Non–Abelian discrete R symmetries

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This work is based on my Diploma thesis and will be published soon.

Outline

Basics
- What is an R symmetry?
- Motivation for R symmetries

Superpotential and discrete symmetries
- R symmetry solution to the μ Problem
- Discrete flavor symmetries

Non–Abelian discrete R symmetries
- Symmetry scan
- A minimal example model
- Spontaneous CP violation

Conclusion

What is an R symmetry?

Internal symmetry that does *not* commute with SUSY

SUSY:

$$Q\,|\text{Boson}\rangle = |\text{Fermion}\rangle\;,$$
$$Q\,|\text{Fermion}\rangle = |\text{Boson}\rangle\;.$$

In general: Q_i $(i = 1, \ldots, \mathcal{N})$ can be charged under any internal symmetry B, i.e.

$$[Q_i, B] = (b)_i^j Q_j .$$

If so, we call B an R symmetry.

$\Leftrightarrow$ Does not commute with SUSY.

$\Leftrightarrow$ Superpartners charged differently.

throughout this talk: $\mathcal{N} = 1$ SUSY.

R symmetries in superspace

Chiral 'superfield' $\Phi \supset (\phi, \psi, F)$ on 'superspace' $(x, \theta, \theta^\dagger)$.
For a global R symmetry:

$$\Phi \to \mathrm{e}^{iq_\Phi \alpha}\Phi, \quad \theta \to \mathrm{e}^{iq_\theta \alpha}\theta.$$

For $\mathcal{L}$ being invariant under the symmetry, the superpotential $\mathcal{W}(\Phi_i)$ itself has to be charged, since:

$$\mathcal{L}(x) \supset \int \underbrace{d^2\theta}_{\to \mathrm{e}^{-i2q_\theta \alpha}} \underbrace{\mathcal{W}(\Phi_i)}_{\Rightarrow \mathrm{e}^{i2q_\theta \alpha}} + \mathrm{c.c.}\ .$$

This is the main distinction to non–R symmetries.

Motivation for R symmetries

Why are R symmetries interesting?

- Tight connection to SUSY breaking. [Nelson and Seiberg, 1994]
- 'Natural' solution to the μ problem with R symmetries.
- Not available in 4D GUTs. [Fallbacher et al., 2011]
  - But: Lorentz invariance of compact extra dimensions can lead to (anomaly free, discrete) R symmetries in 4D effective theory. e.g. [Lee et al., 2011b]
  - And: there are string theory examples which can reproduce the exact MSSM spectrum + discrete R symmetries at low energies. [Kappl et al., 2011]
- Instrumental in many aspects of model building (e.g. flavon VEV alignement).

R symmetry solution to the μ Problem

The MSSM superpotential, with effective B–L violating terms up to mass dimension 5:

$$
\begin{aligned}
\mathcal{W} = {} & \mu H_u H_d + \kappa_i L_i H_u \\
& + Y_e^{ij} H_d L_i E_j + Y_d^{ij} H_d Q_i \bar{D}_j + Y_u^{ij} H_u Q_i \bar{U}_j \\
& + \lambda^{(1)}_{ijk} L_i L_j \bar{E}_k + \lambda^{(2)}_{ijk} L_i Q_j \bar{D}_k + \lambda^{(3)}_{ijk} \bar{U}_i \bar{D}_j \bar{D}_k \\
& + \kappa^{(1)}_{ijkl} Q_i Q_j Q_k L_l + \kappa^{(2)}_{ijkl} \bar{U}_i \bar{U}_j \bar{D}_k \bar{E}_l \\
& + \kappa^{(3)}_{ijk} Q_i Q_j Q_k H_d + \kappa^{(4)}_{ijk} Q_i \bar{U}_j \bar{E}_k H_d + \kappa^{(5)}_i L_i H_u H_u H_d \\
& + \kappa^{(6)}_{ij} L_i H_u L_j H_u .
\end{aligned}
$$

- Severely constrained by absence of proton decay.
- Prohibited by R parity.
- Unexplained small.

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- Severely constrained by absence of proton decay.
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R Symmetry solution to the μ Problem

- 'Natural' solution: smallness of μ explained by symmetry.['t Hooft, 1980]
- Requiring:
  - Gauge coupling unification,
  - Anomaly freedom (allowing for Green–Schwarz mechanism),

  $\Rightarrow$ Only R symmetries can prohibit μ term.[Babu et al., 2003]
  There is a *unique* SO(10) commuting solution: $\mathbb{Z}_4^R$.

  [Lee et al., 2011a, Chen et al., 2012]
- The $\mathbb{Z}_4^R$ is broken non–pertubatively and one finds $\mu \sim m_{\not{SUSY}} \sim m_{3/2} \Rightarrow \mu$ problem solved.
- Also: $\kappa^{(1)}, \kappa^{(2)} \sim \frac{m_{3/2}}{M_P^2} \Rightarrow$ proton decay under control.
- R parity is contained as an unbroken $\mathbb{Z}_2$ subgroup of the $\mathbb{Z}_4^R$.
- Yukawa couplings and (effective) neutrino mass still allowed.

Other discrete Symmetries?

Discrete flavor symmetries:

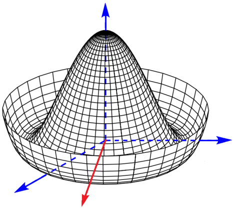
- Typically: non–Abelian discrete symmetry used as flavor symmetries G_f, e.g. $\mathrm{S}_3, \mathrm{A}_4, \mathrm{T}', \dots$

$$
\begin{aligned}
\mathcal{W} = {} & \mu H_u H_d + \kappa_i L_i H_u \\
& + Y_e^{ij} H_d L_i E_j + Y_d^{ij} H_d Q_i \bar{D}_j + Y_u^{ij} H_u Q_i \bar{U}_j \\
& + \lambda_{ijk}^{(1)} L_i L_j \bar{E}_k + \dots \\
& + \dots \\
& + \kappa_{ij}^{(6)} L_i H_u L_j H_u .
\end{aligned}
$$

Other discrete Symmetries?

Discrete flavor symmetries:

- Typically: non–Abelian discrete symmetry used as flavor symmetries $G_{\rm f}$, e.g. $\rm S_3, A_4, T', \dots$
- Spontaneous breaking of $G_{\rm f}$ by (MS)SM singlets ('flavons')
- To obtain mixing patterns (e.g. tri–bi–maximal), VEV alignment is crucial.



- For the dynamical generation of VEV alignment, there are two ways known:
  - $\mathrm{U}(1)_R$,[Altarelli and Feruglio, 2006]
  - compact extra dimensions.[Altarelli and Feruglio, 2005]

Discrete symmetries of the superpotential

$$\begin{aligned}\mathcal{W} = &\ \mu H_u H_d + \kappa_i L_i H_u \\ &+ Y_e^{ij} H_d L_i E_j + Y_d^{ij} H_d Q_i \bar{D}_j + Y_u^{ij} H_u Q_i \bar{U}_j \\ &+ \lambda^{(1)}_{ijk} L_i L_j \bar{E}_k + \lambda^{(2)}_{ijk} L_i Q_j \bar{D}_k + \lambda^{(3)}_{ijk} \bar{U}_i \bar{D}_j \bar{D}_k \\ &+ \kappa^{(1)}_{ijkl} Q_i Q_j Q_k L_l + \kappa^{(2)}_{ijkl} \bar{U}_i \bar{U}_j \bar{D}_k \bar{E}_l \\ &+ \kappa^{(3)}_{ijk} Q_i Q_j Q_k H_d + \kappa^{(4)}_{ijk} Q_i \bar{U}_j \bar{E}_k H_d + \kappa^{(5)}_i L_i H_u H_u H_d \\ &+ \kappa^{(6)}_{ij} L_i H_u L_j H_u .\end{aligned}$$

- Mainly affected by discrete flavor symmetry.
- Mainly affected by discrete R symmetry.

? Can we combine the discrete symmetries **?**

$$\left.\begin{array}{l}\text{Non–Abelian discrete} \\ + \ \text{discrete } R\end{array}\right\} \Rightarrow \text{Non–Abelian discrete } R \text{ symmetry.}$$

Non–Abelian discrete R symmetries

- We require the symmetry to:
  - Contain the succesfull $\mathbb{Z}_4^R$ solution of the μ problem,
  - Act non–trivially in flavor space.
- In order not to break SUSY at the flavor scale, the R symmetry needs to overstay the flavor symmetry breaking.
  - From mixing phenomenology: unbroken $\mathbb{Z}_4^R$ *cannot* be family dependent.
  - $\Rightarrow$ Only possible way: $\mathbb{Z}_4^R$ lies in the center of the non–Abelian group!

`GAP` scan:

$\mathcal{O}(G)$	G_f
24	$\mathbb{Z}_3 \rtimes \mathbb{Z}_8$
24	$\mathrm{S}_3 \times \mathbb{Z}_4$
32	$(\mathbb{Z}_8 \times \mathbb{Z}_2) \rtimes \mathbb{Z}_2$
32	$(\mathbb{Z}_4 \times \mathbb{Z}_4) \rtimes \mathbb{Z}_2$
32	$\mathbb{Z}_8 \rtimes \mathbb{Z}_4$
	...

A $\mathbb{Z}_3 \rtimes \mathbb{Z}_8^R$ minimal example model

The group $\mathbb{Z}_3 \rtimes \mathbb{Z}_8$:

- 8 singlet and 4 doublet representations $\mathbf{1}_{1-8}$, $\mathbf{2}_{1-4}$.
- Possible breaking:

$$\mathbb{Z}_3 \rtimes \mathbb{Z}_8$$

$$\langle \mathbf{1}_2 \rangle, \langle \mathbf{2}_2 \rangle \longrightarrow \mathbb{Z}_4 \qquad\qquad \langle \mathbf{2}_2 \rangle \propto (1,1) \longrightarrow \mathbb{Z}_8$$

Minimal example model:

- Just one set of SSB flavons: ϕ in $\mathbf{1}_2$, χ in $\mathbf{2}_2$.
- Neutrino masses from (effective) Weinberg operator.
- Spare any additional ‘shaping’ symmetries (commonly used to generate hierarchies and split quarks & leptons).

$\Rightarrow$ Do not expect perfect phenomenology, but able to discuss general issues such as:

- Consistent assignment of fields to representations,
- Symmetry breaking,
- Calculation of leading order mass matrices,
- discussion of $\not{CP}$,
- anomaly cancellation.

A $\mathbb{Z}_3 \rtimes \mathbb{Z}_8^R$ minimal example model

The possible assignments are fixed by the assignment of the underlying $\mathbb{Z}_4^R$.

- $\mathbb{Z}_4^R$ charges: θ and matter: 1, Higgses: 0.[Chen et al., 2012]

$\Rightarrow$ θ in $\mathbf{1}_5$ or $\mathbf{1}_8$,
Matter in $\mathbf{1}_5, \mathbf{1}_8$ or $\mathbf{2}_4$,
Higgses in $\mathbf{1}_1$ and $\mathbf{1}_2$.

- Assuming SU(5) GUT compability and neglecting 'anarchic' assignments: 24 possible assignments.
- One example:

$(Q, \bar{U}, \overline{E})_{1,2}$	$(Q, \bar{U}, \bar{E})_3$	$(L, \bar{D})_{1,2}$	$(L, \bar{D})_3$	H_u	H_d	χ	ϕ	θ
$\mathbf{1}_8$	$\mathbf{1}_5$	$\mathbf{2}_4$	$\mathbf{1}_5$	$\mathbf{1}_1$	$\mathbf{1}_2$	$\mathbf{2}_2$	$\mathbf{1}_2$	$\mathbf{1}_5$

As usual: $(Q_i, \bar{U}_i, \overline{E}_i) = \mathbf{10}, (L_i, \bar{D}_i) = \bar{\mathbf{5}}$ of SU(5)

A $\mathbb{Z}_3 \rtimes \mathbb{Z}_8^R$ minimal example model

Spontaneous symmetry breaking:

- Assume VEVs:

$$\langle\chi\rangle \;=\; v\begin{pmatrix}\cos\theta_\chi\\ \sin\theta_\chi\end{pmatrix} \quad \text{and} \quad \langle\phi\rangle \;=\; v\,r_\phi\;,$$

  in general: no alignment necessary for $G_\mathrm{f} \to \mathbb{Z}_4^R$.

- Identify terms consistent with all symmetries ($\mathcal{W}$ charged!).

↷ Yukawa coupling (mass) matrix structures

$$Y_e \sim Y_d^T \sim \begin{pmatrix}\varepsilon\delta & \varepsilon & 1\\ \varepsilon\delta & \varepsilon & 1\\ \varepsilon & \varepsilon\delta & \varepsilon\end{pmatrix}, Y_u \sim \begin{pmatrix}1 & 1 & \varepsilon\\ 1 & 1 & \varepsilon\\ \varepsilon & \varepsilon & 1\end{pmatrix}, M_\nu \sim \begin{pmatrix}\varepsilon & \varepsilon\delta & \varepsilon\delta\\ \varepsilon\delta & \varepsilon & \varepsilon\\ \varepsilon\delta & \varepsilon & 1\end{pmatrix}.$$

with $\varepsilon := v/\Lambda$ and $\delta := \theta_\chi - \pi/4$

Model features:

- 28 model parameters for (max.) 22 physical parameters (no shaping symmetries and no GUT relations used so far).
- Anomaly free by Green–Schwarz mechanism.

Spontaneous CP violation

Generalized CP transformation (gCP):

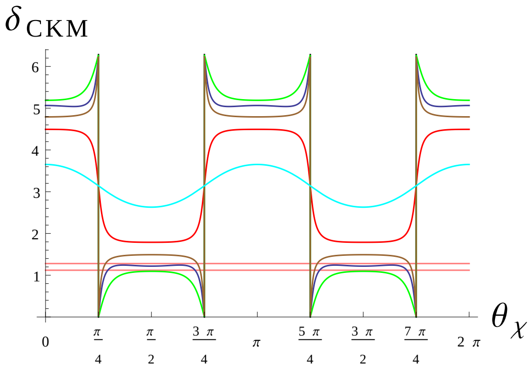
- $\equiv$ Usual CP trafo + non-trivial permutation of fields

$$\Phi \xrightarrow{\text{gCP}} V\,\Phi^* \,,$$

- gCP transformations represented by the *outer automorphism group* 'Out' of a flavor symmetry. [Holthausen et al., 2012]
  ('Out' is the symmetry of the character table)
- $\mathrm{Out}(\mathbb{Z}_3 \rtimes \mathbb{Z}_8^R) = \mathbb{Z}_2 \times \mathbb{Z}_2$, in our model: *both* $\mathbb{Z}_2$'s are broken explicitly and maximally.
- But: the combined transformation *can* be conserved.
- gCP is conserved (by assumption) $\Leftrightarrow$ phase relations for Yukawa couplings.
- Crucial: the gCP trafo includes a swap symmetry $\chi_1 \leftrightarrow \chi_2$, this symmetry is spontaneously broken by a misaligned VEV.
- From mixing and $\not{C}\!\not{P}$: VEV has to be misaligned!

Spontaneous CP violation

Check this statement numerically (e.g. for quark sector):



five random set of parameters with $y \in [0.01 \ldots 0.5]$, $\varepsilon = 0.2$.

Conclusion

- We have introduced non–Abelian discrete R symmetries in the framework of $\mathcal{N} = 1$ SUSY GUTs.
- A search for viable symmetries that include
  - solution to the μ and proton decay problem of the MSSM,
  - possible explanation of the flavor structure,

  reveals $\mathbb{Z}_3 \rtimes \mathbb{Z}_8^R$ as the smallest possible group with non–trivial embedding of the R symmetry.
- Generic features of such models are:
  - the Abelian R symmetry lies in the center of the group,
  - VEVs have to be misaligned to explain family mixing and $\not{CP}$.

Outlook:

- The construction of possibly realistic models seems feasible. In such models, one can discuss:
  - The dynamics of the spontaneous symmetry breaking,
  - Implications for $\not{\text{SUSY}}$, i.e. scalar sector soft masses,
  - Possible UV origin of the symmetry.

Thank You!

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Backup slides

R symmetries in superspace

Chiral 'superfield' $\Phi \supset (\phi, \psi, F)$ on 'superspace' $(y^\mu, \theta^\alpha, \theta^\dagger_{\dot\alpha})$

- $\theta^\alpha, \theta^\dagger_{\dot\alpha}$ fermionic coordinates - Grassmann numbers $(\alpha = 1, 2)$,
- $\Phi = \Phi(y, \theta, \theta^\dagger) = \phi(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y)$,

For a global R symmetry:

$$\Phi \rightarrow \mathrm{e}^{iq_\Phi \alpha}\Phi, \quad \theta \rightarrow \mathrm{e}^{iq_\theta \alpha}\theta.$$

This implies

$$\phi \rightarrow e^{iq_\Phi \alpha}\phi,\ \psi \rightarrow e^{i(q_\Phi - q_\theta)\alpha}\psi,\ F \rightarrow e^{i(q_\Phi - 2q_\theta)\alpha}F,$$

i.e. superpartners are charged differently.

Anomaly constraints for non–Abelian discrete R symmetries

Chiral superfields $\Phi^{(s)}$ transforming

- in representation $\boldsymbol{d}^{(s)}$ of a non–Abelian discrete R symmetry D,

$$\text{for } \Phi^{(s)} \;\to\; U_\mathsf{a}(\boldsymbol{d}^{(s)})\,\Phi^{(s)} \;=\; \mathrm{e}^{2\pi\,\mathrm{i}\,\lambda_\mathsf{a}(\boldsymbol{d}^{(s)})/M}\,\Phi^{(s)}\;,$$

$$\text{we define } \delta_\mathsf{a}^{(f)} \;:=\; \mathrm{tr}[\lambda_\mathsf{a}(\boldsymbol{d}^{(f)})] \;=\; \frac{M}{2\pi\,\mathrm{i}}\ln\det U_\mathsf{a}(\boldsymbol{d}^{(f)})\;.$$

- with charge $Q^{(s)}$ under Abelian factors of a gauge group G,
- in representation $\boldsymbol{r}^{(s)}$ under non–Abelian factors of G.

Anomaly coefficients of the a–generated subgroup $\mathbb{Z}_M$ of D are given as

$$A_{G-G-\mathbb{Z}^R_{M(\mathsf{a})}} \;=\; \sum_s \ell(\boldsymbol{r}^{(s)})\cdot\left[\delta^{(s)} - \mathsf{dim}(\boldsymbol{d}^{(s)})\,\delta^{(\theta)}\right] + \ell(\mathrm{adj}\,G)\cdot\delta^{(\theta)}\;,$$

$$A_{\mathrm{U}(1)-\mathrm{U}(1)-\mathbb{Z}^R_{M(\mathsf{a})}} \;=\; \sum_s (Q^{(s)})^2\,\dim(\boldsymbol{r}^{(s)})\cdot\left[\delta^{(s)} - \mathsf{dim}(\boldsymbol{d}^{(s)})\,\delta^{(\theta)}\right]\;,$$

$$A_{\mathrm{grav}-\mathrm{grav}-\mathbb{Z}^R_{M(\mathsf{a})}} \;=\; -21\,\delta^{(\theta)} + \delta^{(\theta)}\sum_G \dim(\mathrm{adj}\,G)$$

$$+\sum_s \dim(\boldsymbol{r}^{(s)})\cdot\left[\delta^{(s)} - \mathsf{dim}(\boldsymbol{d}^{(s)})\,\delta^{(\theta)}\right]\;.$$

Outer automorphism group ‘Out’

‘Out’ is the symmetry of the character table, i.e. the invariance under permutations of representations while at the same time permuting conjugacy classes.

	e	b	b^2	b^4	a	b^3	b^5	b^6	ab^2	ab^4	b^7	ab^6
	1	3	1	1	2	3	3	1	2	2	3	2
$\mathbb{Z}_3 \rtimes \mathbb{Z}_8$	1a	8a	4a	2a	3a	8b	8c	4b	12a	6a	8d	12b
$\mathbf{1}_1$	1	1	1	1	1	1	1	1	1	1	1	1
$\mathbf{1}_2$	1	-1	1	1	1	-1	-1	1	1	1	-1	1
$\mathbf{1}_3$	1	$-i$	-1	1	1	i	$-i$	-1	-1	1	i	-1
$\mathbf{1}_4$	1	i	-1	1	1	$-i$	i	-1	-1	1	$-i$	-1
$\mathbf{1}_5$	1	$-\tau$	i	-1	1	τ^*	τ	$-i$	i	-1	$-\tau^*$	$-i$
$\mathbf{1}_6$	1	τ^*	$-i$	-1	1	$-\tau$	$-\tau^*$	i	$-i$	-1	τ	i
$\mathbf{1}_7$	1	$-\tau^*$	$-i$	-1	1	τ	τ^*	i	$-i$	-1	$-\tau$	i
$\mathbf{1}_8$	1	τ	i	-1	1	$-\tau^*$	$-\tau$	$-i$	i	-1	τ^*	$-i$
$\mathbf{2}_1$	2	0	-2	2	-1	0	0	-2	1	-1	0	1
$\mathbf{2}_2$	2	0	2	2	-1	0	0	2	-1	-1	0	-1
$\mathbf{2}_3$	2	0	$-2i$	-2	-1	0	0	$2i$	i	1	0	$-i$
$\mathbf{2}_4$	2	0	$2i$	-2	-1	0	0	$-2i$	$-i$	1	0	i

Welche R Symmetrien sind möglich?

- Zusammen:

$$\begin{array}{l} \quad\text{Anomalienuniversalität} \\ +\ \text{verbotener } \mu\text{-Term} \\ +\ \text{re-induzierung, } q_R(H_u H_d) \stackrel{!}{=} 0 \\ \hline \end{array}$$

$$\Rightarrow\ \mathbb{Z}_M^R \text{ mit } M = 4 \times \mathbb{N} \text{ und } q_\theta = \tfrac{M}{4} \times \mathbb{N}$$
$$\Rightarrow\ q_{\mathcal{W}} = 2q_\theta = M/2 \text{ mod } M$$

$$\mathbb{Z}_4^R, \mathbb{Z}_8^R, \mathbb{Z}_{16}^R, \ldots$$

- Nur $M = 2^n$, alle anderen zerfallen z.B. $\mathbb{Z}_{12}^R \cong \mathbb{Z}_4^R \times \mathbb{Z}_3$

Backup

Für eine Grassmann Integration gilt

$$\int d^2\theta \ 1 = 0 \ \text{ und } \ \int d^2\theta \ \theta\theta = 1.$$

Wir können ein allgemeines Superfeld $S(y, \theta, \theta^\dagger)$ in den fermionischen Koordinaten θ^α und $\theta^\dagger_{\dot\alpha}$ entwickeln. Zum Beispiel für ein chirales Superfeld

$$\Phi(y, \theta, \theta^\dagger) = \phi(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y).$$

Eine Integration $\int d^2\theta\Phi$ würde also $F(y)$ isolieren. Man nennt dies einen F-Term Beitrag zu $\mathcal{L}$ und schreibt $[\Phi]_F$. Es stellt sich heraus, dass $[\mathcal{W}(\Phi_i)]_F$ genau die Terme eines herkömmlichen supersymmetrischen Potentiales enthält, d.h.

$$\mathcal{L}(x) \supset [\mathcal{W}(\Phi_i)]_F = \int d^2\theta \mathcal{W}(\Phi_i).$$

Mechanismen zur (wieder) Erzeugung des μ-Terms

i) Giudice-Masiero Giudice & Masiero

Mechanismen zur (wieder) Erzeugung des μ-Terms

i) Giudice-Masiero Giudice & Masiero

- μ-Term Aus dem Kähler-Potential

$$K \supset \kappa \frac{X^\dagger}{M_P} H_u H_d + h.c.$$

$$X^\dagger \to \langle X \rangle \Rightarrow \mu_{eff} = \kappa \frac{\langle X \rangle}{M_P} \sim m_{3/2}$$

- Entscheidende Annahme: F_X entspringt aus einem „Hidden-Sektor“, der die Gravitinomasse $m_{3/2}$ erzeugt, d.h. X ist das Feld, welches SUSY bricht

$$\Rightarrow \int d^2\theta X W_\alpha W^\alpha \text{Operator muss exisitieren} \Rightarrow q_R(X) = 0$$

- Und damit der obige Kählerterm erlaubt ist damit auch

$$\Rightarrow q_R(H_u H_d) = 0$$

Mechanismen zur (wieder) Erzeugung des μ-Terms

i) Giudice-Masiero Giudice & Masiero

ii) Kim-Nilles Kim & Nilles

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- μ-Term durch effektive Terme im Superpotential

$$\mathcal{W} \supset c_\Omega \frac{\Omega}{M_P^2} H_u H_d,$$

wobei Ω das Superpotential eines Hidden-Sektors ist. ($[\Omega] = 3$)

- Dieser Hidden-Sektor ist ebenfalls R-geladen, d.h. damit ein solcher Term zustande kommt muss auch hier gelten

$$q_R(H_u H_d) = 0.$$

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Mechanismen zur (wieder) Erzeugung des μ-Terms

i) Giudice-Masiero Giudice & Masiero

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iii) Nicht-perturbativ Dine et al., Lee et al.

- R-Symmetrie ist anomal, die Anomalie wird aufgehoben durch den Green-Schwarz Mechanismus
- $\Rightarrow$ es existiert ein Green-Schwarz Axion, als direkte Konsequenz tauchen Terme des Superpotentials mit $q_R = 0$ im nicht-perturbativen Teil des Superpotentials auf und werden so wiedereingeführt. („Retrofitting“)

Mechanismen zur (wieder) Erzeugung des μ-Terms

i) Giudice-Masiero Giudice & Masiero

ii) Kim-Nilles Kim & Nilles

iii) Nicht-perturbativ Dine et al., Lee et al.

Das heißt, unabhängig vom konkreten Mechanismus gilt

$$q_R(H_u H_d) = 0$$

Mechanismus zur (wieder) Erzeugung des μ-Terms

- R-Symmetrie ist anomal, die Anomalie wird aufgehoben durch ein Green-Schwarz Axion
- Dieses Axion koppelt nicht-perturbativ an Terme mit $q_R = 0$
- Brechung der R-Symmetrie falls das Axion einen VEV bekommt, z.B. durch einen Hidden Sektor
- Terme mit $q_R = 0$, welche zuvor verboten waren, werden re-induziert $\Rightarrow q_R(H_u H_d) \stackrel{!}{=} 0$
- Verbindung zu dynamischer SUSY Brechung, z.B. über Gaugino Kondensat

$$\mathcal{W} \supset \frac{\langle \mathcal{W}_h \rangle}{M_P^2} H_u H_d \Rightarrow \mu \sim \frac{\langle \mathcal{W}_h \rangle}{M_P^2} \sim m_{3/2}$$ Lee et al.

- μ-Problem gelöst, $\mu \sim m_{\cancel{SUSY}} \sim m_{3/2}$ auf natürliche Weise
- R-Parität als Untergruppe der gebrochenen R-Symmetrie erhalten

Einschränkungen für M

$$A_3 - A_2 = 0 \text{ mod } \eta \quad \Rightarrow \quad q_{H_u} + q_{H_d} = 4q_\theta \text{ mod } 2\eta$$

mit $q_{H_u} + q_{H_d} = 0$ mod η folgt,

$$4q_\theta \text{ mod } M = 0.$$

Für ungerade M: q_θ muss ein vielfaches von M oder 0 sein, d.h. θ und das Superpotential ist ungeladen und der μ-Term erlaubt.
Für gerade M folgt hingegen (alle Ladungen sind ganzzahllig):

$$q_\theta = \frac{M}{GCD(M,4)} \times \mathbb{Z}.$$

Die Einzigen wirklichen R-Symmetrien ergeben sich damit für θ-Ladungen

$$q_\theta = \frac{M}{4} \times \mathbb{Z} \Big\backslash \left\{ \frac{M}{2} \times \mathbb{Z} \right\}$$