \hbar\omega\rangle + \frac{1}{\sqrt{3}} \cdot |\mathrm{I} \downarrow 2\hbar\omega\rangle$$

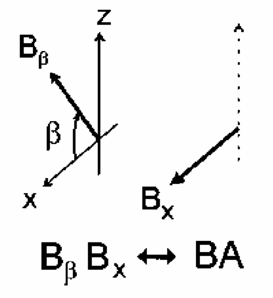
GHZ-state: $$|\Psi\rangle_{\mathrm{GHZ}} = \frac{1}{\sqrt{2}} \cdot |\mathrm{II} \downarrow \hbar\omega\rangle + \frac{1}{\sqrt{2}} \cdot |\mathrm{I} \uparrow 0\rangle$$

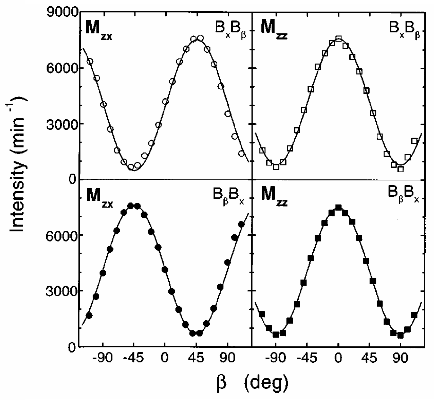


D. Erdösi et al. New J. Phys. 15 (2013) 023033

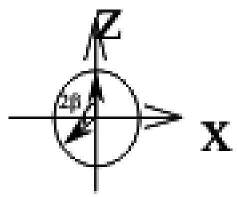
Neutron polarimetry







$$\hat{O}|\pm Y\rangle = -e^{\pm i\beta}\cdot|\pm Y\rangle \Rightarrow\Rightarrow$$



$$\hat{O}|+Z\rangle = \hat{O}\left\{\frac{1}{\sqrt{2}}\left(|+Y\rangle + |-Y\rangle\right)\right\} = \frac{1}{\sqrt{2}}\left(e^{i\beta}|+Y\rangle + e^{-i\beta}|-Y\rangle\right) = \frac{e^{i\beta}}{\sqrt{2}}\left(|+Y\rangle + e^{-2i\beta}|-Y\rangle\right)$$

- Bell-Test, PLA (2010)
- Leggett-Test, NJP (2012)
- GHZ-entanglement, NJP (1012)

Neutron Detector

High efficiency (>99%)

1. BF_3-Detectors

$$n + {}^{10}_{5}B \longrightarrow {}^{11}_{5}B \longrightarrow^{93\%}_{7\%} \begin{cases} \nearrow {}^{7}_{3}Li^{*} + {}^{4}_{2}He \longrightarrow {}^{7}_{3}Li + {}^{4}_{2}He + 2.31MeV + \gamma(0.48MeV) \\ \searrow {}^{7}_{3}Li + {}^{4}_{2}He + 2.79MeV \end{cases}$$

2. ${}^{3}He$-Detectors

$$n + {}^{3}He \Rightarrow {}^{3}H + p + 0.764MeV$$

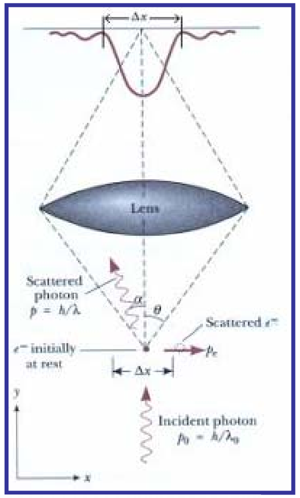
Uncertainty relation: historical 1

- In 1927 Heisenberg postulated an uncertainty principle:

  γ-ray thought experiment

  → $p_1 q_1 \sim h$

  with q_1 (mean error) & p_1 (discontinuous change)



- Sei q_1 die Genauigkeit, mit der der Wert q bekannt ist (q_1 ist etwa der mittlere Fehler von q), also hier die Wellenlänge des Lichtes, p_1 die Genauigkeit, mit der der Wert p bestimmbar ist, also hier die unstetige Änderung von p beim Comptoneffekt, so stehen nach elementaren Formeln des Comptoneffekts p_1 und q_1 in der Beziehung

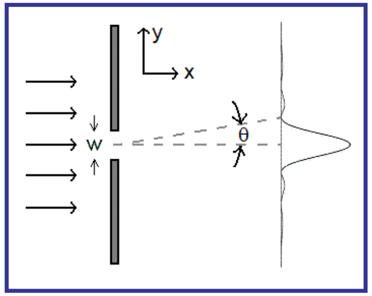
$$p_1 q_1 \sim h. \qquad (1)$$

Uncertainty relation: historical 2

- Kennard considered the spread of a wave function ψ

$$\sigma(Q)\sigma(P) \geq \frac{\hbar}{2}$$

σ: standard deviations



- Robertson generalized the relation to arbitrary pairs of observables in any states ψ

$$\sigma(A)\ \sigma(B) \geq \frac{1}{2}\left|\langle\psi|[A,B]|\psi\rangle\right|$$

→ dependent on the state but independent of the appartus

$$Is\ \ \varepsilon(A)\eta(B) \geq \frac{1}{2}\left|\langle\psi|[A,B]|\psi\rangle\right|\ \ generally\ \ valid\,?$$

Ozawa's Universally Valid Uncertainty Relation 1

PHYSICAL REVIEW A **67**, 042105 (2003)

Universally valid reformulation of the Heisenberg uncertainty principle on noise and disturbance in measurement

Masanao Ozawa
Graduate School of Information Sciences, Tôhoku University, Aoba-ku, Sendai, 980-8579, Japan
(Received 9 October 2002; published 11 April 2003)

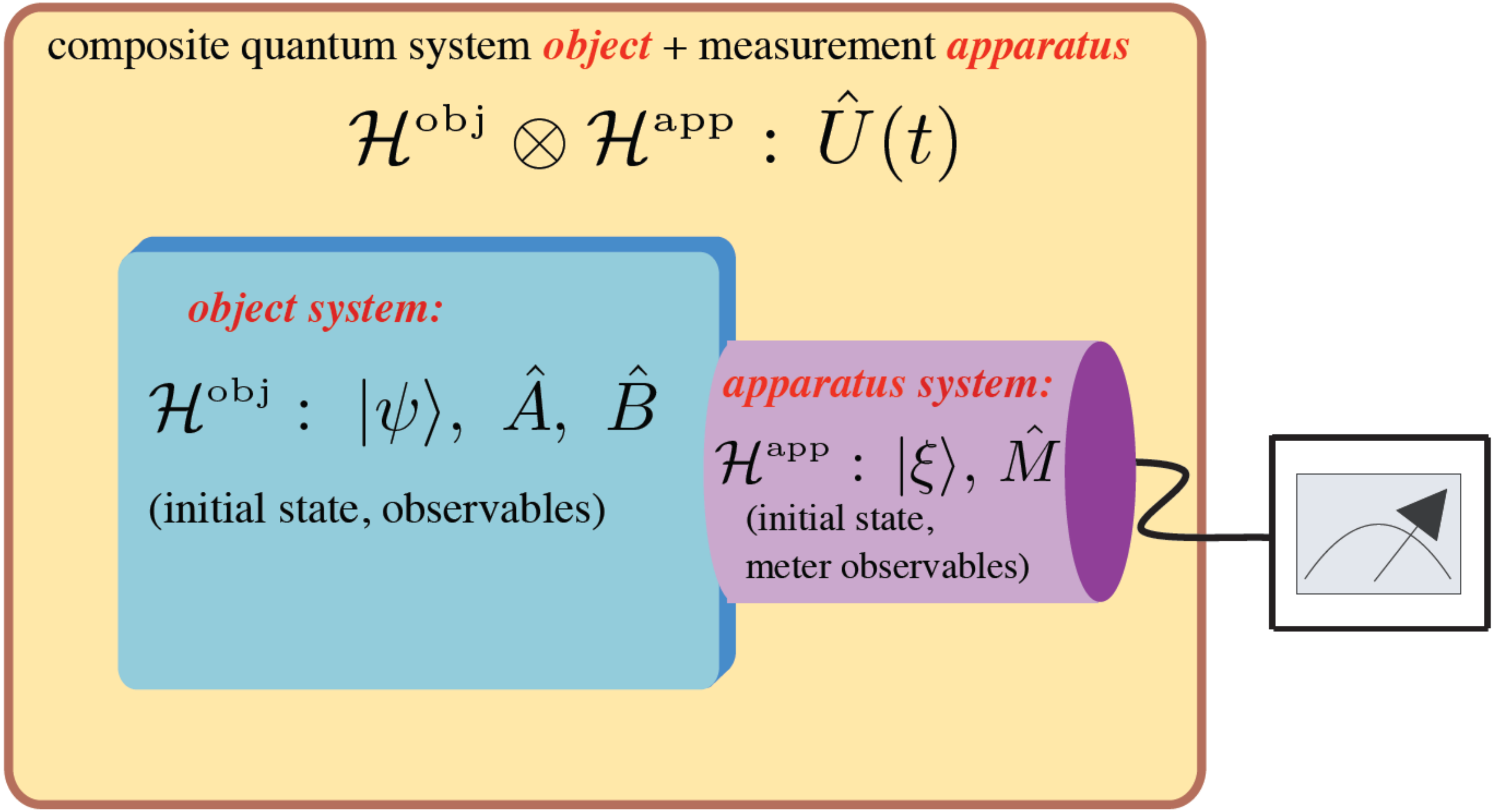
The Heisenberg uncertainty principle states that the product of the noise in a position measurement and the momentum disturbance caused by that measurement should be no less than the limit set by Planck's constant $\hbar/2$ as demonstrated by Heisenberg's thought experiment using a γ-ray microscope. Here it is shown that this common assumption is not universally true: a universally valid trade-off relation between the noise and the disturbance has an additional correlation term, which is redundant when the intervention brought by the

$$\epsilon(A)\eta(B) + \epsilon(A)\sigma(B) + \sigma(A)\eta(B) \geq \frac{1}{2}|\langle\psi|[A,B]|\psi\rangle|$$

- rigorous theoretical treatments of quantum measurements:
  - ***first term:*** error of the first measuremt, disturbance on the second measurement
  - ***second and third terms:*** crosstalks between spreads of wavefunctions and error/disturbance

FWF

Ozawa's Universally Valid Uncertainty Relation 2



Definition of error & disturbance

- **Error** is defined as the root-mean-square (rms):

$$\epsilon(A) = \langle\psi| \otimes \langle\xi| \big(U^\dagger (I \otimes M) U - A \otimes I\big)^2 |\xi\rangle \otimes |\psi\rangle^{1/2}$$

describes how accurate the value of the observable *A* before the measurement is transferred to the apparatus's meter observable *M*

meter observable *M has* orthonormal basis $|\lambda\rangle$: $M = \sum_\lambda \lambda |\lambda\rangle\langle\lambda|$

family of measurement ***operators***: $O_\lambda = \langle\lambda|U|\xi\rangle$ acting on object-system $\mathcal{H}^{\mathrm{obj}}$ (*U* on $\mathcal{H}^{\mathrm{obj}} \otimes \mathcal{H}^{\mathrm{app}}$)

- $\epsilon(A)^2 = \sum_\lambda ||O_\lambda (\lambda - A)|\psi\rangle||^2$

$||...|| = ||X|\psi\rangle|| = \langle\psi|X^\dagger X|\psi\rangle^{\frac{1}{2}}$

- **Disturbance** is defined in the same manner:

$$\eta(B) = \langle\psi| \otimes \langle\xi| \big(U^\dagger (B \otimes I) U - B \otimes I\big)^2 |\xi\rangle \otimes |\psi\rangle^{1/2}$$

defined by the rms difference between the observable *B* at time $t = 0$ and at time $t = \Delta t$

- $\eta(B)^2 = \sum ||[O_\lambda, B]|\psi\rangle||^2$

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Error and disturbance for projective measurement

- **Error**:

$$\epsilon(A)^2 = || \sum_\lambda O_\lambda(\lambda - A)|\psi||^2$$

If the O_λ are mutually orthogonal projection operators sum and norm can be exchanged

$$\epsilon(A)^2 = ||(O_A - A)|\psi\rangle||^2$$

output operator: $O_A = \sum_\lambda \lambda O_\lambda$

different expression for measurement (5 expectation values):

$$\epsilon(A)^2 = \langle\psi|A^2|\psi\rangle + \langle\psi|O_A^2|\psi\rangle + \langle\psi|O_A|\psi\rangle + \underbrace{\langle\psi|AO_AA|\psi\rangle}_{\langle\psi'|O_A|\psi'\rangle} - \underbrace{\langle\psi|(A+I)O_A(A+I)|\psi\rangle}_{\langle\psi''|O_A|\psi''\rangle}$$

with $O_A^2 = \sum_\lambda \lambda^2 O_\lambda^\dagger O_\lambda$

- **Disturbance**: $\eta(B)^2 = \sum_\lambda ||[O_\lambda, B]|\psi\rangle||^2$

$$\eta(B)^2 = \langle\psi|B^2|\psi\rangle + \langle\psi|X_B^2|\psi\rangle + \langle\psi|X_B|\psi\rangle + \overbrace{\langle\psi|BX_BB|\psi\rangle}^{\langle\psi'''|X_B|\psi'''\rangle} - \overbrace{\langle\psi|(B+I)X_B(B+I)|\psi\rangle}^{\langle\psi''''|X_B|\psi''''\rangle}$$

with $X_B^2 = \sum_\lambda O_\lambda^\dagger B^2 O_\lambda$, and modified output operator: $X_B = \sum_\lambda O_\lambda^\dagger B O_\lambda$

Universally valid uncertainty relation by Ozawa

$$\epsilon(A)\eta(B) + \epsilon(A)\sigma(B) + \sigma(A)\eta(B) \geq \frac{1}{2}|\langle\psi|[A,B]|\psi\rangle|$$

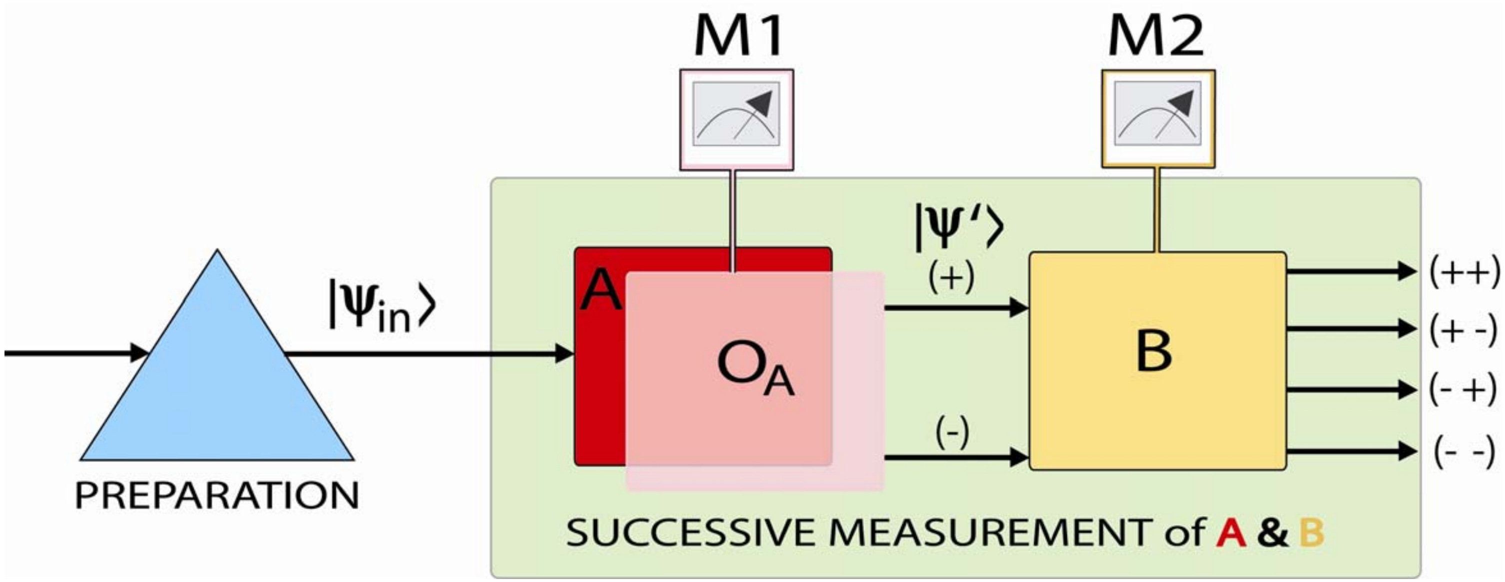
$$\begin{cases} \varepsilon : \text{error of the first measurmen} \, (A) \\ \eta : \text{disturbance on the second measurement} \, (B) \\ \sigma : \text{standard deviations} \end{cases}$$

First term: error of the first measuremt, disturbance on the second measurement

second and third terms: crosstalks between spreads of wavefunctions and error/disturbance

M. Ozawa, Phys. Rev. A **67**, 042105 (2003).

Experimental scheme



- Successively measurement of 2 noncommuting observables A and B

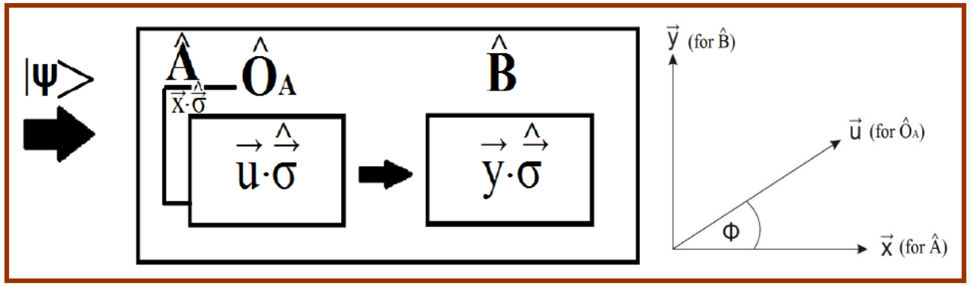
- Apparatus 1 measures O_A, Apparatus 2 measures B

Theoretical predictions 1

For error and disturbance:

$$\epsilon^2(A) = 2 - 2\,(\vec{x}\cdot\vec{u})$$

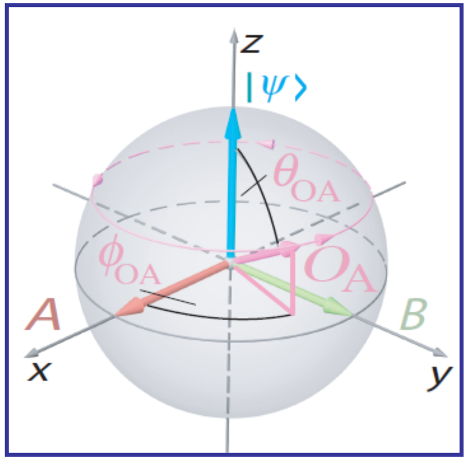
$$\eta^2(B) = 2 - 2\,(\vec{u}\cdot\vec{y})^2$$



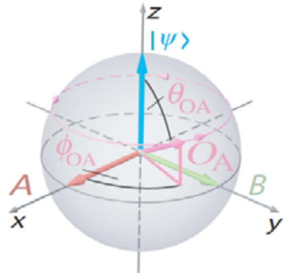
For the standard deviations:

$$\sigma^2(A) = \underbrace{\langle\psi|\,A^2\,|\psi\rangle}_{1} - (\langle\psi|\,A\,|\psi\rangle)^2$$

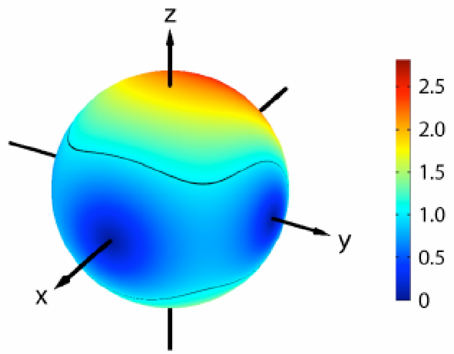
$$\sigma^2(B) = \underbrace{\langle\psi|\,B^2\,|\psi\rangle}_{1} - (\langle\psi|\,B\,|\psi\rangle)^2$$



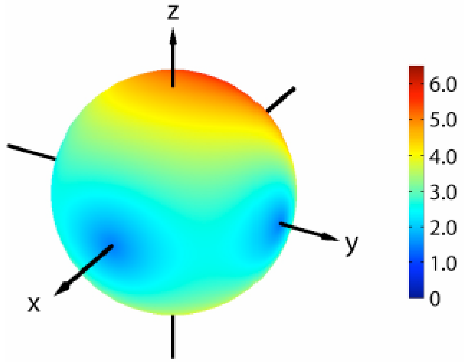
Theoretical predictions 2



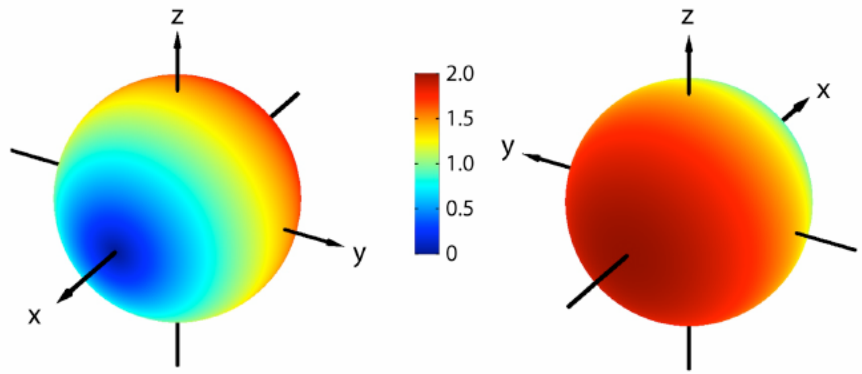
-Heisenberg's relation



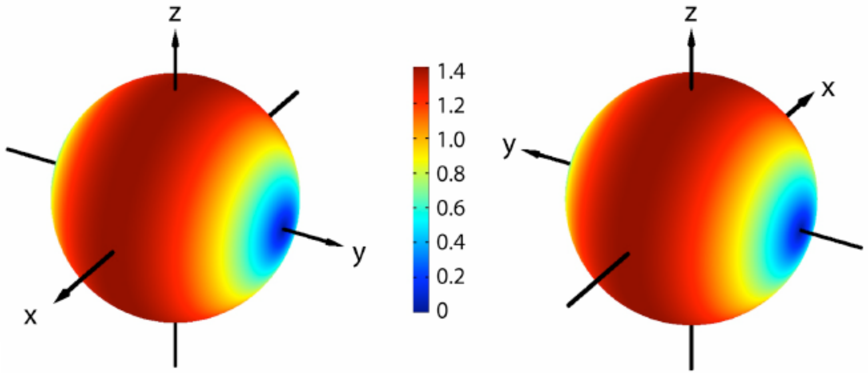
- new uncertainty relation



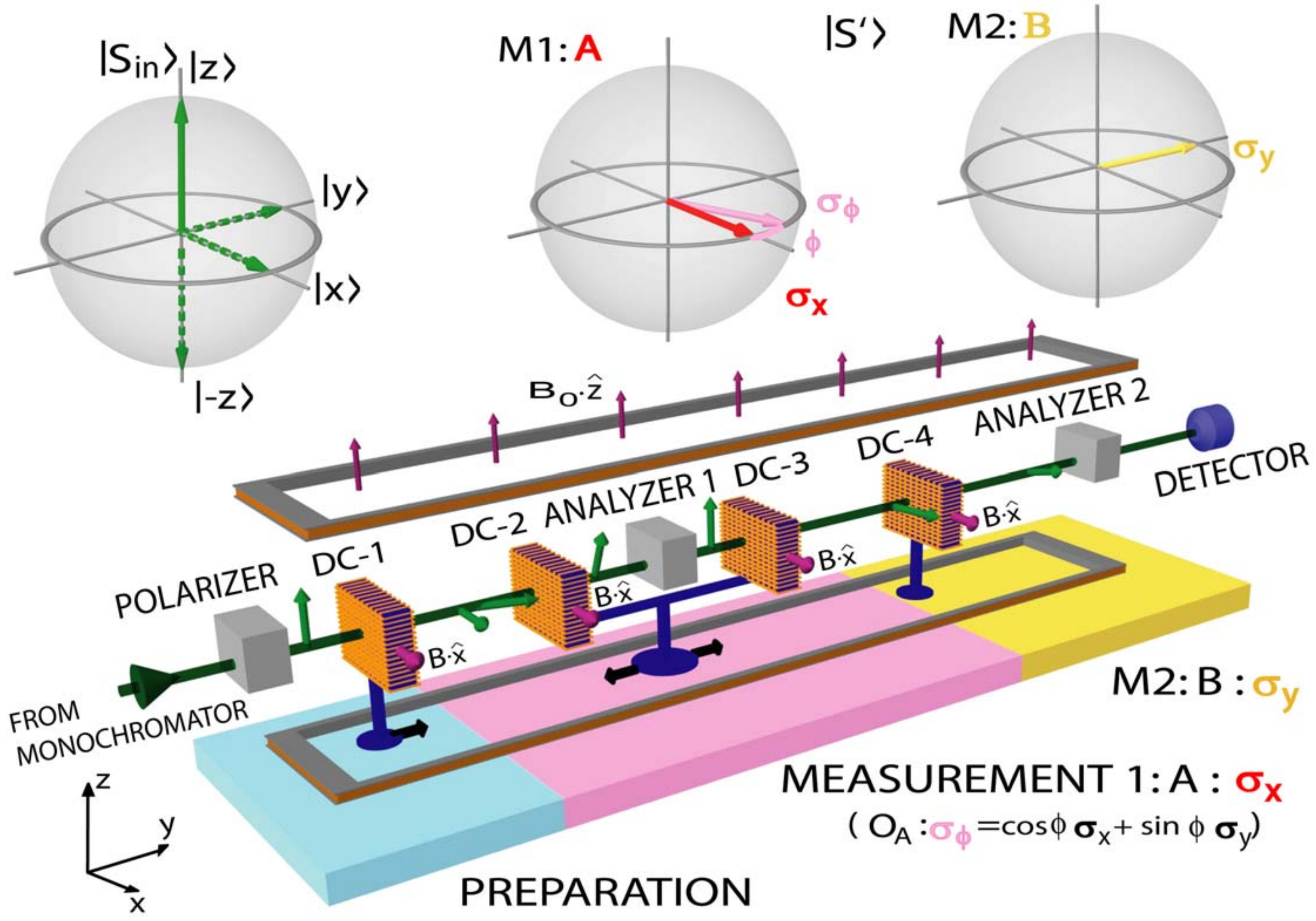
Error: $\epsilon(A)$



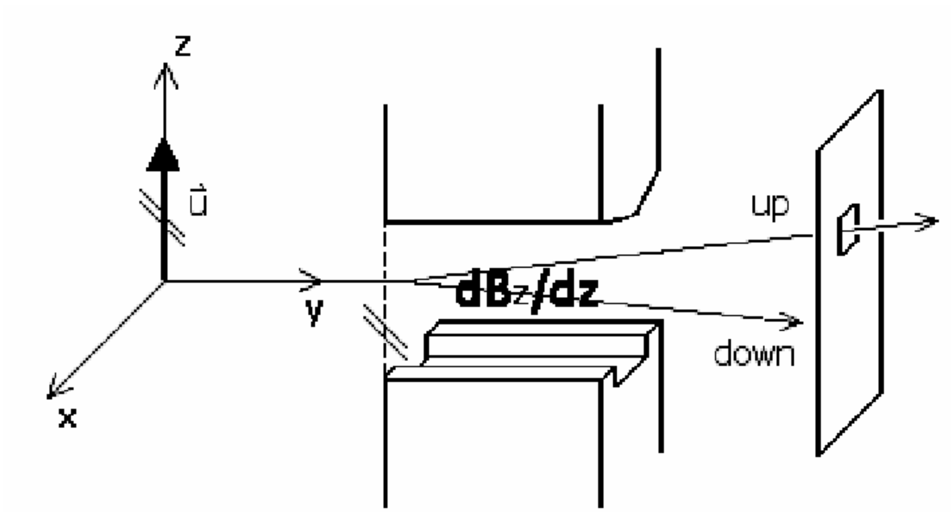
Disturbance: $\eta(B)$



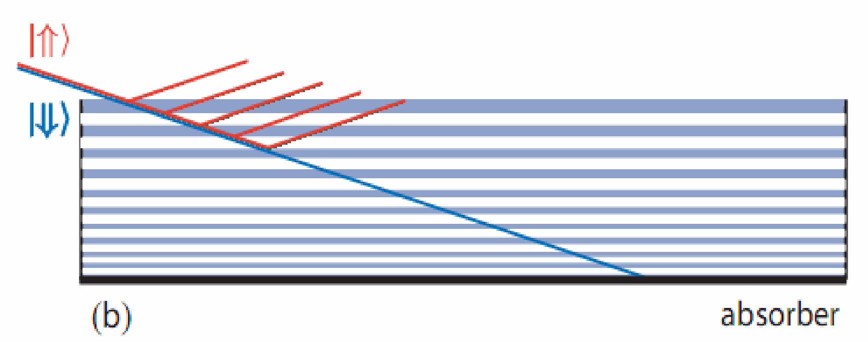
Experimental setup



Polarizer for neutrons 1

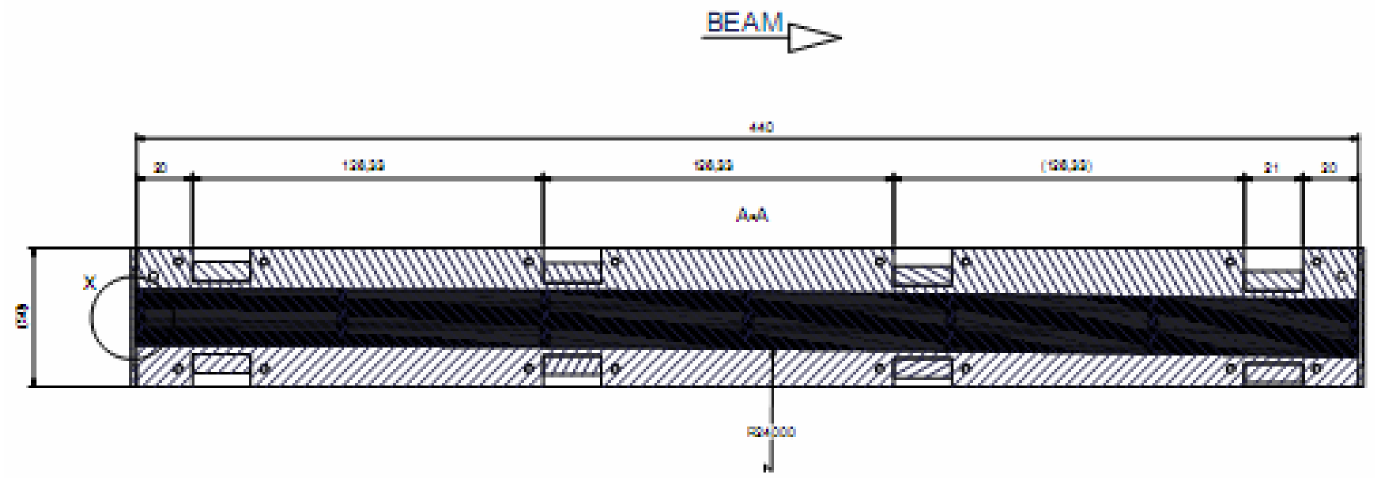


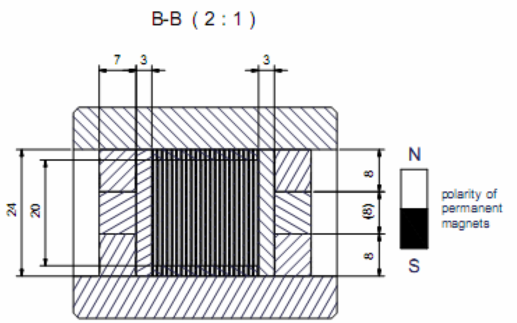
Stern-Gerlach apparatus

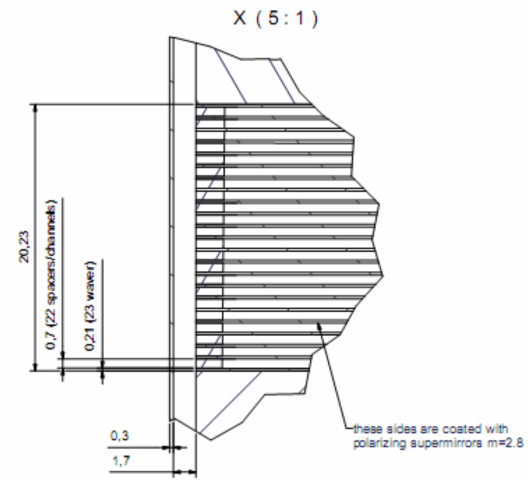


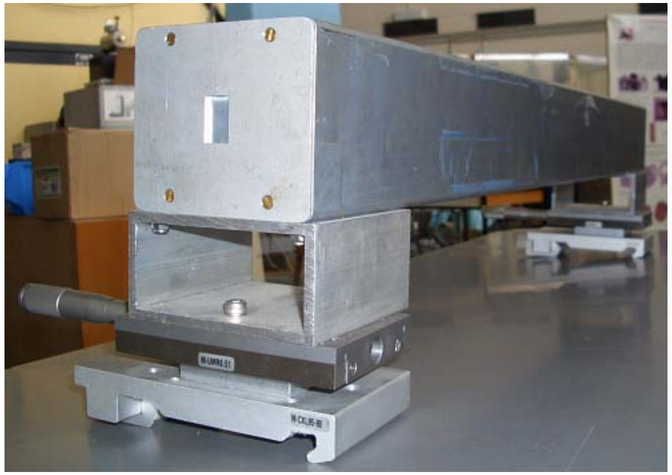
Multilayer polarizer

Polarizer for neutrons 2



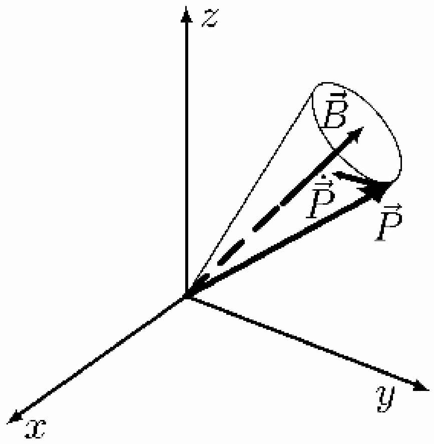


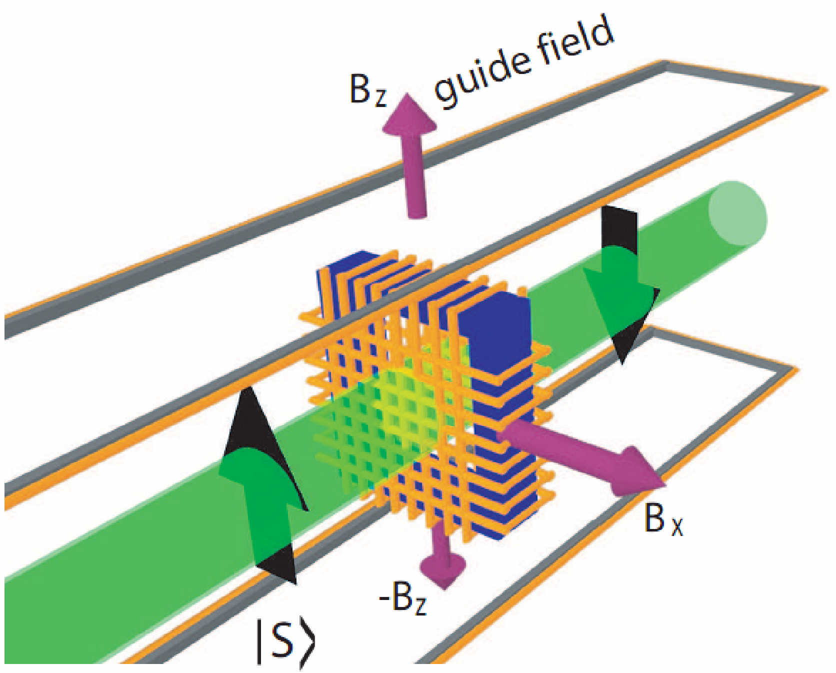


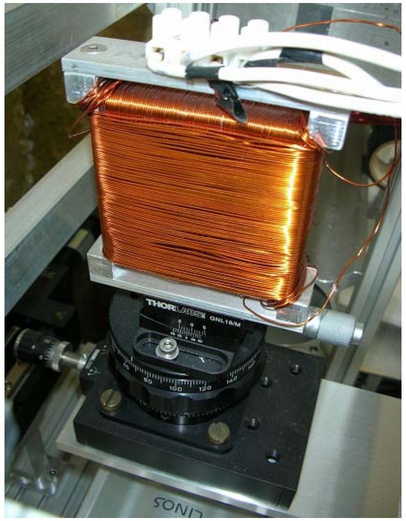


Spin rotator: Larmor precession

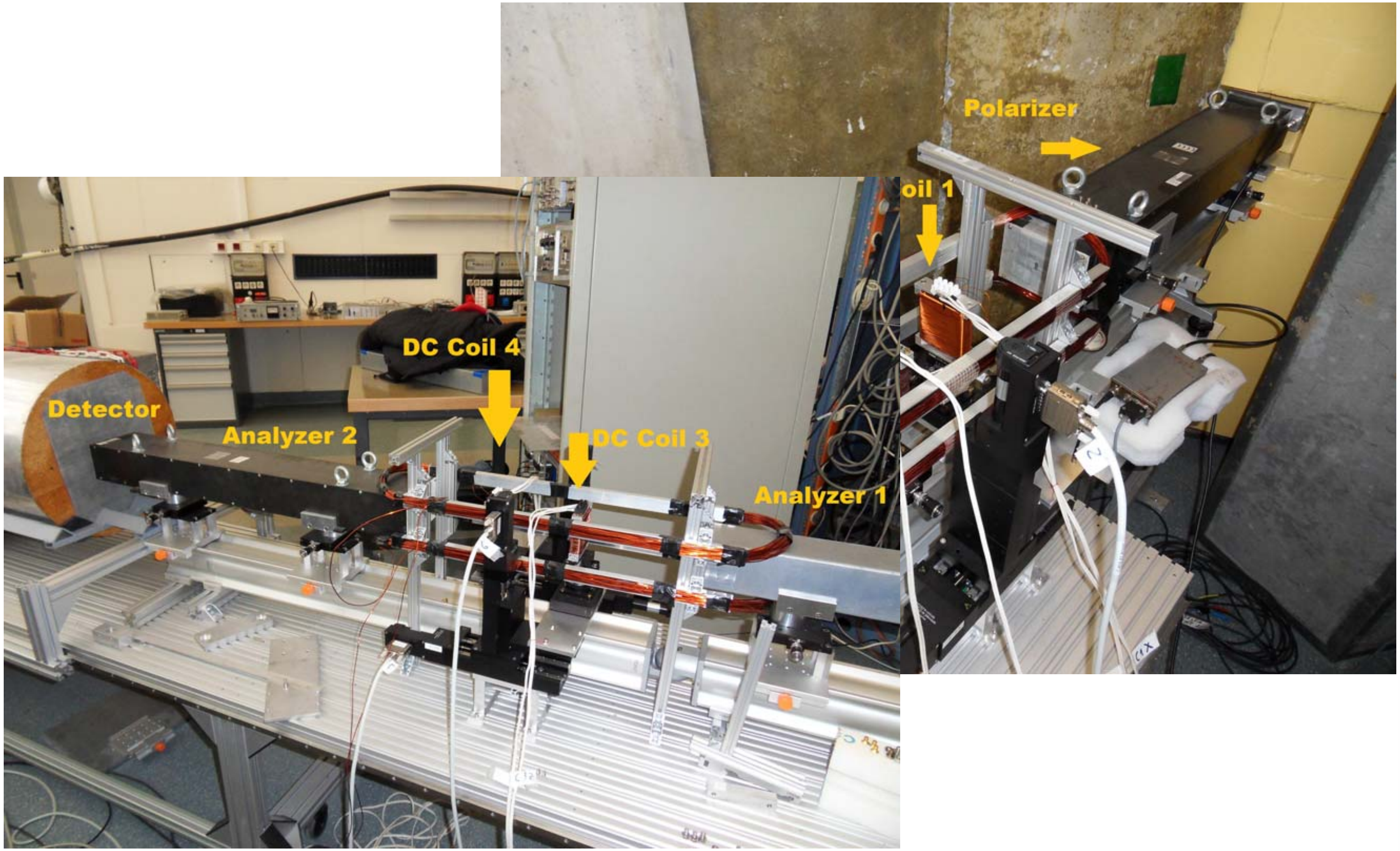
$$\dot{\vec{P}}(t) = \vec{P}(t) \times \gamma \vec{B}(t)$$







Experimental setup

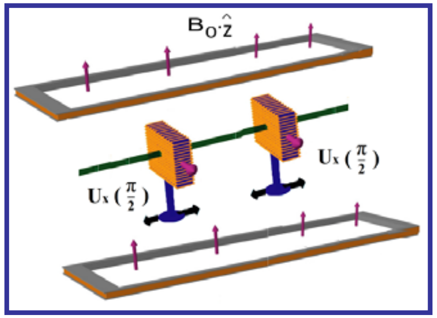


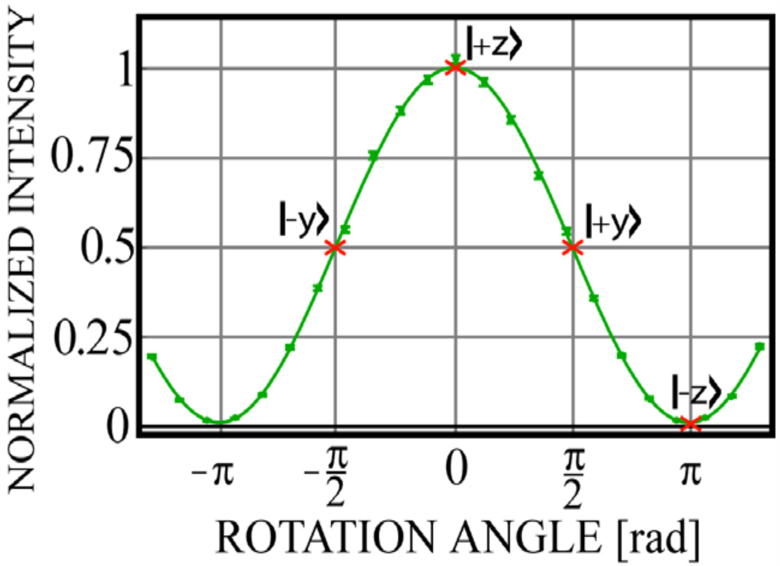
Adjustment

Unitary transformation: U=e$^{i\alpha\sigma/2}$

Determination of U($\pm\pi$)
- Scanning of the current in x-direction

Determination of U($\pm\pi/2$)
- Position scanning

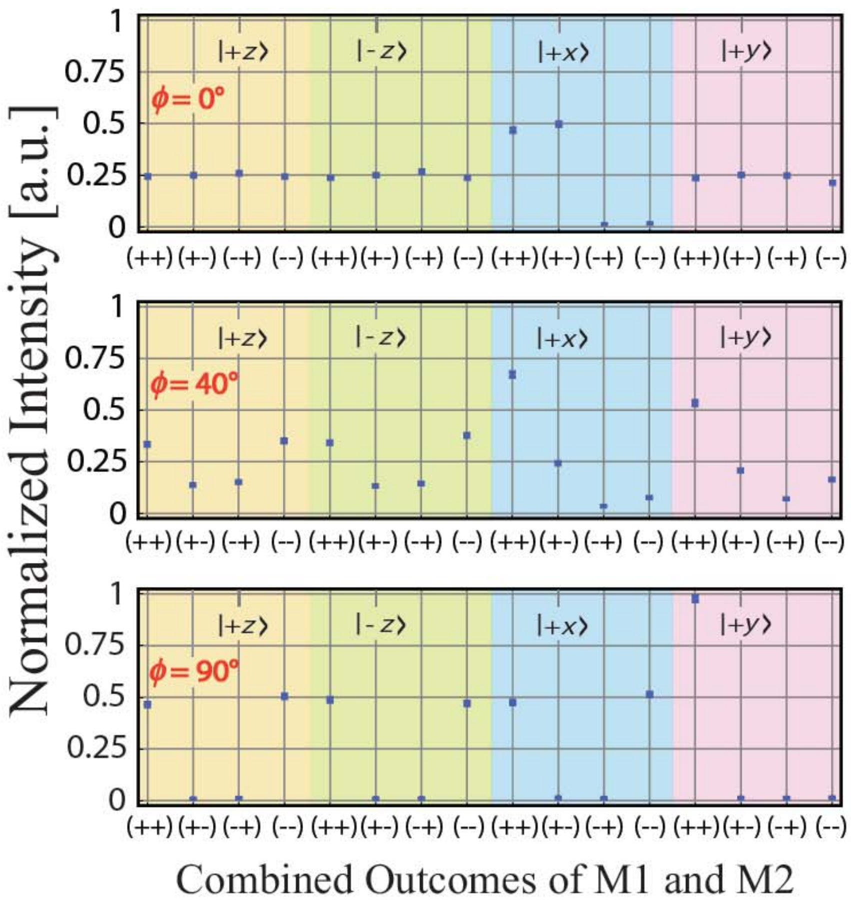




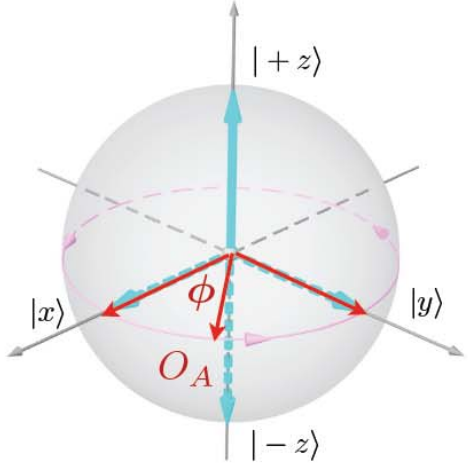
Contrast of each DC coil adjustement: ~98%

Contrast of the whole system: ~96%

Experimental data



$$A = \sigma_x$$
$$B = \sigma_y$$



$$O_A = \sigma_\phi = \sigma_x \sin\phi + \sigma_y \cos\phi$$

$$\frac{(I_{++} + I_{+-}) - (I_{-+} + I_{--})}{I_{++} + I_{-+} + I_{+-} + I_{--}} = \langle\psi_i|O_A|\psi_i\rangle$$

$$\frac{(I_{++} + I_{-+}) - (I_{+-} + I_{--})}{I_{++} + I_{-+} + I_{+-} + I_{--}} = \langle\psi_i|X_B|\psi_i\rangle$$

$$|\psi_i\rangle = |+z\rangle,\ |-z\rangle,\ |x\rangle,\ |y\rangle$$

$$\epsilon(A)^2 = 2 + \langle +z|\sigma_\phi|+z\rangle + \langle -z|\sigma_\phi|-z\rangle - \langle x|\sigma_\phi|x\rangle$$
$$\eta(B)^2 = 2 + \langle +z|X_B|+z\rangle + \langle -z|X_B|-z\rangle - \langle y|X_B|y\rangle$$

Experimental determination

where I_+ and I_- represent the positive and negative projections

$$\sigma^2(\hat{\sigma_j}) = \langle \hat{\sigma_j}^2 \rangle - \langle \hat{\sigma_j} \rangle^2 = 1 - (\langle \sigma_+ \rangle - \langle \sigma_- \rangle)^2 = 1 - \left(\frac{I_+ - I_-}{I_+ + I_-} \right)^2$$

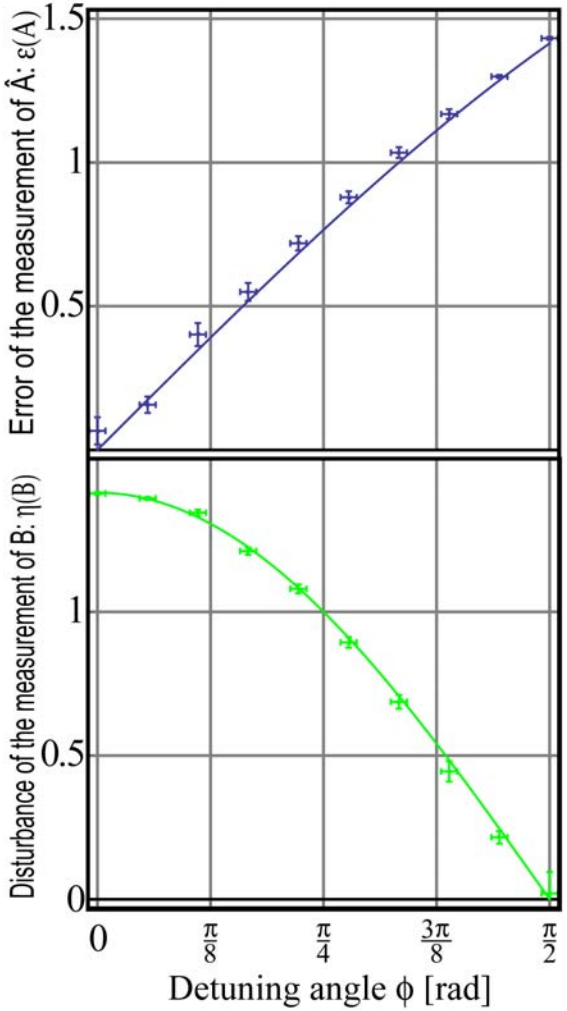
Projection operator: $\vec{P} := \langle \psi | \vec{\sigma} | \psi \rangle$

Error of A (projective measurment):

$$\epsilon(A)^2 = \underbrace{\langle \psi | A^2 | \psi \rangle}_{1} + \underbrace{\langle \psi | O_A^2 | \psi \rangle}_{1} + \langle \psi | O_A | \psi \rangle + \langle A\psi | O_A | A\psi \rangle - \langle (A + \mathbb{I})\psi | O_A | (A + \mathbb{I})\psi \rangle$$

$$\langle \psi | O_A | \psi \rangle = \frac{(I_{++} + I_{+-}) - (I_{-+} + I_{--})}{\sum I}$$

Results: error-disturbance trade-off



$$|\psi_i\rangle = |+z\rangle$$

$$\hat{A} = \hat{\sigma_x} \quad \hat{O}_A = \hat{\sigma}_\phi = \cos(\phi)\hat{\sigma}_x + \sin(\phi)\hat{\sigma}_y$$

$$\hat{B} = \hat{\sigma_y}$$

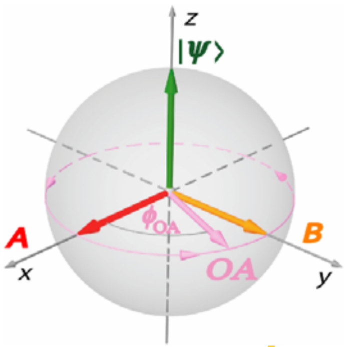
$$\varepsilon(A)^2 = \langle\psi|A^2|\psi\rangle + \langle\psi|O_A^2|\psi\rangle + \langle\psi|O_A|\psi\rangle + \langle A\psi|O_A|A\psi\rangle - \langle(A+I)\psi|O_A|(A+I)\psi\rangle$$

$$|\psi\rangle = |+z\rangle \qquad |\psi\rangle = |+z\rangle$$

$$|A\psi\rangle = |-z\rangle \qquad |B\psi\rangle = |-z\rangle$$

$$|(A+\mathbb{I})\psi\rangle = |+x\rangle \qquad |(B+\mathbb{I})\psi\rangle = |+y\rangle$$

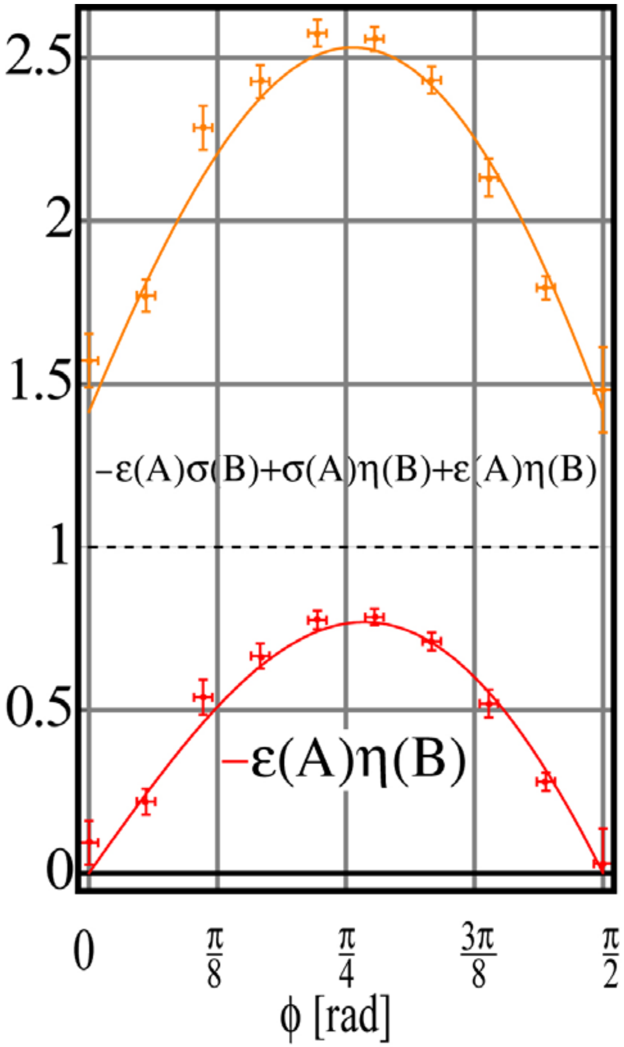
Results: new/old uncertainty relation



New uncertainty principle

$$\begin{cases} \varepsilon : \text{error of the first measurmen} \, (A) \\ \eta : \text{disturbance on the second measurement} \, (B) \\ \sigma : \text{standard deviations} \end{cases}$$

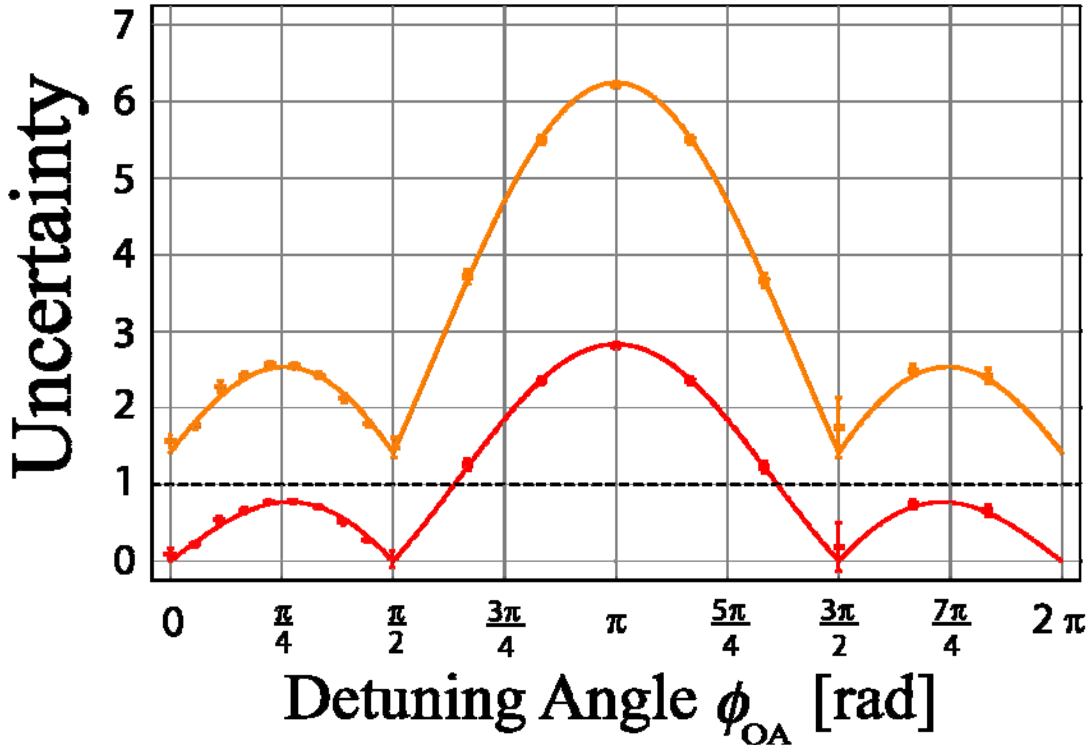
standard deviations:
σ(B)= 0.9999(1)
σ(A)= 0.9994(3)

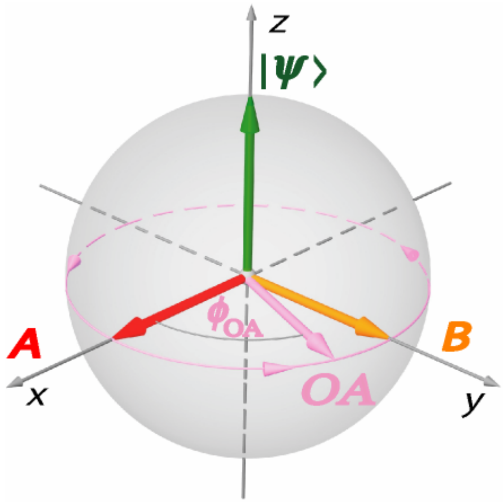


Heisenberg product

J. Erhart et al., Nature Phys. **8**, 185-189 (2012)

Results 1: incident spin-state (|s>=|θ,ϕ>)





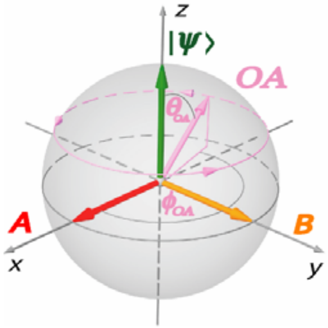
$$\left|s\right\rangle = \left|\theta = 0, \phi = 0\right\rangle$$

— $\varepsilon(A)\,\eta(B)$

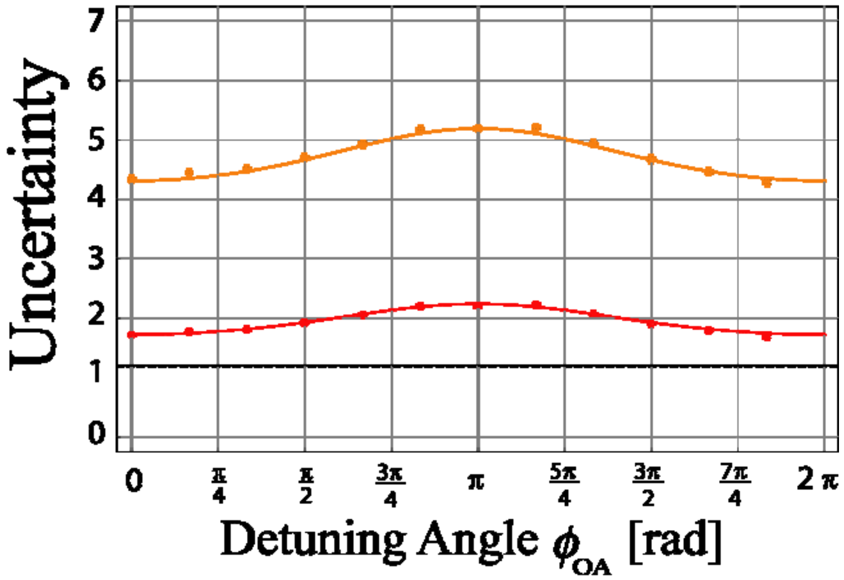
— $\varepsilon(A)\,\eta(B)+\varepsilon(A)\sigma(B)+\sigma(A)\eta(B)$

--- Heisenberg lower limit

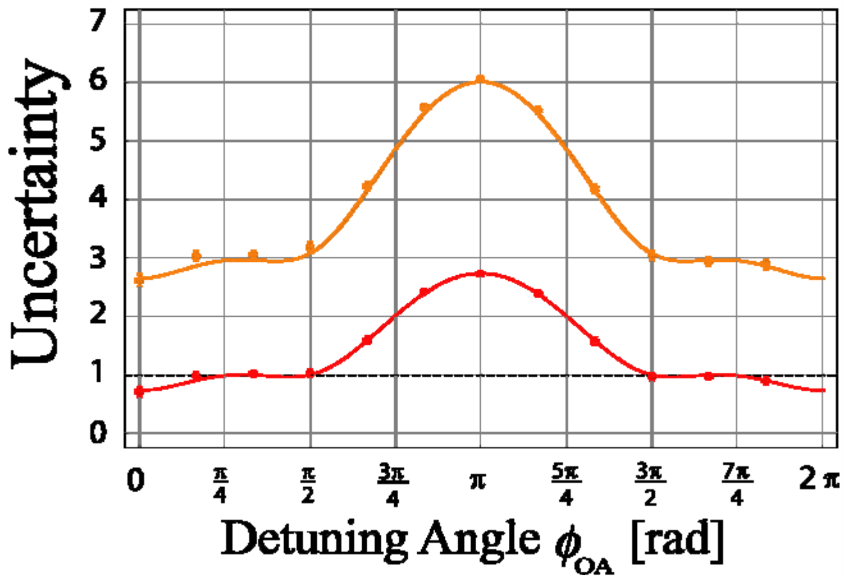
Results 2: polar angle of O_A [$\theta(O_A)$]



$\theta(O_A) = \pi/12$



$\theta(O_A) = \pi/3$

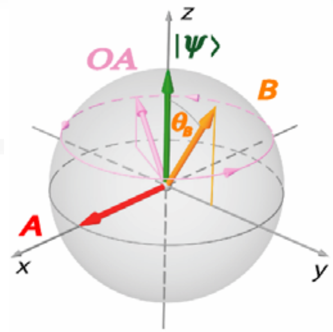


New sum is always above border!

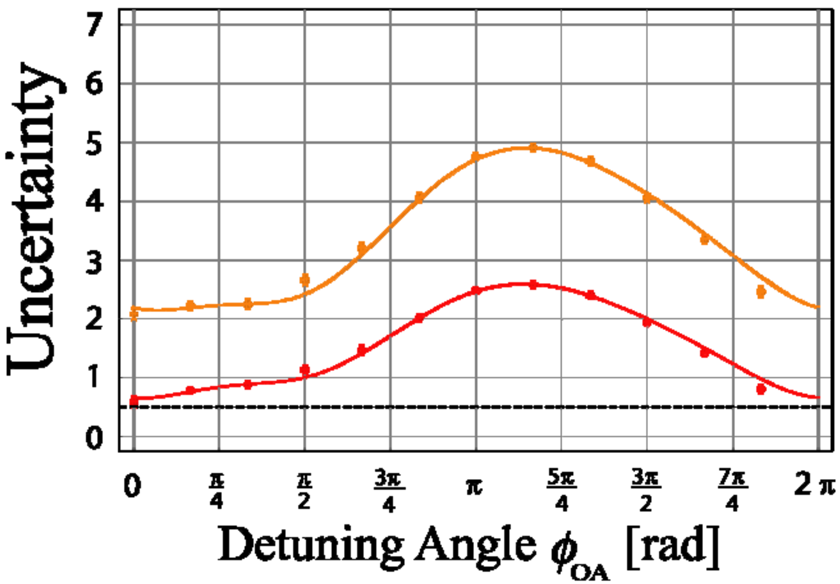
— $\varepsilon(A)\,\eta(B)$
— $\varepsilon(A)\,\eta(B) + \varepsilon(A)\sigma(B) + \sigma(A)\eta(B)$
--- Heisenberg lower limit

FWF

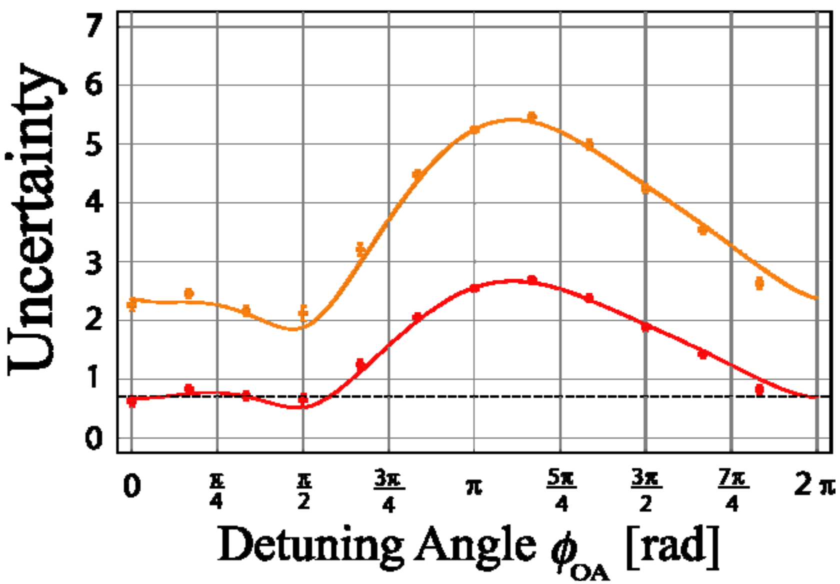
Results 3: polar angle of B [θ(B)]



$\theta(B) = \pi/6$



$\theta(B) = \pi/4$

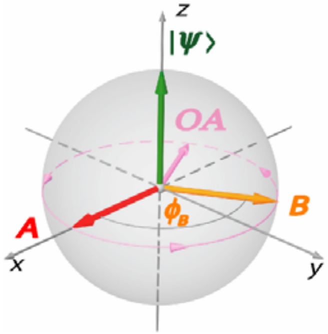


Asymmetry appears!

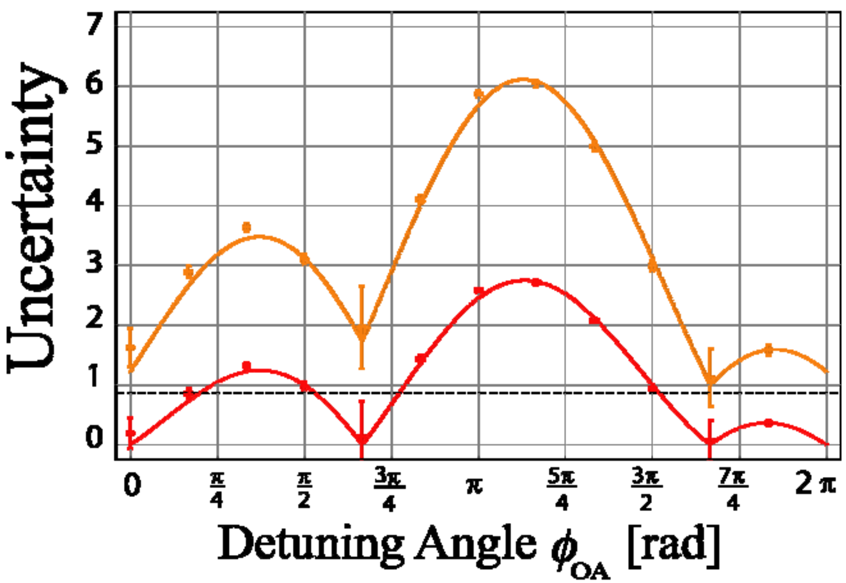
— $\varepsilon(A)\eta(B)$
— $\varepsilon(A)\eta(B)+\varepsilon(A)\sigma(B)+\sigma(A)\eta(B)$
--- Heisenberg lower limit

ATI FWF

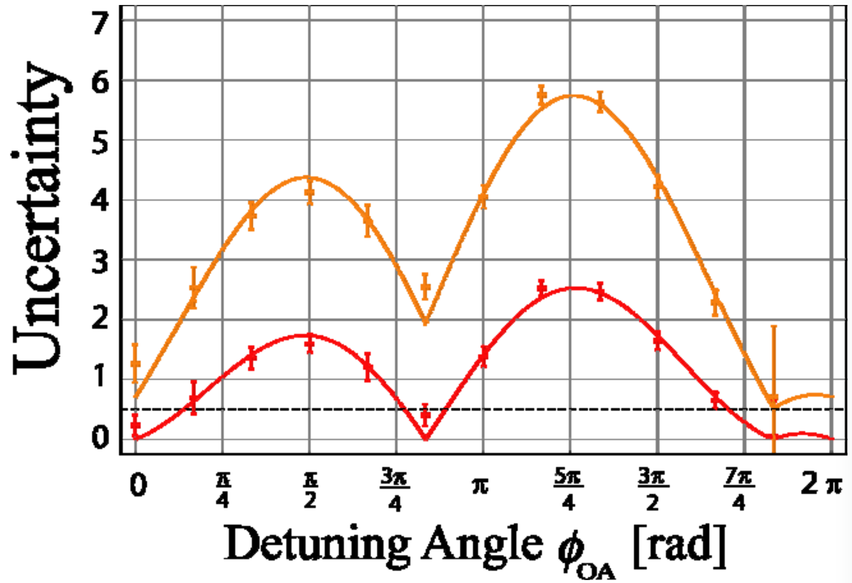
Results 4: azimuthal angle of B [ϕ(B)]



ϕ(B) = 2π/3



ϕ(B) = 5π/6



Sum touches the border!

— $\varepsilon(A)\,\eta(B)$
— $\varepsilon(A)\,\eta(B)+\varepsilon(A)\sigma(B)+\sigma(A)\eta(B)$
--- Heisenberg lower limit

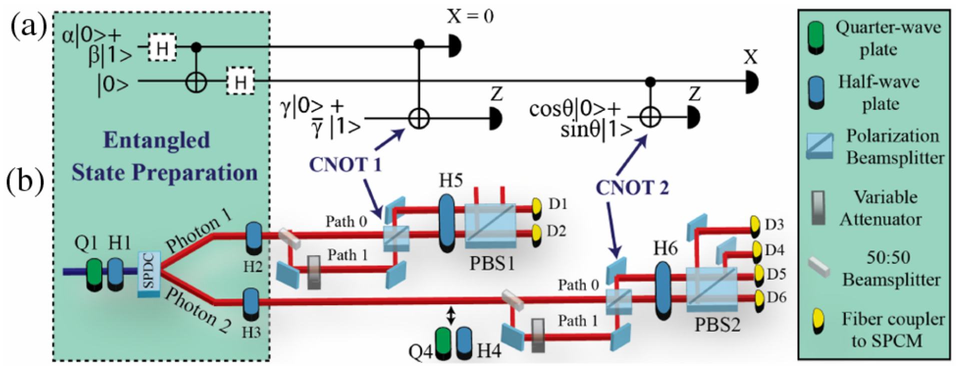
Publications by other groups 1

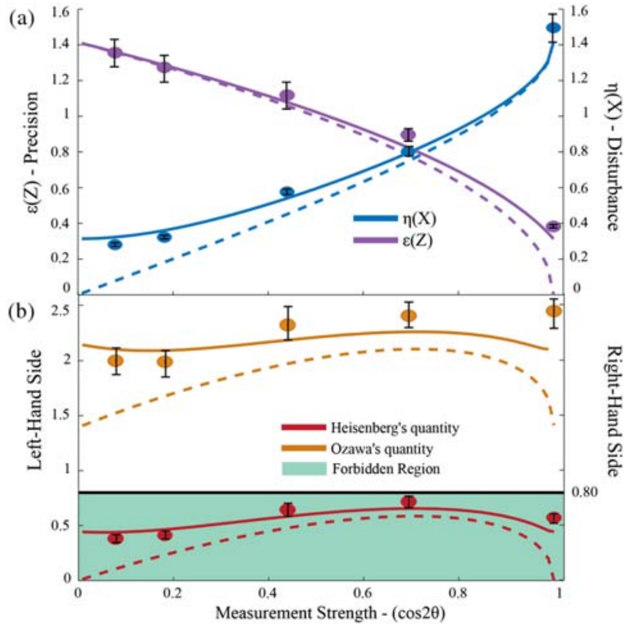
PRL **109,** 100404 (2012) PHYSICAL REVIEW LETTERS week ending 7 SEPTEMBER 2012

Violation of Heisenberg's Measurement-Disturbance Relationship by Weak Measurements

Lee A. Rozema, Ardavan Darabi, Dylan H. Mahler, Alex Hayat, Yasaman Soudagar, and Aephraim M. Steinberg
Centre for Quantum Information & Quantum Control and Institute for Optical Sciences, Department of Physics, 60 St. George Street, University of Toronto, Toronto, Ontario, Canada M5S 1A7

(Received 4 July 2012; published 6 September 2012)





Publications by other groups 2

ArXiv; 1211.0370

Experimental test of universal joint measurement uncertainty relations

Morgan M. Weston, Michael J. W. Hall, Matthew S. Palsson, Howard M. Wiseman and Geoff J. Pryde
Centre for Quantum Computation and Communication Technology (Australian Research Council),
Centre for Quantum Dynamics, Griffith University, Brisbane, QLD 4111, Australia

The principle of complementarity is fundamental to quantum mechanics, and restricts
of accuracy with which incompatible quantum observables can be jointly measured. Des
concep
we rep
known
tarity r
qubits,
the uni
the me
relation

ArXiv; 1304.2071

How well can one jointly measure two incompatible observables on a given quantum state?

Cyril Branciard
Centre for Engineered Quantum Systems and School of Mathematics and Physics,
The University of Queensland, St Lucia, QLD 4072, Australia

(Dated: April 9, 2013)

Heisenberg's uncertainty principle is one of the main tenets of quantum theory. Nevertheless, and despite its fundamental importance for our understanding of quantum foundations, there has been some confusion in its interpretation: although Heisenberg's first argument was that the measurement of one observable on a quantum state necessarily disturbs another incompatible observable, standard uncertainty relations typically bound the indeterminacy of the outcomes when either one or the other observable is measured. In this paper, we quantify precisely Heisenberg's intuition. Even if two incompatible observables cannot be measured together, one can still approximate their joint measurement, at the price of introducing some errors with respect to the ideal measurement of each of them. We present a new, tight relation characterizing the optimal trade-off between the error on one observable versus the error on the other. As a particular case, our approach allows us to characterize the disturbance of an observable induced by the approximate measurement of another one; we also derive a stronger error-disturbance relation for this scenario.

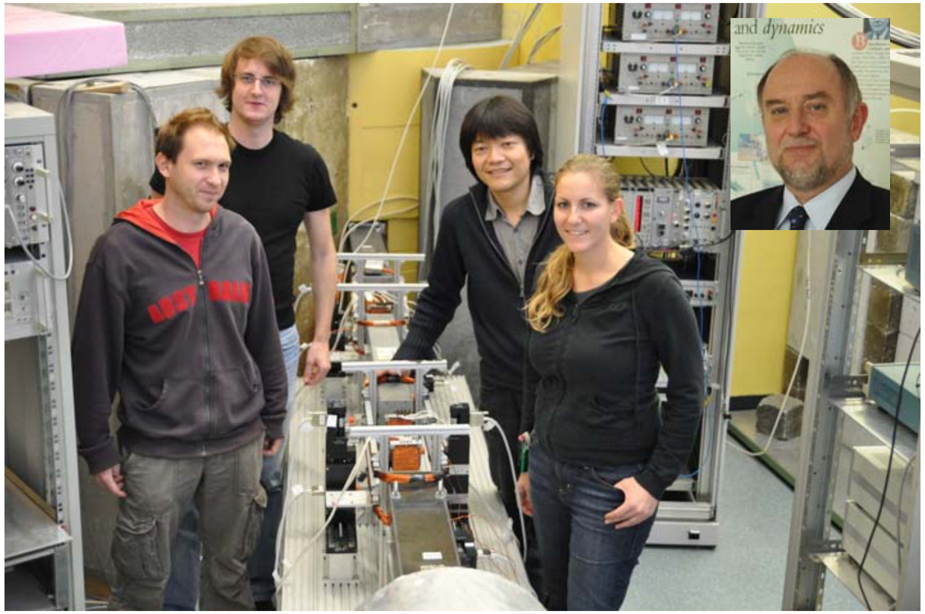
Concluding remarks: error-disturbance uncertain relation

Universally valid uncertainty-relation by Ozawa is experimentally tested!

- **Neutron's spin measurement confirmed the new error-disturbance uncertainty relation.**

- **New sum is always above the limit!**
  Heisenberg product is often below the limit!

- **Error & disturbance are determined from data.**
  Projective measurements are exploited.



Thank you very much for your attention

J. Erhart et al., Nature Phys. **8**, 185-189 (2012)