How research is done in AdS/CFT

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Ringberg Schloß

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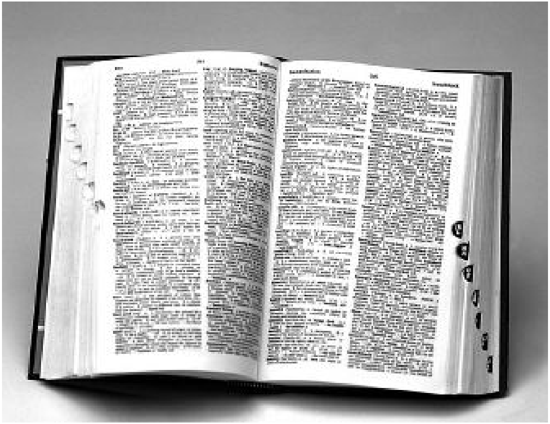
Outline

1. Present AdS/CFT both in an illustrative and in a rather technical way.
2. Explain why people work on AdS/CFT.
3. Explain how people work on AdS/CFT.
4. Provide concrete example.

What is AdS/CFT about?



AdS/CFT in a nutshell

Equivalent descriptions

Two different descriptions of one and the same thing provide two analogies.

$$A = B \quad \& \quad A = C \Rightarrow B = C$$

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AdS/CFT conjecture

$$\begin{array}{c} \mathcal{N} = 4 \ SU(N) \text{ SYM} \\ \text{Conformal field theory} \end{array} = \begin{array}{c} \text{Type IIB superstring theory} \\ \text{on } AdS_5 \times S^5 \end{array}$$

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$\mathcal{N} = 4$ $SU(N)$ SYM Conformal field theory	$=$	Type IIB superstring theory on $AdS_5 \times S^5$

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Take limit to make life simpler:

Easy and cool version of AdS/CFT

$$\begin{array}{ccc} \text{Einstein gravity} & & \mathcal{N}=4\ SU(N)\ \text{SYM} \\ \text{(consistent truncation)} & = & N \to \infty,\ \text{strongly coupled} \\ AdS\ \text{5d} & & \partial AdS\ \text{4d} \end{array}$$

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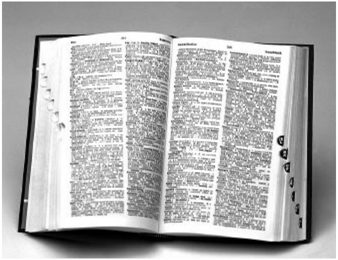
AdS/CFT dictionary

GRAVITY	GAUGE
A_μ	J^μ
$g_{\mu\nu}$	$T^{\mu\nu}$



AdS/CFT:how things are done

Matching of parameters

Which point of view is mathematically tractable depends on whether the gauge theory is strongly or weakly coupled:

GRAVITY THEORY	GAUGE THEORY
☺ easy	☹ hard
☹ hard	☺ easy

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There are two possible ways:

Top-down

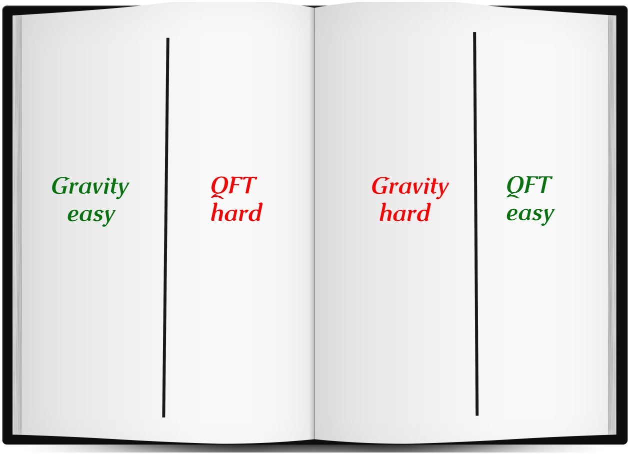
GRAVITY THEORY $\Longrightarrow$ QFT

Bottom-up

QFT $\Longrightarrow$ GRAVITY THEORY

AdS/CFT: bottomline

If nature provides such a beautiful dictionary, why not using it!?



AdS/CFT at work

Exploit duality to translate difficult unexplored physics into tractable problems.
AdS/CFT requires a large amount of symmetry, so either you explore very few things or you start being creative.
Different phenomena being explored: superconductivity, superfluidity, hydrodynamics, Kondo effect, quantum Hall effect,...

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Holography at work

1. Select a set up (tractable model which has appealing complicated translation):
2. Solve model, i.e. solve equations of motion.

$$\frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) = 0$$

3. Analyse results to extract information.

Actions in action

The dynamics is given by the action:

Superconductivity

$$\mathcal{L} \propto \sqrt{-g}\left(R - \Lambda - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - |\partial_\mu\psi - iqA_\mu\psi|^2 - m^2|\psi|^2\right)$$

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p-wave superconductivity, superconductor in $\vec{B}$

$$\mathcal{L} \propto \sqrt{-g}\left(R - \Lambda - \frac{1}{4}F^{(i)}_{\mu\nu}F^{\mu\nu}_{(i)}\right)$$

Always have metric (consistent truncation), gauge-charge + stuff

EOMs to solve

```
EqA =
  Collect[
   (2 R^2 ρ^3 f[ρ] h[ρ] f′[ρ] (χ^(0,1)[ρ, x] (-χ^(0,1)[ρ, x] A0^(1,0)[ρ, x] + 2 A0^(0,1)[ρ, x] χ^(1,0)[ρ, x]) + R^2 ρ^2 h[ρ] A0^(1,0)[ρ, x] (-1 + χ[ρ, x]^2 + ρ^2 χ^(1,0)[ρ, x]^2)) +
      h[ρ] (4 R^2 ρ^4 h[ρ]^2 (-1 + χ[ρ, x]^2) A0^(1,0)[ρ, x]^3 -
         ρ h′[ρ] A0^(1,0)[ρ, x] (ρ^2 χ^(0,1)[ρ, x]^2 A0^(1,0)[ρ, x]^2 - 2 ρ^2 A0^(0,1)[ρ, x] χ^(0,1)[ρ, x] A0^(1,0)[ρ, x] χ^(1,0)[ρ, x] + A0^(0,1)[ρ, x]^2 (3 - 3 χ[ρ, x]^2 + ρ^2 χ^(1,0)[ρ, x]^2)) +
         2 h[ρ] (ρ A0^(1,0)[ρ, x]^2 ((-1 + χ[ρ, x]^2) A0^(0,2)[ρ, x] + ρ (R^2 ρ^3 (-1 + χ[ρ, x]^2) h′[ρ] - χ^(0,1)[ρ, x]^2) A0^(1,0)[ρ, x]) +
            2 ρ A0^(0,1)[ρ, x] A0^(1,0)[ρ, x] (ρ χ^(0,1)[ρ, x] A0^(1,0)[ρ, x] χ^(1,0)[ρ, x] - (-1 + χ[ρ, x]^2) A0^(1,1)[ρ, x]) +
            A0^(0,1)[ρ, x]^2 (A0^(1,0)[ρ, x] (-4 + 4 χ[ρ, x]^2 - ρ^2 χ^(1,0)[ρ, x]^2) + ρ (-1 + χ[ρ, x]^2) A0^(2,0)[ρ, x]))) +
      R^2 ρ f[ρ]^2 (2 ρ^2 h′[ρ] χ^(0,1)[ρ, x]^2 A0^(1,0)[ρ, x] + 2 R^2 ρ^3 h[ρ]^2 (A0^(1,0)[ρ, x] (2 - 2 χ[ρ, x]^2 + 3 ρ^2 χ^(1,0)[ρ, x]^2) - ρ (-1 + χ[ρ, x]^2) A0^(2,0)[ρ, x]) +
         h[ρ] (A0^(0,2)[ρ, x] (2 - 2 χ[ρ, x]^2 + 2 ρ^2 χ^(1,0)[ρ, x]^2) +
            ρ (R^2 ρ^3 h′[ρ] A0^(1,0)[ρ, x] (3 - 3 χ[ρ, x]^2 + ρ^2 χ^(1,0)[ρ, x]^2) + 2 χ^(0,1)[ρ, x] (2 χ^(1,0)[ρ, x] (2 A0^(0,1)[ρ, x] - ρ A0^(1,1)[ρ, x]) +
                  χ^(0,1)[ρ, x] (A0^(1,0)[ρ, x] + ρ A0^(2,0)[ρ, x])))))) /. {R → 1}, ρ, Simplify];
EqC =
  Collect[
   (R^3 ρ^2 (-1 + χ[ρ, x]^2) (2 R^2 ρ^4 f[ρ] h[ρ] f′[ρ] χ^(1,0)[ρ, x] (χ^(0,1)[ρ, x]^2 + R^2 ρ^2 h[ρ] (1 - χ[ρ, x]^2 + ρ^2 χ^(1,0)[ρ, x]^2)) +
        h[ρ] (4 R^2 ρ^4 h[ρ]^2 (-1 + χ[ρ, x]^2) A0^(1,0)[ρ, x]^2 (χ[ρ, x] + ρ χ^(1,0)[ρ, x]) -
           ρ^2 h′[ρ] (ρ^2 χ^(0,1)[ρ, x]^2 A0^(1,0)[ρ, x]^2 χ^(1,0)[ρ, x] + A0^(0,1)[ρ, x]^2 χ^(1,0)[ρ, x] (1 - χ[ρ, x]^2 + ρ^2 χ^(1,0)[ρ, x]^2) -
              2 A0^(0,1)[ρ, x] χ^(0,1)[ρ, x] A0^(1,0)[ρ, x] (-1 + χ[ρ, x]^2 + ρ^2 χ^(1,0)[ρ, x]^2)) +
           2 h[ρ] (2 χ[ρ, x]^3 A0^(0,1)[ρ, x]^2 - χ[ρ, x] (3 ρ^2 χ^(0,1)[ρ, x]^2 A0^(1,0)[ρ, x]^2 - 6 ρ^2 A0^(0,1)[ρ, x] χ^(0,1)[ρ, x] A0^(1,0)[ρ, x] χ^(1,0)[ρ, x] +
                 A0^(0,1)[ρ, x]^2 (2 + 3 ρ^2 χ^(1,0)[ρ, x]^2)) + ρ χ[ρ, x]^2 (ρ A0^(1,0)[ρ, x]^2 (χ^(0,2)[ρ, x] + R^2 ρ^4 h′[ρ] χ^(1,0)[ρ, x]) +
                 2 A0^(0,1)[ρ, x] A0^(1,0)[ρ, x] (χ^(0,1)[ρ, x] - ρ χ^(1,1)[ρ, x]) + A0^(0,1)[ρ, x]^2 (2 χ^(1,0)[ρ, x] + ρ χ^(2,0)[ρ, x])) -
              ρ (ρ A0^(1,0)[ρ, x]^2 (χ^(0,2)[ρ, x] + ρ (R^2 ρ^3 h′[ρ] + χ^(0,1)[ρ, x]^2) χ^(1,0)[ρ, x]) - 2 A0^(0,1)[ρ, x] A0^(1,0)[ρ, x] (χ^(0,1)[ρ, x] (-1 + ρ^2 χ^(1,0)[ρ, x]^2) + ρ χ^(1,1)[ρ, x]) +
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         (2 ρ^2 h′[ρ] χ^(0,1)[ρ, x]^2 χ^(1,0)[ρ, x] + 2 R^2 ρ^2 h[ρ]^2 (-2 χ[ρ, x]^3 + χ[ρ, x] (2 + 3 ρ^2 χ^(1,0)[ρ, x]^2) - ρ χ[ρ, x]^2 (4 χ^(1,0)[ρ, x] + ρ χ^(2,0)[ρ, x]) +
              ρ (4 χ^(1,0)[ρ, x] + 3 ρ^2 χ^(1,0)[ρ, x]^3 + ρ χ^(2,0)[ρ, x])) +
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              ρ (R^2 ρ^3 h′[ρ] χ^(1,0)[ρ, x] (1 + ρ^2 χ^(1,0)[ρ, x]^2) + 2 χ^(0,1)[ρ, x] (-2 ρ χ^(1,0)[ρ, x] χ^(1,1)[ρ, x] + χ^(0,1)[ρ, x] (5 χ^(1,0)[ρ, x] + ρ χ^(2,0)[ρ, x]))))))) /. {R → 1},
  ρ, Simplify];
```
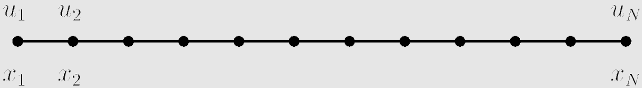

Solve PDEs: numerics

The analytic approach is not feasable so we resort to numerics.

Discretization



- Discretize spacetime.
- Functions take values at grid points (seed).
- **Derivatives are substituted by operators (matrices).**
- Once equation has turned into numbers use favourite resolution method (Newton-Raphson).

Differentiation matrices

Easy example

3 grid points: x_1, x_2, x_3
2nd order finite difference for derivative: $x_i' = \frac{x_{i+1} - x_{i-1}}{2h}$
Boundary conditions: periodic.

$$\begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} = \frac{1}{h} \begin{pmatrix} 0 & 1/2 & -1/2 \\ -1/2 & 0 & 1/2 \\ 1/2 & -1/2 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \tag{1}$$

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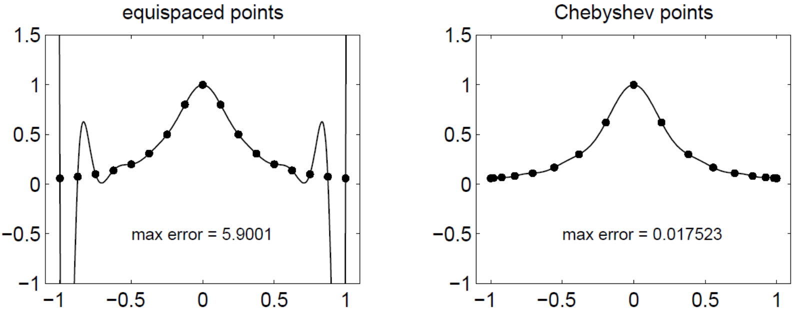
Spectral methods

Use unevenly distributed points: Chebyshev points.

$$x_j = \cos(j\pi/N) \qquad j = 0, 1, ...N$$

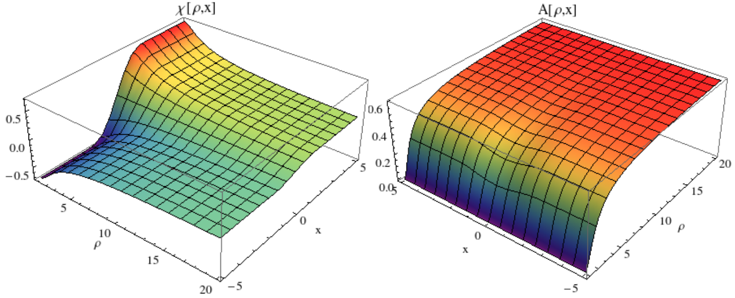
Use N-th order polynomial interpolation.

Even vs uneven



Put machinery to work!

1. Choose your set-up.
2. Numerize
3. Get results: functions, perturbations,...
4. Look for interpretation



Interpretation

This is where the dictionary comes into play. Compute different quantities,e.g.:

Conductivity

Remember Ohm's law (back in Kindergarten):

$$V = IR$$

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Turn to grown-up version:

$$\vec{j}(\omega) = \sigma(\omega)\vec{E}(\omega)$$

Holographic conductivity

- Gauge field present in set-up.

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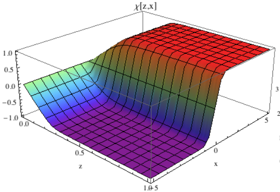
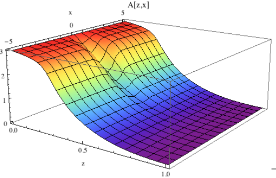
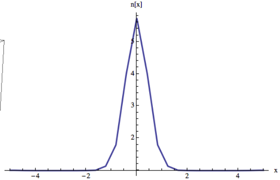
- Clever choice

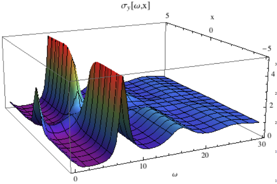
$$A_x = \frac{E}{i\omega} e^{i\omega t} + \langle J_x \rangle z + \dots \qquad \dot{A}_x = E e^{i\omega t} \qquad \sigma(\omega) = \left. \frac{A_x'}{i\omega A_x} \right|_{z=0}$$

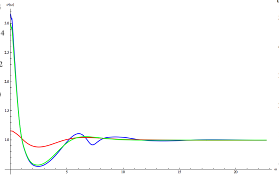
Conductivities

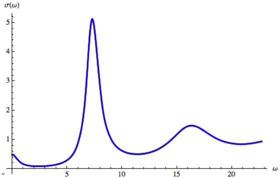












Conclusion

Following this lines, lots of things have been done and are being done:
Strongly coupled phenomena which are not well-understood

- Heavy ion collisions
- Superconductor physics (high Tc, curates...)
- Topological insulators
- Magnetic field induced lattice ground states

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REVIEW: AdS/CFT at work

1. Pick up tractable gravity model with interesting features to explore.
$$\mathcal{L} = R - \Lambda + ...$$
2. Solve equations of motion (probably numerically)
$$\frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) = 0$$
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$$g_{\mu\nu} \leftrightarrow T^{\mu\nu} \qquad A_\mu \leftrightarrow J^\mu$$

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