Supersymmetric Holography on AdS_3

Michael Kech

IMPRS workshop

March 18, 2013

1. Introduction

2. Higher Spin Duality on AdS_3
   - The Original Proposal
   - Supersymmetric Holography on AdS_3

3. Partition Functions
   - CFT Partition Function
   - Gravity Partition Function

Section 1

Introduction

Higher Spin Duality

Duality between higher spin theories and CFTs

- Simplified version of AdS/CFT duality

Higher Spin Duality

Duality between higher spin theories and CFTs

- Simplified version of AdS/CFT duality
- Conceptual insight in AdS/CFT

Higher Spin Duality

Duality between higher spin theories and CFTs

- Simplified version of AdS/CFT duality
- Conceptual insight in AdS/CFT
- CFT methods become available for quantum gravity

Section 2

Higher Spin Duality on AdS_3

3d Version

Consider higher spin duality in three dimensions

- 2d CFTs are well understood

3d Version

Consider higher spin duality in three dimensions

- 2d CFTs are well understood
- Higher spin theories simplify in three dimensions

Subsection 1

The Original Proposal

3d Duality

Duality between one of Vasiliev's higher spin theories on AdS_3 containing a complex scalar and the $\mathcal{W}_N^k$ minimal models in the t'Hooft limit

- CFT coupling given by t'Hooft parameter λ

3d Duality

Duality between one of Vasiliev's higher spin theories on AdS_3 containing a complex scalar and the $\mathcal{W}_N^k$ minimal models in the t'Hooft limit

- CFT coupling given by t'Hooft parameter λ
- Mass of the scalar given by $M^2 = -(1 - \lambda^2)$

Minimal models

Minimal models are the coset theories

$$\mathcal{W}_N^k = \frac{\mathfrak{su}(N)_k \oplus \mathfrak{su}(N)_1}{\mathfrak{su}(N)_{k+1}}$$

where $\lambda = \frac{N}{N+k}$.

Higher Spin Theory

There is a description of higher spin theories analogous to the Chern-Simons formulation of gravity

$$S_{CS} = S[A] - S[\bar{A}], \quad S[A] = \frac{\hat{k}}{4\pi} \int Tr[A \wedge dA + \frac{2}{3} A \wedge A \wedge A]$$

with gauge group $hs(\lambda)$ instead of $\mathfrak{sl}(2)$.

Evidence

Much non-trivial evidence in favour of proposal based on

- Symmetry analysis

Evidence

Much non-trivial evidence in favour of proposal based on

- Symmetry analysis
- Correlation functions

Evidence

Much non-trivial evidence in favour of proposal based on

- Symmetry analysis
- Correlation functions
- Spectrum

Subsection 2

Supersymmetric Holography on AdS_3

The CFT

CFT is based on the supersymmetric coset

$$\frac{\mathfrak{su}(N+1)^1_{k+N+1}}{\mathfrak{su}(N)^1_{k+N+1} \oplus \mathfrak{u}(1)^1_\kappa}$$

This coset is one of the $\mathcal{N} = 2$ Kazama-Suzuki cosets.

The Gravity Theory

Higher spin theory is based on the superalgebra $shs(\lambda)$. $shs(\lambda)$ can be obtained from the associative algebra $A(\nu)$ generated by

$$[y_\alpha, y_\beta] = 2i\epsilon_{\alpha\beta}(1+\nu k), \quad \{k, y_\alpha\} = 0, \quad \alpha, \beta = 1, 2$$

by introducing a grading according to the powers of $y_{\alpha,\beta}$. ν is related to λ by $\lambda = \frac{1+\nu}{2}$.

Section 3

Partition Functions

Subsection 1

CFT Partition Function

Othogonal Version

Orthogonal version of supersymmetric coset is

$$\frac{\mathfrak{so}(2N+2)^1_{k+2N+2}}{\mathfrak{so}(2N)^1_{k+2N+2} \oplus \mathfrak{u}(1)^1_\kappa}.$$

This is isomorphic to the bosonic subalgebras and free Majorana fermions

$$\frac{\mathfrak{so}(2N+2)_k \oplus \mathfrak{so}(4N)_1}{\mathfrak{so}(2N)_{k+2} \oplus \mathfrak{u}(1)_\kappa}.$$

The Hilbert Space

The coset representations are given by the multiplicity spaces (Λ, Φ, I) in

$$\Lambda \otimes NS = \bigoplus_{I,\Phi} (\Lambda, \Phi, I) \otimes \Phi \otimes I.$$

The Hilbert space is defined by charge conjugation

$$\mathcal{H}_{CFT} = \bigoplus_{\Lambda,\Phi,I} (\Lambda, \Phi, I) \otimes \overline{(\Lambda, \Phi, I)}.$$

Branching Functions

To calculate partition function one has to find the characters of the multiplicity spaces (Λ, Φ, I).

Main ideas

- In t'Hooft limit affine characters simplify

Branching Functions

To calculate partition function one has to find the characters of the multiplicity spaces (Λ, Φ, I).

Main ideas

- In t'Hooft limit affine characters simplify
- Problem reduces to counting $\mathfrak{so}(2N)$ representations in a free theory

Branching Functions

To calculate partition function one has to find the characters of the multiplicity spaces (Λ, Φ, I).

Main ideas

- In t'Hooft limit affine characters simplify
- Problem reduces to counting $\mathfrak{so}(2N)$ representations in a free theory
- Result can be expressed in terms of $\mathfrak{gl}(\infty|\infty)_+$ characters

Subsection 2

Gravity Partition Function

Spectrum

- CFT partition function determines field content of higher spin theory

Spectrum

- CFT partition function determines field content of higher spin theory
- In the gauge sector the dual higher spin theory has to contain two $N = 2$ multiplets with lowest spin s for each spin $s = 1, 2, 3, \ldots$ and $s = 1/2, 5/2, 9/2, \ldots$

Spectrum

- CFT partition function determines field content of higher spin theory
- In the gauge sector the dual higher spin theory has to contain two $N = 2$ multiplets with lowest spin s for each spin $s = 1, 2, 3, ...$ and $s = 1/2, 5/2, 9/2, ...$
- Sums over $\mathfrak{gl}(\infty|\infty)_+$ characters can be interpreted in terms of massive fields. There are two complex scalars, two real scalars, two complex fermions and two real fermions

Matching the Spectrum

Goal is to construct a higher spin algebra that posses this spectrum.
Such an algebra can be obtained by restricting the Lie superalgebra obtained from

$$\mathfrak{gl}(2) \otimes A(\nu)$$

by an automorphism.

Conclusion

- Higher spin dualities on AdS_3 allow for strong consistency tests

Conclusion

- Higher spin dualities on AdS_3 allow for strong consistency tests
- Prove duality in t'Hooft limit (better understand scalar on gravity side)

Conclusion

- Higher spin dualities on AdS_3 allow for strong consistency tests
- Prove duality in t'Hooft limit (better understand scalar on gravity side)
- Understand relation between higher spin theories and string theories