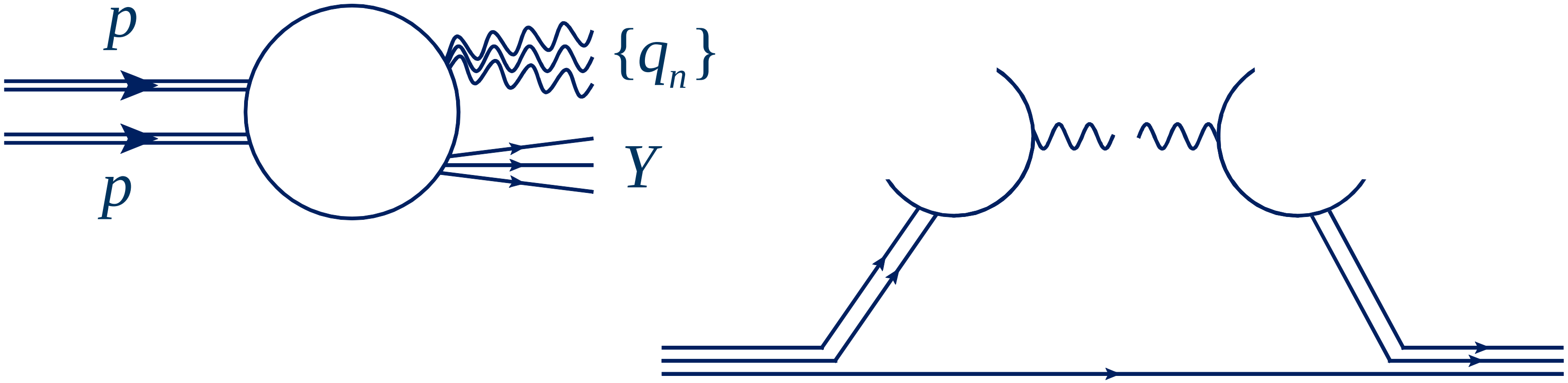

Double PDFs and new positivity constraints

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Content

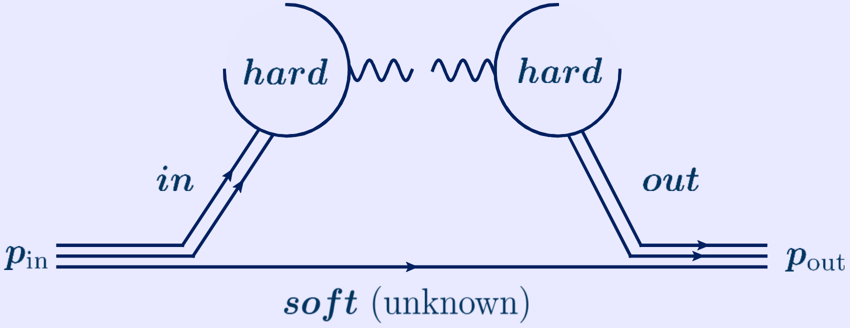
- The **double-parton interaction** in contrast to the single-parton interaction
  - → Motivation
  - → Correlators | Generalization from single to double
- New **double-parton Soffer bounds (positivity constraints)**
  - → Review on single-parton Soffer bounds
  - → Derivation of double-parton Soffer bounds

Motivation

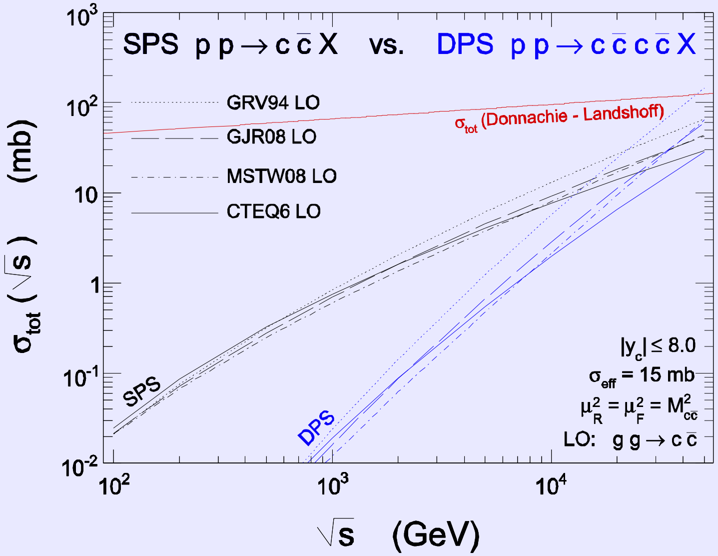
- Proton-proton scattering and optical theorem

- Double Parton Scattering (DPS) at $\sqrt{s} \approx 10^4 \mathrm{GeV}$ becomes more prominent

  → typical scale at LHC



Soft part of the proton-proton scattering after the optical theorem



Total LO cross section for $pp \to c\bar{c}X$ for single parton scattering (SPS) and double parton scattering (DPS)

[Łuszczak, Maciuła, Szczurek | 2012]

Single-Parton Correlator

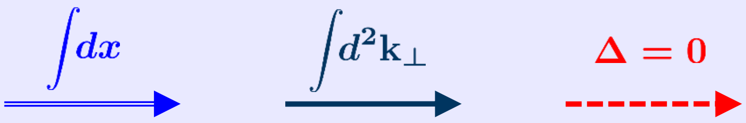
- Single-parton correlator | proton-proton scattering

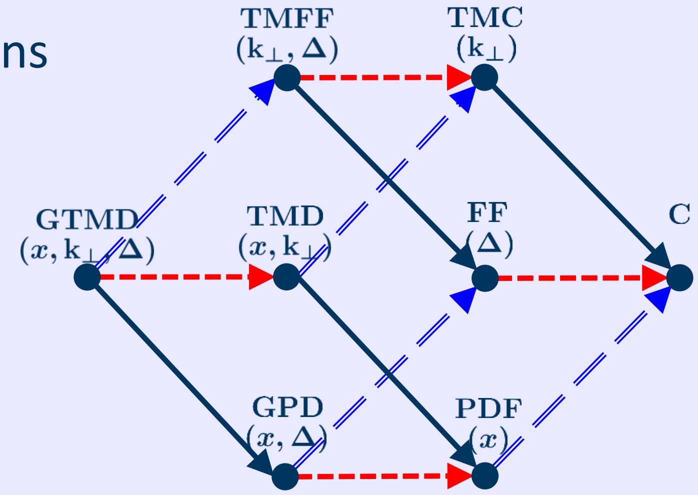
$$f_{\alpha\beta}^{[\Gamma]}(x,\mathbf{k}_\perp,\Delta) \equiv \frac{1}{2}\int\frac{dz^-}{2\pi}\int\frac{d^2\mathbf{z}_\perp}{(2\pi)^2}\, e^{ixz^-\bar{p}^+}e^{-i\ell_\perp\mathbf{z}_\perp}\langle p',\Lambda'|:\bar{\psi}_\alpha(-\tfrac{z}{2})\Gamma\psi_\beta(\tfrac{z}{2}):|p,\Lambda\rangle$$

→ Longitudinal **momentum fraction** $x \equiv k^+/\bar{p}^+$
→ Relative **transverse momentum** $\mathbf{k}_\perp \equiv \bar{\ell}_\perp - x\bar{\mathbf{p}}_\perp$
→ Proton **momentum transfer** $\Delta \equiv p' - p$
→ Expansion in **large p^+** $\Gamma \in \{\gamma^+, \sigma^{+1}, \sigma^{+2}, \gamma^+\gamma^5\}$

- Different reductions lead to different functions

e.g. [Lorcé, Pasquini, Vanderhaeghen | 2011]

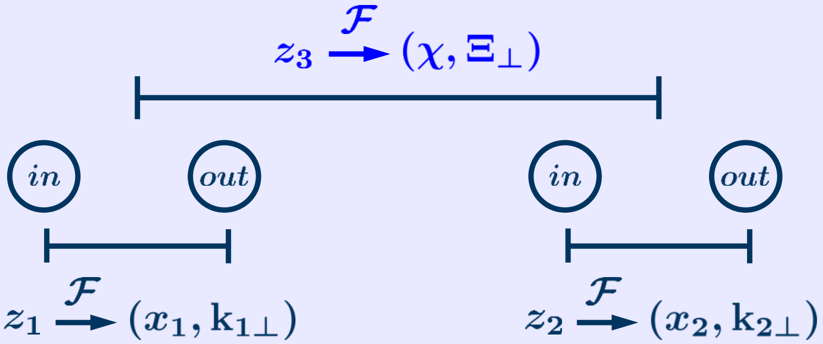




Double-Parton Correlator

- **Double-parton correlator** $f^{[\Gamma_1\Gamma_2]}_{\alpha\beta\gamma\delta}(x_1, x_2, \mathrm{k}_{1\perp}, \mathrm{k}_{2\perp}, \chi, \Xi_\perp, \xi, \Delta_\perp)$

- New: **momentum transfer** between **active partons**



$z_3 \xrightarrow{\mathcal{F}} (\chi, \Xi_\perp)$

in *out* *in* *out*

$z_1 \xrightarrow{\mathcal{F}} (x_1, \mathrm{k}_{1\perp})$ $z_2 \xrightarrow{\mathcal{F}} (x_2, \mathrm{k}_{2\perp})$

- **Symmetry** under the exchange of the active partons

$$\begin{aligned} &f^{[\Gamma_1\Gamma_2]}_{\alpha\beta\gamma\delta}(x_1, x_2, \mathrm{k}_{1\perp}, \mathrm{k}_{2\perp}, \chi, \Xi_\perp, \xi, \Delta_\perp) \\ =&f^{[\Gamma_2\Gamma_1]}_{\gamma\delta\alpha\beta}(x_2, x_1, \mathrm{k}_{2\perp}, \mathrm{k}_{1\perp}, -\chi, -\Xi_\perp, \xi, \Delta_\perp) \end{aligned}$$

- **Double PDFs**: $f^{[\Gamma_1\Gamma_2]}_{\alpha\beta\gamma\delta}(x_1, x_2)$

Single-Parton Soffer Bound

- Three independent Single PDFs

$$f_1^q(x) \equiv f_{qq}^{[\gamma^+]}(x) \qquad g_1^q(x) \equiv f_{qq}^{[\gamma^+\gamma^5]}(x) \qquad h_1^q(x) \equiv \frac{1}{2}\left(f_{qq}^{[\sigma^{+1}]}(x) + f_{qq}^{[\sigma^{+2}]}(x)\right)$$

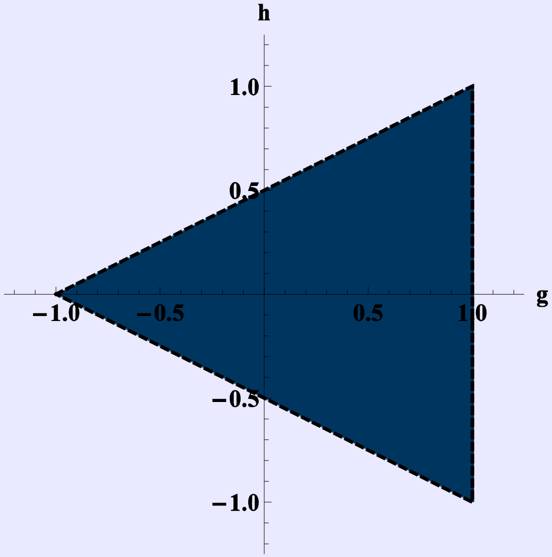
- Rewrite PDFs with **helicity amplitudes** $|\lambda, \Lambda\rangle$

→ positive semi-definite matrix $|+,+\rangle$, $|+,-\rangle$, $|-,+\rangle$, $|-,-\rangle$

$$\mathcal{A}^q = \frac{1}{4}\begin{pmatrix} f_1^q + g_1^q & 0 & 0 & 2h_1^q \\ 0 & f_1^q - g_1^q & 0 & 0 \\ 0 & 0 & f_1^q - g_1^q & 0 \\ 2h_1^q & 0 & 0 & f_1^q + g_1^q \end{pmatrix}$$

- Constraints on PDFs → **“Soffer bound”**

$$f_1^q(x) \geq 0$$
$$f_1^q(x) \geq |g_1^q(x)|$$
$$2|h_1^q(x)| \leq f_1^q(x) + g_1^q(x)$$



Graphical representation of the Soffer bound for $f_1^q(x) = 1$

Double-Parton Bounds I

- Seven (almost) independent Double PDFs

$$f_2(x_1,x_2) \equiv f^{[\gamma^+,\gamma^+]}(x_1,x_2) \qquad \tilde{f}_2(x_1,x_2) \equiv f^{[\gamma^+\gamma^5,\gamma^+\gamma^5]}(x_1,x_2)$$

$$f_{T2}(x_1,x_2) \equiv \frac{1}{2}\left(f^{[\sigma^{+1},\sigma^{+1}]}(x_1,x_2) + f^{[\sigma^{+2},\sigma^{+2}]}(x_1,x_2)\right)$$

$$g_2^{\alpha}(x_1,x_2) \equiv f^{[\gamma^+,\gamma^+\gamma^5]}(x_1,x_2) \qquad \beta : \gamma^+ \leftrightarrow \gamma^+\gamma^5$$

$$h_2^{\alpha}(x_1,x_2) \equiv \frac{1}{2}\left(f^{[\gamma^+,\sigma^{+1}]}(x_1,x_2) + f^{[\gamma^+,\sigma^{+2}]}(x_1,x_2)\right) \qquad \beta : \gamma^+ \leftrightarrow \sigma^{+i}$$

- Rewrite DPDFs with **helicity amplitudes**

→ 8 × 8 positive semi-definite matrix → use $\mathcal{P}$

$$|{+}{+},{+}\rangle\,,\ |{-}{+},{-}\rangle\,,\ |{-}{-},{+}\rangle\,,\ |{+}{-},{-}\rangle\,,$$

$$\cancel{|{-}{-},{-}\rangle}\,,\ \cancel{|{+}{-},{+}\rangle}\,,\ \cancel{|{+}{+},{-}\rangle}\,,\ \cancel{|{-}{+},{+}\rangle}$$

$$\mathcal{O}^{qq} := \begin{pmatrix} f_2 + g_2^{\alpha} + g_2^{\beta} + \tilde{f}_2 & 2h_2^{\alpha} & 2h_2^{\beta} & 0 \\ 2h_2^{\alpha} & f_2 + g_2^{\alpha} - g_2^{\beta} - \tilde{f}_2 & 2f_{T2} & 0 \\ 2h_2^{\beta} & 2f_{T2} & f_2 - g_2^{\alpha} + g_2^{\beta} - \tilde{f}_2 & 0 \\ 0 & 0 & 0 & f_2 - g_2^{\alpha} - g_2^{\beta} + \tilde{f}_2 \end{pmatrix}$$

Double-Parton Bounds II

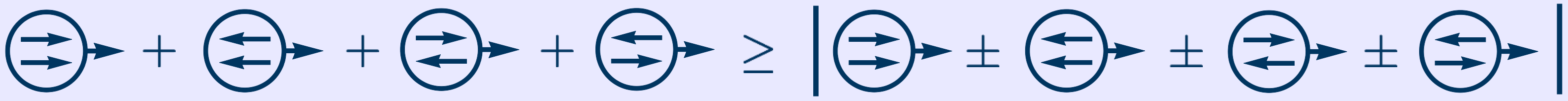
- Reduced 4 × 4 positive semi-definite matrix

$$|{+}{+},{+}\rangle\,,\ |{-}{+},{-}\rangle\,,\ |{-}{-},{+}\rangle\,,\ |{+}{-},{-}\rangle$$

$$\mathcal{O}^{qq} := \begin{pmatrix} \boxed{f_2 + g_2^\alpha + g_2^\beta + \tilde{f}_2} & 2h_2^\alpha & 2h_2^\beta & 0 \\ 2h_2^\alpha & \boxed{f_2 + g_2^\alpha - g_2^\beta - \tilde{f}_2} & 2\,f_{T2} & 0 \\ 2h_2^\beta & 2f_{T2} & \boxed{f_2 - g_2^\alpha + g_2^\beta - \tilde{f}_2} & 0 \\ 0 & 0 & 0 & \boxed{f_2 - g_2^\alpha - g_2^\beta + \tilde{f}_2} \end{pmatrix}$$

- **“naïve bounds”** for DPDFs

$$f_2(x_1,x_2) \geq 0,$$
$$f_2(x_1,x_2) \geq |g_2^\alpha(x_1,x_2)| \text{ and } f_2(x_1,x_2) \geq |g_2^\beta(x_1,x_2)|,$$
$$f_2(x_1,x_2) \geq |\tilde{f}_2(x_1,x_2)|$$



Graphical representation of the naïve bounds for DPDFs

Double-Parton Bounds III

- Reduced 4 × 4 positive semi-definite matrix

$$|{+}{+},{+}\rangle\,,\ |{-}{+},{-}\rangle\,,\ |{-}{-},{+}\rangle\,,\ |{+}{-},{-}\rangle$$

$$\mathcal{O}^{qq} := \begin{pmatrix} f_2 + g_2^\alpha + g_2^\beta + \tilde{f}_2 & 2h_2^\alpha & 2h_2^\beta & 0 \\ 2h_2^\alpha & f_2 + g_2^\alpha - g_2^\beta - \tilde{f}_2 & 2f_{T2} & 0 \\ 2h_2^\beta & 2f_{T2} & f_2 - g_2^\alpha + g_2^\beta - \tilde{f}_2 & 0 \\ 0 & 0 & 0 & f_2 - g_2^\alpha - g_2^\beta + \tilde{f}_2 \end{pmatrix}$$

- **New bounds** for double- and single-PDFs

$$4(f_{T2}(x_1,x_2))^2 \leq (f_2(x_1,x_2) - \tilde{f}_2(x_1,x_2))^2 - (g_2^\alpha(x_1,x_2) - g_2^\beta(x_1,x_2))^2$$

Double-Parton Bounds III

- Reduced 4 × 4 positive semi-definite matrix

$$|{+}{+},{+}\rangle\,,\ |{-}{+},{-}\rangle\,,\ |{-}{-},{+}\rangle\,,\ |{+}{-},{-}\rangle$$

$$\mathcal{O}^{qq} := \begin{pmatrix} f_2 + g_2^\alpha + g_2^\beta + \tilde{f}_2 & 2h_2^\alpha & 2h_2^\beta & 0 \\ 2h_2^\alpha & f_2 + g_2^\alpha - g_2^\beta - \tilde{f}_2 & 2f_{T2} & 0 \\ 2h_2^\beta & 2f_{T2} & f_2 - g_2^\alpha + g_2^\beta - \tilde{f}_2 & 0 \\ 0 & 0 & 0 & f_2 - g_2^\alpha - g_2^\beta + \tilde{f}_2 \end{pmatrix}$$

- **New bounds** for double- and single-PDFs

$$4(f_{T2}(x_1,x_2))^2 \leq (f_2(x_1,x_2) - \tilde{f}_2(x_1,x_2))^2 - (g_2^\alpha(x_1,x_2) - g_2^\beta(x_1,x_2))^2$$

$$4(h_2^{\alpha,\beta}(x_1,x_2))^2 \leq (f_2(x_1,x_2) + g_2^{\alpha,\beta}(x_1,x_2))^2 - (\tilde{f}_2(x_1,x_2) + g_2^{\beta,\alpha}(x_1,x_2))^2$$

- Remember: single-parton Soffer bound $2|h_1^q(x)| \leq f_1^q(x) + g_1^q(x)$

Double-Parton Bounds III

- Reduced 4 × 4 positive semi-definite matrix
  $|{+}{+},{+}\rangle\,,\ |{-}{+},{-}\rangle\,,\ |{-}{-},{+}\rangle\,,\ |{+}{-},{-}\rangle$

$$\mathcal{O}^{qq} := \begin{pmatrix} \boxed{f_2 + g_2^\alpha + g_2^\beta + \tilde{f}_2} & 2h_2^\alpha & \boxed{2h_2^\beta} & 0 \\ 2h_2^\alpha & f_2 + g_2^\alpha - g_2^\beta - \tilde{f}_2 & 2f_{T2} & 0 \\ \boxed{2h_2^\beta} & 2f_{T2} & \boxed{f_2 - g_2^\alpha + g_2^\beta - \tilde{f}_2} & 0 \\ 0 & 0 & 0 & f_2 - g_2^\alpha - g_2^\beta + \tilde{f}_2 \end{pmatrix}$$

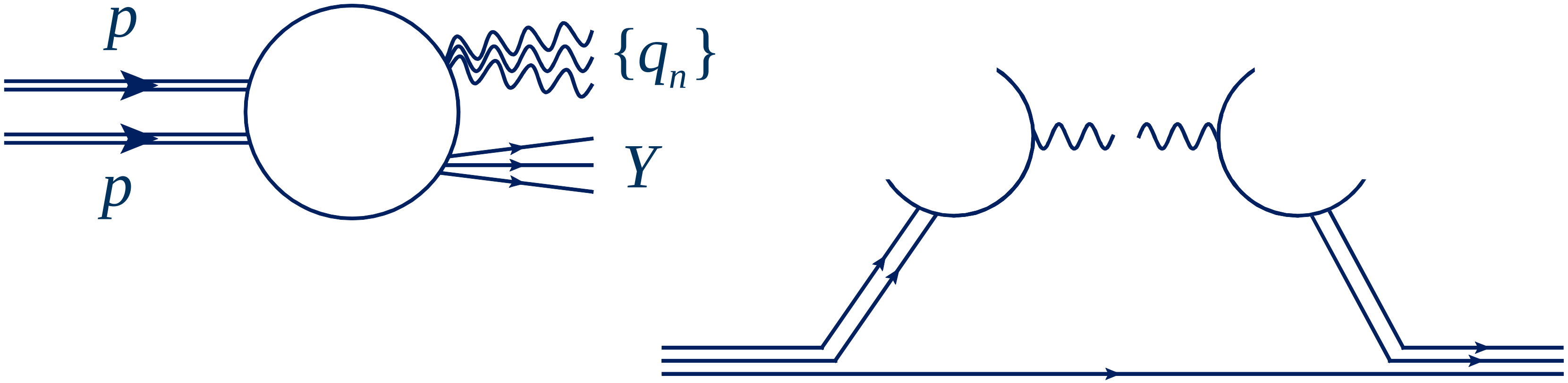
- **New bounds** for double- and single-PDFs

$$4(f_{T2}(x_1,x_2))^2 \leq (f_2(x_1,x_2) - \tilde{f}_2(x_1,x_2))^2 - (g_2^\alpha(x_1,x_2) - g_2^\beta(x_1,x_2))^2$$
$$4(h_2^{\alpha,\beta}(x_1,x_2))^2 \leq (f_2(x_1,x_2) + g_2^{\alpha,\beta}(x_1,x_2))^2 - (\tilde{f}_2(x_1,x_2) + g_2^{\beta,\alpha}(x_1,x_2))^2$$

- Remember: single-parton Soffer bound $2|h_1^q(x)| \leq f_1^q(x) + g_1^q(x)$

Summary

- **Double-parton correlator**
  - → New double-parton functions
  - → **Relations** between **single-parton** and **double-parton functions**
- New **Soffer-like positivity constraints**
  - → Reduction of the single-parton positivity constraints
  - → Eventually smaller region for PDFs



Thanks for your attention!

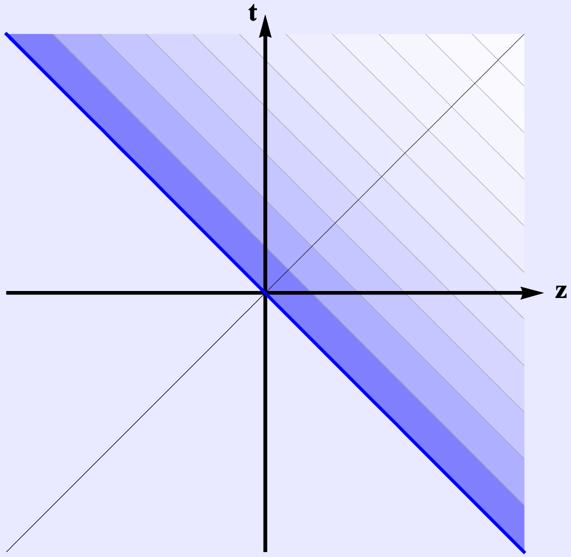
Light-Front Notations

- Other arrangement of variables | mixture of known time and space coordinates

$$x^{\pm} \equiv \frac{1}{\sqrt{2}}(x^0 \pm x^3) \qquad x_{\perp} \equiv (x_1, x_2)$$

- x^+ chosen as light-front time
- New metric to raise and lower indices

$$\eta^{\mu\nu} \xrightarrow{LF} \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$



Graphical representation of the front form

- Different scalar product

$$a^{\mu}b_{\mu} = a^+b^- + a^-b^+ - a_{\perp}b_{\perp}$$

- Positivity for massive particles | no square roots

$$p^- = \frac{M^2 + p_{\perp}^2}{2p^+} \geq 0$$

$$\rightarrow \gamma^+ \equiv \frac{1}{\sqrt{2}}(\gamma^0 + \gamma^3)$$

Lepage-Brodsky Spinors

- Lepage-Brodsky spinors | Dirac equation $(m - \not{p})u = 0 \;\; (m + \not{p})v = 0$

$$u(p,\lambda) \equiv \left(\sqrt{2}p^+\right)^{-\frac{1}{2}} \left(\sqrt{2}p^+ + \gamma_0 m + \gamma_0 \gamma_\perp p_\perp\right) \times \begin{cases} \chi(\uparrow) & \text{for } \lambda = +1 \\ \chi(\downarrow) & \text{for } \lambda = -1 \end{cases}$$

$$v(p,\lambda) \equiv \left(\sqrt{2}p^+\right)^{-\frac{1}{2}} \left(\sqrt{2}p^+ - \gamma_0 m + \gamma_0 \gamma_\perp p_\perp\right) \times \begin{cases} \chi(\downarrow) & \text{for } \lambda = +1 \\ \chi(\uparrow) & \text{for } \lambda = -1 \end{cases}$$

- Twist 2 γ-matrices

Γ_2	$\frac{\bar{u}(q,\lambda)\Gamma u(p,\lambda)}{2\sqrt{p^+q^+}}$	$\frac{\bar{v}(q,\lambda)\Gamma v(p,\lambda)}{2\sqrt{p^+q^+}}$	$\frac{\bar{u}(q,-\lambda)\Gamma u(p,\lambda)}{2\sqrt{p^+q^+}}$	$\frac{\bar{v}(q,-\lambda)\Gamma v(p,\lambda)}{2\sqrt{p^+q^+}}$
γ^+	1	1	0	0
σ^{+1}	0	0	$i\lambda$	$-i\lambda$
σ^{+2}	0	0	-1	-1
$\gamma^+\gamma^5$	λ	$-\lambda$	0	0

- For quarks: $\gamma^+ \to 1,\; \sigma^{+1} \to \sigma^1,\; \sigma^{+2} \to \sigma^2,\; \gamma^+\gamma^5 \to \sigma^3$

Relations between the Correlators

- **Forward** correlator **reduction** at 3Q level $(\boldsymbol{\Delta = 0})$

$$\sum_{\lambda_\gamma\lambda_\delta} \delta^{f_\gamma f_\delta}\delta^{c_\gamma c_\delta}\int dx_2 d^2\mathrm{k}_{2\perp}\,{}^3f^{[\Gamma\gamma^+]}_{\alpha\beta\gamma\delta}(x,x_2,\mathrm{k}_\perp,\mathrm{k}_{2\perp},0,0_\perp) = {}^3f^{[\Gamma]}_{\alpha\beta}(x,\mathrm{k}_\perp)$$

- Constraints on longitudinal momentum

$$\begin{aligned}\tfrac{1}{2}|\chi-\xi| &\leq x_1 < 1-\tfrac{1}{2}|\chi+\xi|\\ \tfrac{1}{2}|\chi+\xi| &\leq x_2 < 1-\tfrac{1}{2}|\chi-\xi|\\ 0 &\leq x_3 < 1\end{aligned}$$

- **Off-forward** correlator **reduction** at 3Q level

$$\underbrace{\chi \equiv \frac{1}{2}((x_2-x_2')-(x_1-x_1')) \qquad\qquad \xi = \frac{1}{2}((x_2-x_2')+(x_1-x_1'))}$$

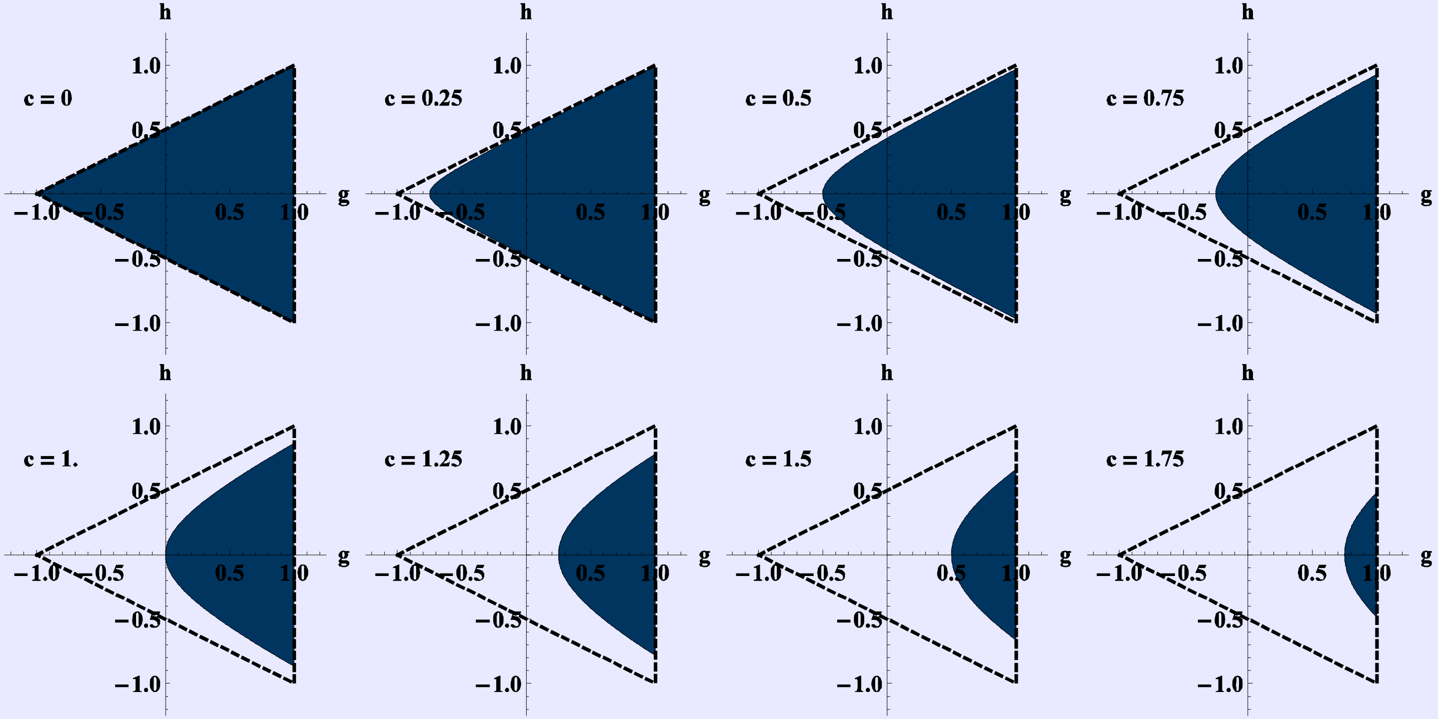
→ longitudinal $\chi \to -\xi$

→ transverse $\Xi_\perp \to \dfrac{1-\bar{x}_1+\bar{x}_2}{1-\xi^2}\dfrac{\Delta_\perp}{2}$

Relation between Single- and Double-Bounds

- Possible reduction of single-parton constraints

→ $4(h_2^\alpha)^2 \leq (f_2 + g_2^\alpha)^2 - (\tilde{f}_2 + g_2^\beta)^2 \leq (f_2 + g_2^\alpha)^2 - (cf_2)^2$



Graphical representation how the double-parton bounds reduce the possible region at $f_2^q(x_1, x_2) = 1$