NLO Calculations with GoSam

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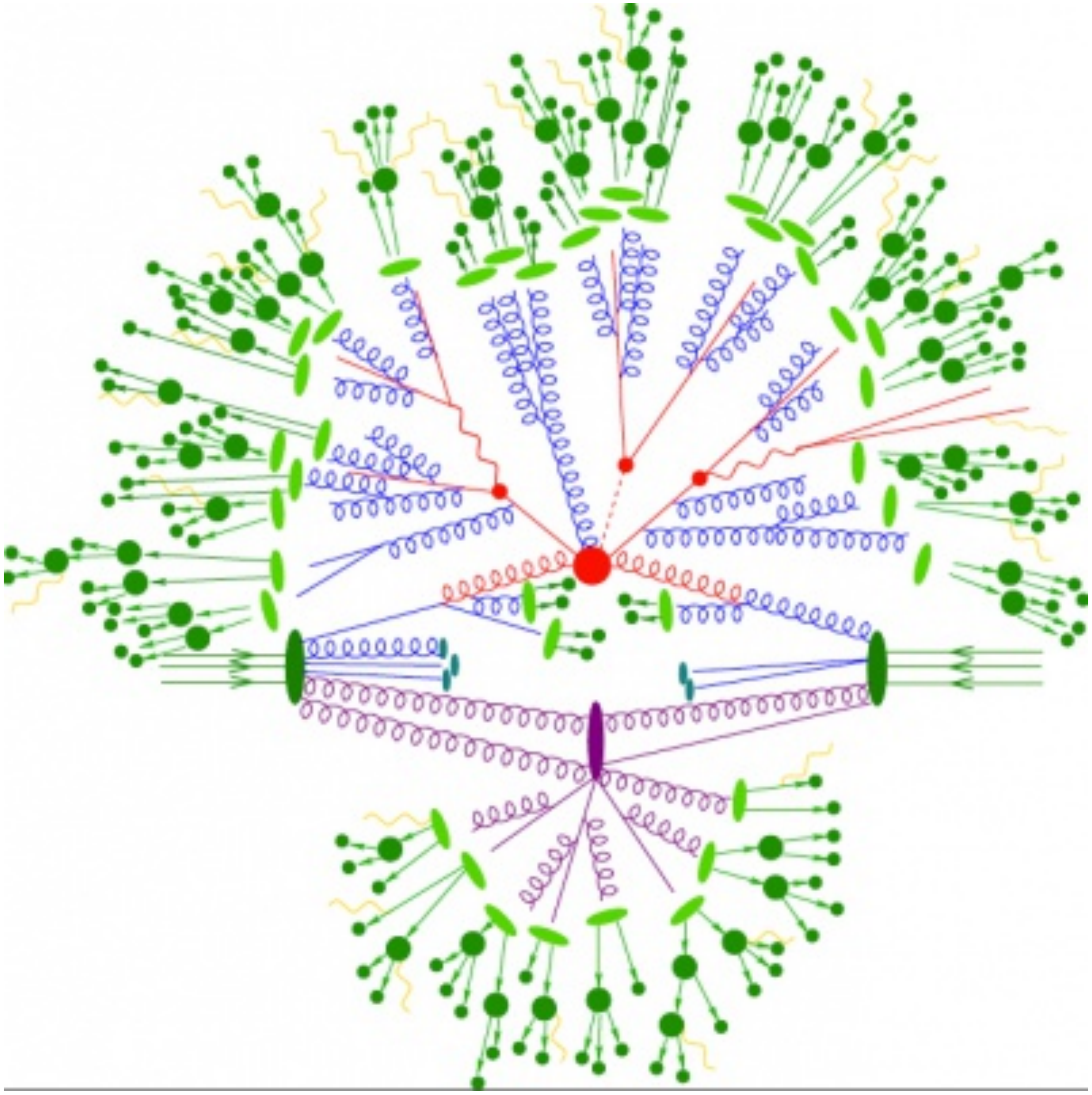
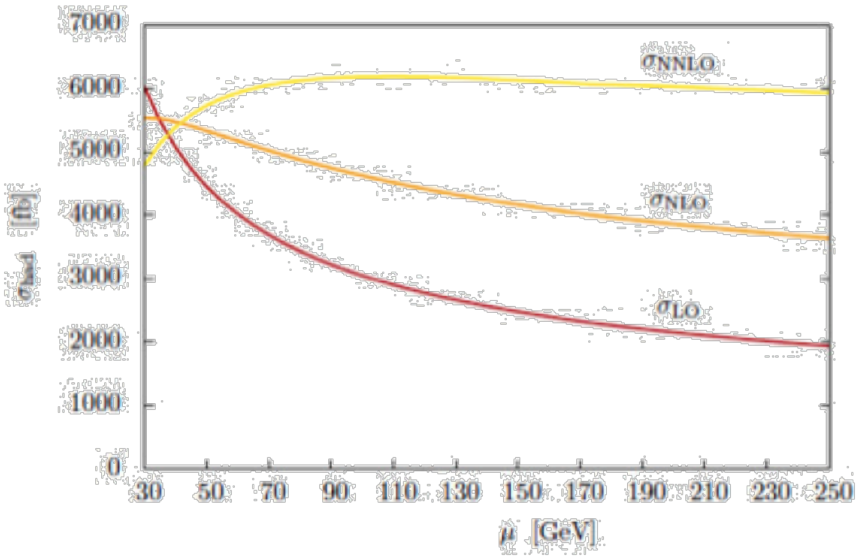
LHC

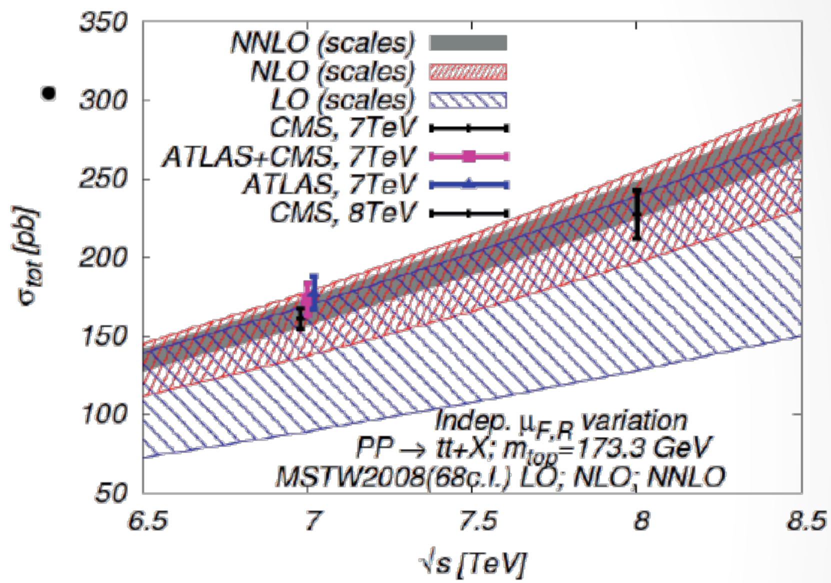


Motivation

- Higgs found, need properties
  - Spin
  - CP properties
  - Couplings to other particles
- BSM physics?

Motivation for NLO





- Reduce theoretical error
- Strong dependence on factorization and renormalization scale

NLO Calculations

$$\sigma^{NLO} = \int_m \left[d^{(4)}\sigma^B + \int_{loop} d^{(d)}\sigma^V + \int_1 d^{(d)}\sigma^S \right] + \int_{m+1} \left[d^{(4)}\sigma^R - d^{(4)}\sigma^S \right]$$

- NLO calculations consists of:
  - Leading Order (LO): Born diagrams
  - Virtual corrections: Loop diagrams 🡐 GoSam
  - Real corrections: Additional radiation
  - Subtraction terms to regulate infinities

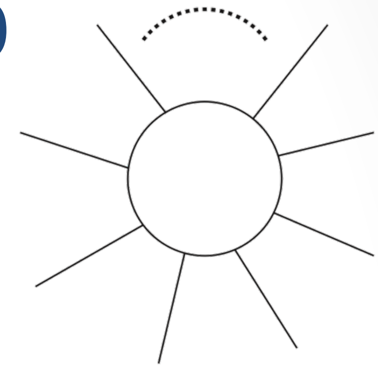
GoSam

Collaboration
Cullen, HvD, Greiner, Heinrich, Luisoni, Mastrolia, Mirabella, Ossola, Peraro, Schlenk, von Soden-Fraunhofen, Tramontano

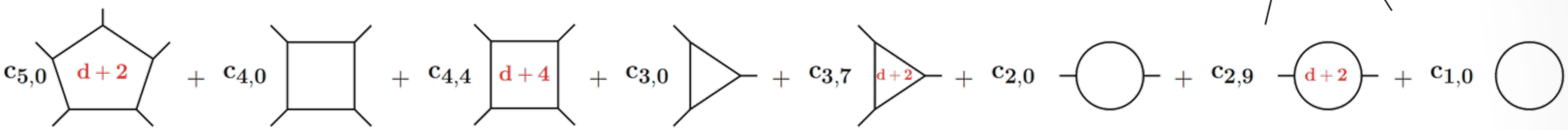
Reduction algorithms

- *Samurai*, *Xsamurai*
  d-dimensional integrand-level reduction
  current default
  [Mastrolia, Ossola, Reiter, Tramontano]; [HvD (2013)]
- *Golem95*, *Golem95 higherrank extension*
  Tensorial reduction
  Numerically stable ➔ rescue system
  [Binoth, Guillet, Heinrich, Pilon, Reiter]; [Guillet, Heinrich, von Soden-Fraunhofen]
- *Ninja*
  Integrand-level+Laurent expansion
  Stable and fast
  [Mastrolia, Mirabella, Peraro]; [HvD, Luisoni, Mastrolia, Mirabella, Ossola, Peraro]; [Peraro]

Amplitudes at one loop



$$\mathcal{M}_n \equiv \int \mathcal{A}_n(\bar{q})\, d\bar{q} \equiv \int d^{-2\epsilon}\mu \int d^4 q \frac{N(q,\mu^2)}{\bar{D}_0 \ldots \bar{D}_{n-1}}$$



$$\mathcal{A}_n = \sum_{ijklm} \frac{\Delta_{ijklm}(q,\mu^2)}{D_i D_j D_k D_l D_m} + \sum_{ijkl} \frac{\Delta_{ijkl}(q,\mu^2)}{D_i D_j D_k D_l} +$$

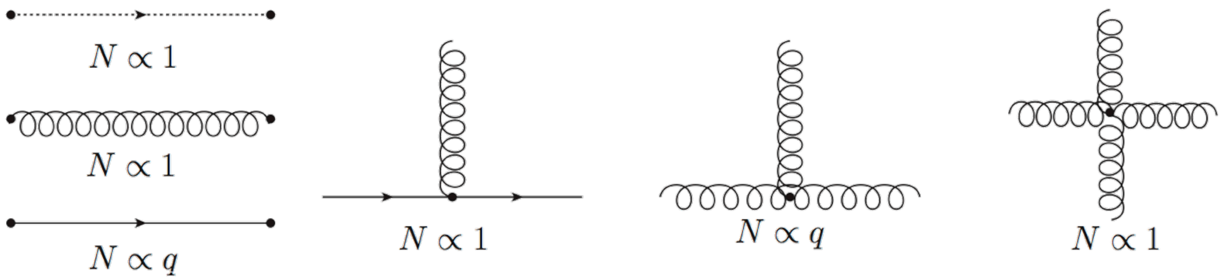
$$+ \sum_{ijk} \frac{\Delta_{ijk}(q,\mu^2)}{D_i D_j D_k} + \sum_{ij} \frac{\Delta_{ij}(q,\mu^2)}{D_i D_j} + \sum_{i} \frac{\Delta_{i}(q,\mu^2)}{D_i}$$

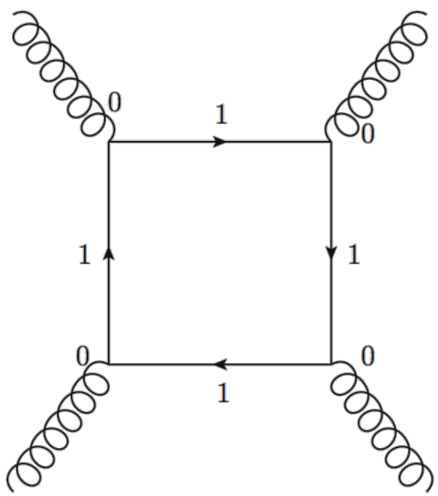
Amplitudes at one loop

$$\mathcal{A}_n = \sum_{ijklm} \frac{\Delta_{ijklm}(q,\mu^2)}{D_i D_j D_k D_l D_m} + \sum_{ijkl} \frac{\Delta_{ijkl}(q,\mu^2)}{D_i D_j D_k D_l} + \sum_{ijk} \frac{\Delta_{ijk}(q,\mu^2)}{D_i D_j D_k} + \sum_{ij} \frac{\Delta_{ij}(q,\mu^2)}{D_i D_j} + \sum_{i} \frac{\Delta_{i}(q,\mu^2)}{D_i}$$

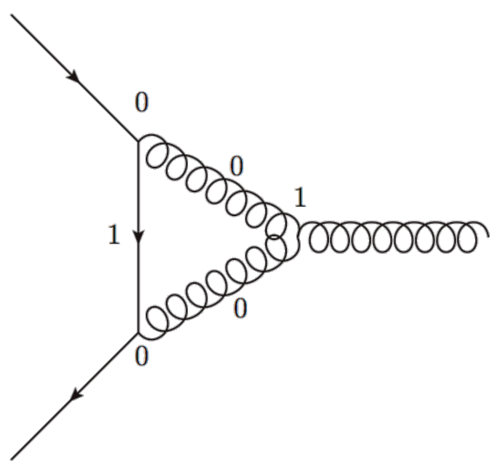
- Residues multivariate polynomials
- Need rank to determine generic form
- Renormalizability requires rank ≤ propagators

Rank of the numerator





$r_N = 4$



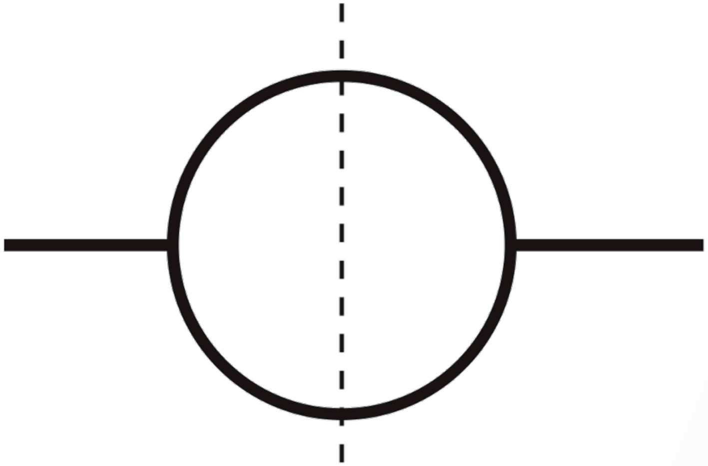
$r_N = 2$

Integrand decomposition

$$\mathcal{A}_n = \sum_{ijklm} \frac{\Delta_{ijklm}(q,\mu^2)}{D_iD_jD_kD_lD_m} + \sum_{ijkl} \frac{\Delta_{ijkl}(q,\mu^2)}{D_iD_jD_kD_l} + \sum_{ijk} \frac{\Delta_{ijk}(q,\mu^2)}{D_iD_jD_k} + \sum_{ij} \frac{\Delta_{ij}(q,\mu^2)}{D_iD_j} + \sum_{i} \frac{\Delta_{i}(q,\mu^2)}{D_i}$$

$$\frac{N}{D_0D_1} = \frac{\Delta_{01}}{D_0D_1} + \frac{\Delta_1}{D_1} + \frac{\Delta_0}{D_0} \quad \Rightarrow N = \Delta_{01} + \Delta_1 D_0 + \Delta_0 D_1$$

$$\Delta_{01} = \mathrm{Res}_{01}\left(\frac{N}{D_0D_1}\right)$$



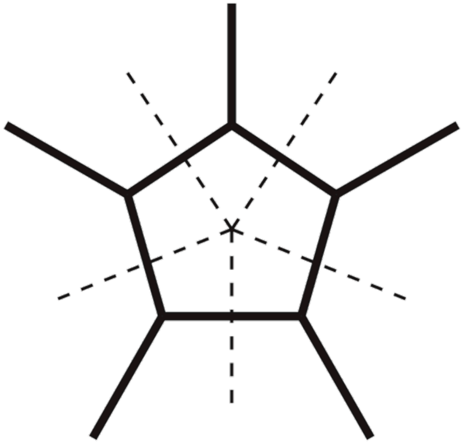
$$\Delta_1 = \mathrm{Res}_1\left(\frac{N-\Delta_{01}}{D_0D_1}\right)$$

$$\Delta_0 = \mathrm{Res}_0\left(\frac{N-\Delta_{01}}{D_0D_1}\right)$$

Integrand decomposition

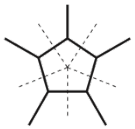
$$\mathcal{A}_n = \sum_{ijklm} \frac{\Delta_{ijklm}(q,\mu^2)}{D_i D_j D_k D_l D_m} + \sum_{ijkl} \frac{\Delta_{ijkl}(q,\mu^2)}{D_i D_j D_k D_l} + \sum_{ijk} \frac{\Delta_{ijk}(q,\mu^2)}{D_i D_j D_k} + \sum_{ij} \frac{\Delta_{ij}(q,\mu^2)}{D_i D_j} + \sum_{i} \frac{\Delta_{i}(q,\mu^2)}{D_i}$$

Quintuple cut



$$\Delta_{ijk\ell m}(\bar{q}) = \mathrm{Res}_{ijk\ell m} \left\{ \frac{N(\bar{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}} \right\}$$

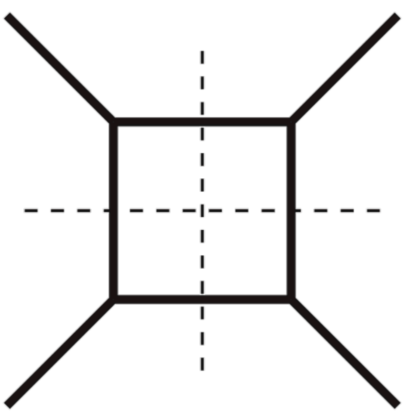
Integrand decomposition



$$\Delta_{ijk\ell m}(\bar{q}) = \mathrm{Res}_{ijk\ell m}\left\{\frac{N(\bar{q})}{\bar{D}_0\cdots\bar{D}_{n-1}}\right\}$$

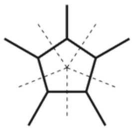
1 coefficient

Quadruple cut



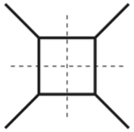
$$\Delta_{ijk\ell}(\bar{q}) = \mathrm{Res}_{ijk\ell}\left\{\frac{N(\bar{q})}{\bar{D}_0\cdots\bar{D}_{n-1}} - \sum_{i<<m}^{n-1}\frac{\Delta_{ijk\ell m}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell\bar{D}_m}\right\}$$

Integrand decomposition



$$\Delta_{ijk\ell m}(\bar{q}) = \text{Res}_{ijk\ell m}\left\{\frac{N(\bar{q})}{\bar{D}_0\cdots\bar{D}_{n-1}}\right\}$$

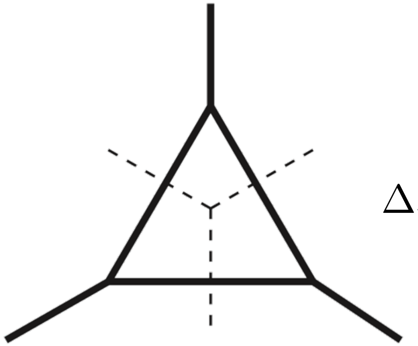
1 coefficient



$$\Delta_{ijk\ell}(\bar{q}) = \text{Res}_{ijk\ell}\left\{\frac{N(\bar{q})}{\bar{D}_0\cdots\bar{D}_{n-1}} - \sum_{i<<m}^{n-1}\frac{\Delta_{ijk\ell m}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell\bar{D}_m}\right\}$$

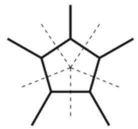
5 coefficients

Triple cut



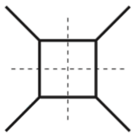
$$\Delta_{ijk}(\bar{q}) = \text{Res}_{ijk}\left\{\frac{N(\bar{q})}{\bar{D}_0\cdots\bar{D}_{n-1}} - \sum_{i<<m}^{n-1}\frac{\Delta_{ijk\ell m}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell\bar{D}_m} - \sum_{i<<\ell}^{n-1}\frac{\Delta_{ijk\ell}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell}\right\}$$

Integrand decomposition



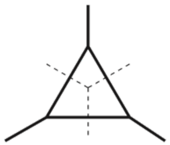
$$\Delta_{ijk\ell m}(\bar{q}) = \mathrm{Res}_{ijk\ell m}\left\{\frac{N(\bar{q})}{\bar{D}_0\cdots\bar{D}_{n-1}}\right\}$$

1 coefficient



$$\Delta_{ijk\ell}(\bar{q}) = \mathrm{Res}_{ijk\ell}\left\{\frac{N(\bar{q})}{\bar{D}_0\cdots\bar{D}_{n-1}} - \sum_{i<<m}^{n-1}\frac{\Delta_{ijk\ell m}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell\bar{D}_m}\right\}$$

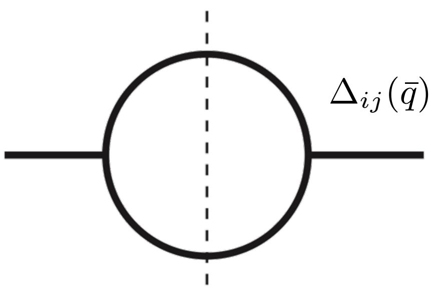
5 coefficients



$$\Delta_{ijk}(\bar{q}) = \mathrm{Res}_{ijk}\left\{\frac{N(\bar{q})}{\bar{D}_0\cdots\bar{D}_{n-1}} - \sum_{i<<m}^{n-1}\frac{\Delta_{ijk\ell m}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell\bar{D}_m} - \sum_{i<<\ell}^{n-1}\frac{\Delta_{ijk\ell}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell}\right\}$$

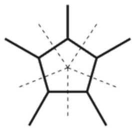
10 coefficients

Double cut



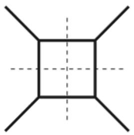
$$\Delta_{ij}(\bar{q}) = \mathrm{Res}_{ij}\left\{\frac{N(\bar{q})}{\bar{D}_0\cdots\bar{D}_{n-1}} - \sum_{i<<m}^{n-1}\frac{\Delta_{ijk\ell m}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell\bar{D}_m} - \sum_{i<<\ell}^{n-1}\frac{\Delta_{ijk\ell}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell} - \sum_{i<<k}^{n-1}\frac{\Delta_{ijk}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k}\right\}$$

Integrand decomposition



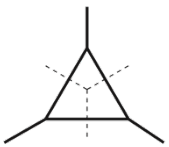
$$\Delta_{ijk\ell m}(\bar{q}) = \mathrm{Res}_{ijk\ell m}\left\{\frac{N(\bar{q})}{\bar{D}_0\cdots\bar{D}_{n-1}}\right\}$$

1 coefficient



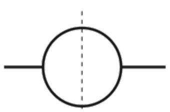
$$\Delta_{ijk\ell}(\bar{q}) = \mathrm{Res}_{ijk\ell}\left\{\frac{N(\bar{q})}{\bar{D}_0\cdots\bar{D}_{n-1}} - \sum_{i<<m}^{n-1}\frac{\Delta_{ijk\ell m}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell\bar{D}_m}\right\}$$

5 coefficients



$$\Delta_{ijk}(\bar{q}) = \mathrm{Res}_{ijk}\left\{\frac{N(\bar{q})}{\bar{D}_0\cdots\bar{D}_{n-1}} - \sum_{i<<m}^{n-1}\frac{\Delta_{ijk\ell m}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell\bar{D}_m} - \sum_{i<<\ell}^{n-1}\frac{\Delta_{ijk\ell}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell}\right\}$$

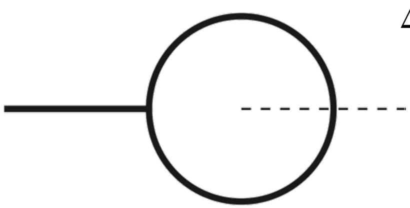
10 coefficients



$$\Delta_{ij}(\bar{q}) = \mathrm{Res}_{ij}\left\{\frac{N(\bar{q})}{\bar{D}_0\cdots\bar{D}_{n-1}} - \sum_{i<<m}^{n-1}\frac{\Delta_{ijk\ell m}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell\bar{D}_m} - \sum_{i<<\ell}^{n-1}\frac{\Delta_{ijk\ell}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell} - \sum_{i<<k}^{n-1}\frac{\Delta_{ijk}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k}\right\}$$

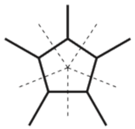
10 coefficients

Single cut



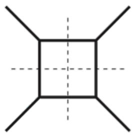
$$\Delta_i(\bar{q}) = \mathrm{Res}_i\left\{\frac{N(\bar{q})}{\bar{D}_0\cdots\bar{D}_{n-1}} - \sum_{i<<m}^{n-1}\frac{\Delta_{ijk\ell m}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell\bar{D}_m} - \sum_{i<<\ell}^{n-1}\frac{\Delta_{ijk\ell}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell} + \right.$$
$$\left. - \sum_{i<<k}^{n-1}\frac{\Delta_{ijk}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k} - \sum_{i<j}^{n-1}\frac{\Delta_{ij}(\bar{q})}{\bar{D}_i\bar{D}_j}\right\}$$

Integrand decomposition



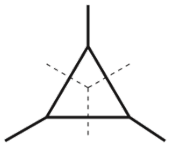
$$\Delta_{ijk\ell m}(\bar{q}) = \text{Res}_{ijk\ell m}\left\{\frac{N(\bar{q})}{\bar{D}_0\cdots\bar{D}_{n-1}}\right\}$$

1 coefficient



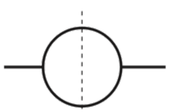
$$\Delta_{ijk\ell}(\bar{q}) = \text{Res}_{ijk\ell}\left\{\frac{N(\bar{q})}{\bar{D}_0\cdots\bar{D}_{n-1}} - \sum_{i<<m}^{n-1}\frac{\Delta_{ijk\ell m}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell\bar{D}_m}\right\}$$

5 coefficients



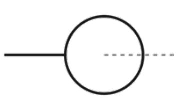
$$\Delta_{ijk}(\bar{q}) = \text{Res}_{ijk}\left\{\frac{N(\bar{q})}{\bar{D}_0\cdots\bar{D}_{n-1}} - \sum_{i<<m}^{n-1}\frac{\Delta_{ijk\ell m}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell\bar{D}_m} - \sum_{i<<\ell}^{n-1}\frac{\Delta_{ijk\ell}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell}\right\}$$

10 coefficients



$$\Delta_{ij}(\bar{q}) = \text{Res}_{ij}\left\{\frac{N(\bar{q})}{\bar{D}_0\cdots\bar{D}_{n-1}} - \sum_{i<<m}^{n-1}\frac{\Delta_{ijk\ell m}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell\bar{D}_m} - \sum_{i<<\ell}^{n-1}\frac{\Delta_{ijk\ell}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell} - \sum_{i<<k}^{n-1}\frac{\Delta_{ijk}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k}\right\}$$

10 coefficients



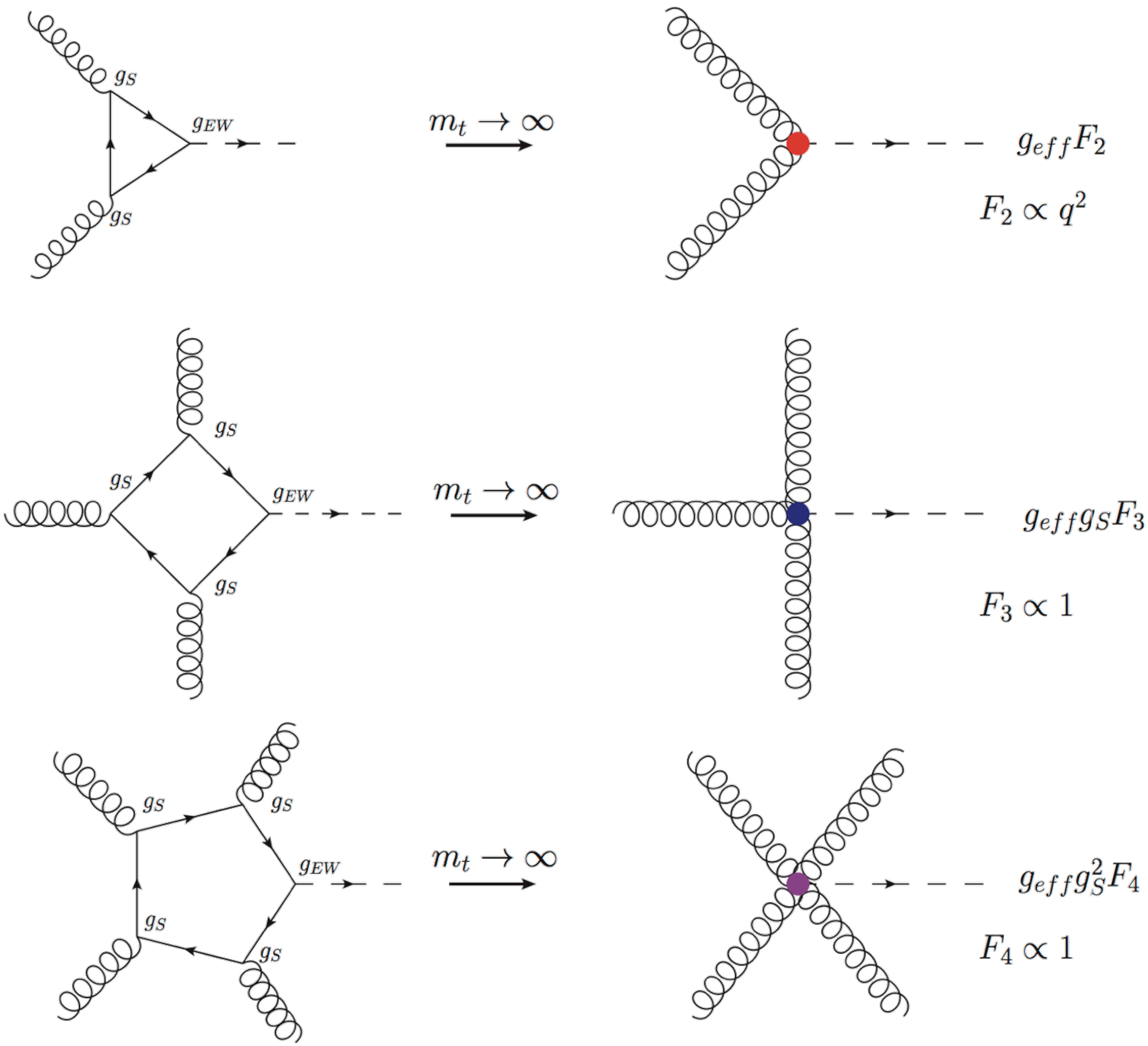
$$\begin{aligned}\Delta_{i}(\bar{q}) \;=\; \text{Res}_{i}\Bigg\{&\frac{N(\bar{q})}{\bar{D}_0\cdots\bar{D}_{n-1}} - \sum_{i<<m}^{n-1}\frac{\Delta_{ijk\ell m}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell\bar{D}_m} - \sum_{i<<\ell}^{n-1}\frac{\Delta_{ijk\ell}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell} + \\ &- \sum_{i<<k}^{n-1}\frac{\Delta_{ijk}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k} - \sum_{i<j}^{n-1}\frac{\Delta_{ij}(\bar{q})}{\bar{D}_i\bar{D}_j}\Bigg\}\end{aligned}$$

5 coefficients

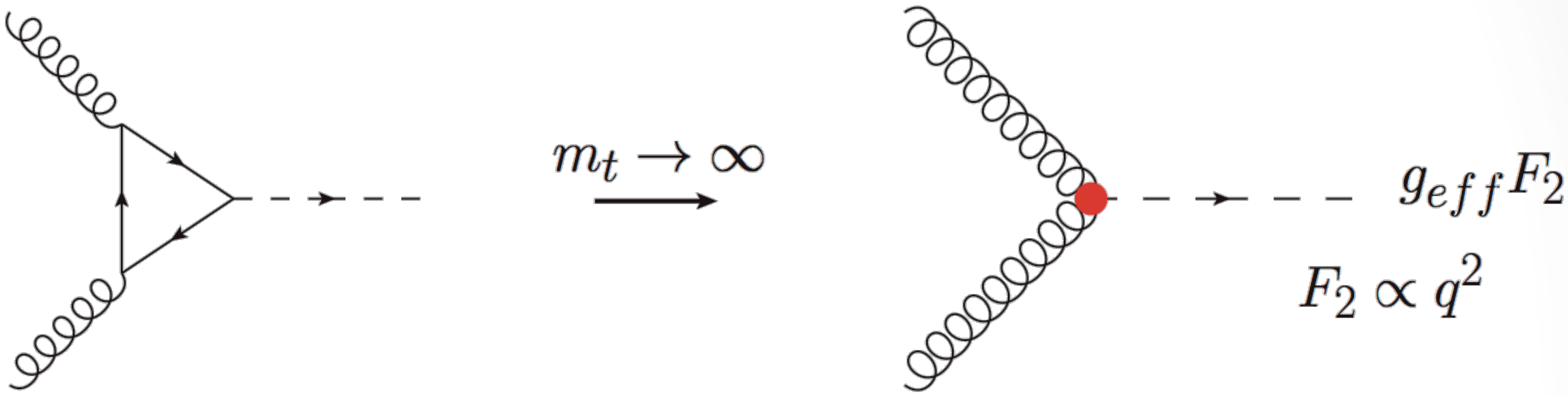
Hexagon:

$$\binom{6}{5}\cdot 1 + \binom{6}{4}\cdot 5 + \binom{6}{3}\cdot 10 + \binom{6}{2}\cdot 10 + \binom{6}{1}\cdot 5 = 461 \text{ coefficients}$$

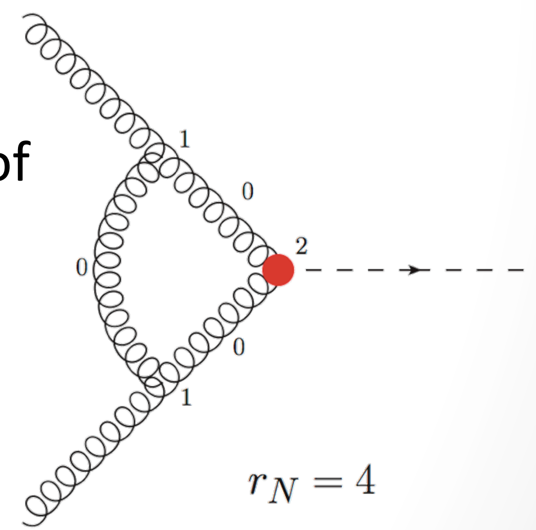
Higher rank


g_S
g_{EW}
g_S
$m_t \to \infty$
$g_{eff}F_2$
$F_2 \propto q^2$
g_S
g_S
g_{EW}
g_S
$m_t \to \infty$
$g_{eff}g_S F_3$
$F_3 \propto 1$
g_S
g_S
g_{EW}
g_S
g_S
$m_t \to \infty$
$g_{eff}g_S^2 F_4$
$F_4 \propto 1$

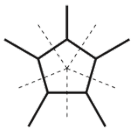
Higher rank



- Effective vertex in loop adds two powers of q
- New rule: rank ≤ propagators + 1

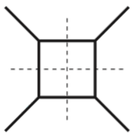


Integrand decomposition



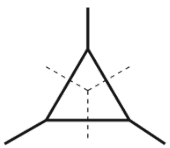
$$\Delta_{ijk\ell m}(\bar{q}) = \text{Res}_{ijk\ell m}\left\{\frac{N(\bar{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}}\right\}$$

1➔1 coefficient



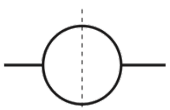
$$\Delta_{ijk\ell}(\bar{q}) = \text{Res}_{ijk\ell}\left\{\frac{N(\bar{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}} - \sum_{i<<m}^{n-1} \frac{\Delta_{ijk\ell m}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell\bar{D}_m}\right\}$$

5➔6 coefficients



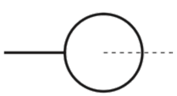
$$\Delta_{ijk}(\bar{q}) = \text{Res}_{ijk}\left\{\frac{N(\bar{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}} - \sum_{i<<m}^{n-1} \frac{\Delta_{ijk\ell m}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell\bar{D}_m} - \sum_{i<<\ell}^{n-1} \frac{\Delta_{ijk\ell}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell}\right\}$$

10➔15 coefficients



$$\Delta_{ij}(\bar{q}) = \text{Res}_{ij}\left\{\frac{N(\bar{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}} - \sum_{i<<m}^{n-1} \frac{\Delta_{ijk\ell m}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell\bar{D}_m} - \sum_{i<<\ell}^{n-1} \frac{\Delta_{ijk\ell}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell} - \sum_{i<<k}^{n-1} \frac{\Delta_{ijk}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k}\right\}$$

10➔20 coefficients



$$\begin{aligned}\Delta_i(\bar{q}) \quad = \quad \text{Res}_i\Bigg\{&\frac{N(\bar{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}} - \sum_{i<<m}^{n-1} \frac{\Delta_{ijk\ell m}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell\bar{D}_m} - \sum_{i<<\ell}^{n-1} \frac{\Delta_{ijk\ell}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_\ell} + \\ &- \sum_{i<<k}^{n-1} \frac{\Delta_{ijk}(\bar{q})}{\bar{D}_i\bar{D}_j\bar{D}_k} - \sum_{i<j}^{n-1} \frac{\Delta_{ij}(\bar{q})}{\bar{D}_i\bar{D}_j}\Bigg\}\end{aligned}$$

5➔15 coefficients

[Mastrolia, Mirabella, Peraro (2012)]; [HvD (2013)]

Hexagon:

$$\binom{6}{5}\cdot 1 + \binom{6}{4}\cdot 6 + \binom{6}{3}\cdot 15 + \binom{6}{2}\cdot 20 + \binom{6}{1}\cdot 15 = (461 \rightarrow)786 \text{ coefficients}$$

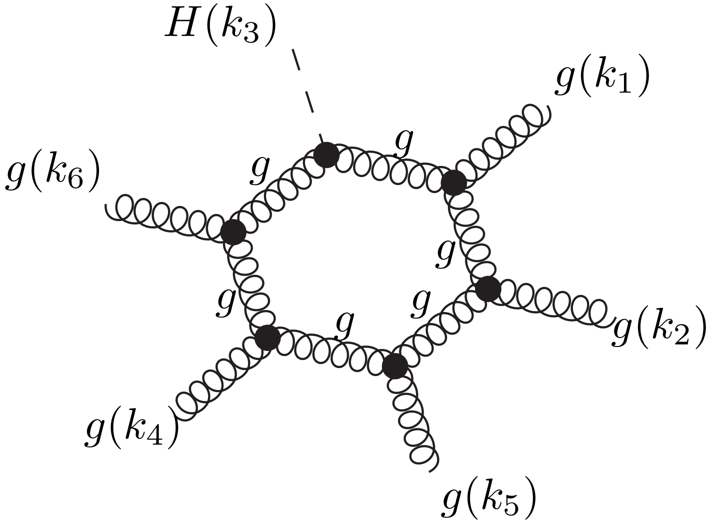
Applications

p p > h j j at NLO in Gluon Fusion

[HvD, Greiner, Luisoni, Mastrolia, Mirabella, Ossola, Peraro, von Soden-Fraunhofen, Tramontano (2013)]

p p > h j j j at NLO in Gluon Fusion

[Cullen, HvD, Greiner, Luisoni, Mastrolia, Mirabella, Ossola, Peraro, Tramontano (2013)]



H+0j	**1 NLO**
$gg \to H$	1 NLO
H+1j	**62 NLO**
$q\bar{q} \to Hg$	14 NLO
$gg \to Hg$	48 NLO
H+2j	**926 NLO**
$qq' \to Hqq'$	32 NLO
$qq \to Hqq$	64 NLO
$q\bar{q} \to Hgg$	179 NLO
$gg \to Hgg$	651 NLO
H+3j	**13179 NLO**
$qq' \to Hqq'g$	467 NLO
$qq \to Hqqg$	868 NLO
$q\bar{q} \to Hggg$	2519 NLO
$gg \to Hggg$	9325 NLO

NLO Calculations

$$\sigma^{NLO} = \int_m \left[d^{(4)}\sigma^B + \int_{loop} d^{(d)}\sigma^V + \int_1 d^{(d)}\sigma^S \right] + \int_{m+1} \left[d^{(4)}\sigma^R - d^{(4)}\sigma^S \right]$$

- NLO calculations consists of:
  - Leading Order (LO): Born diagrams
  - Virtual corrections: Loop diagrams 🡐 GoSam
  - Real corrections: Additional radiation
  - Subtraction terms to regulate infinities

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- NLO calculations consists of:
  - Leading Order (LO): Born diagrams ← MC
  - Virtual corrections: Loop diagrams ← GoSam
  - Real corrections: Additional radiation ← MC
  - Subtraction terms to regulate infinities ← MC

Interface with aMC@NLO

- Recent release: MadGraph5_aMC@NLO (MG5aMC)
- Uses Binoth Les Houches Accord interface (BLHA)
  See next talk by Johann Felix
- Interface tested on several non-trivial processes
- Compared GoSam vs. MadLoop
- Runs smoothly parallelized with Condor

Summary

- GoSam is a framework for the automated computation of one loop diagrams
- Samurai has been extended to deal with higher rank numerators, f.e. effective Higgs couplings: Xsamurai
- Xsamurai has been used to calculate Higgs plus 2 and 3 jets in Gluon Fusion at NLO
- GoSam is being interfaced to MG5aMC for the calculation of even more complicated processes