An effective field theory for electroweak symmetry breaking including a light Higgs

Presentation at the IMPRS workshop Munich

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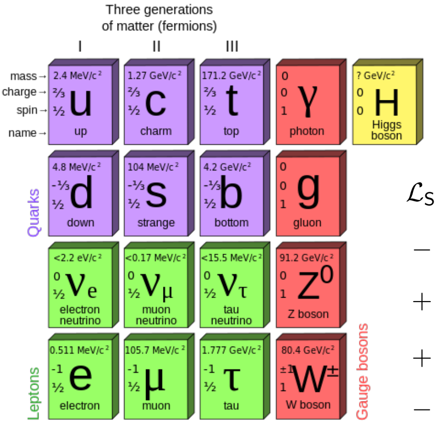
LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

ARNOLD SOMMERFELD CENTER FOR THEORETICAL PHYSICS

Supervisor: Prof. Dr. Gerhard Buchalla

Outline

The Standard Model of Particle Physics



$$
\begin{aligned}
\mathcal{L}_{\mathrm{SM}} = {} & \\
& -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{2}\langle W_{\mu\nu}W^{\mu\nu}\rangle - \frac{1}{2}\langle G_{\mu\nu}G^{\mu\nu}\rangle \\
& + \bar{q}i\not{D}q + \bar{l}i\not{D}l + \bar{u}i\not{D}u + \bar{d}i\not{D}d + \bar{e}i\not{D}e \\
& + \frac{1}{2}(D_\mu\phi^\dagger)(D^\mu\phi) + \frac{1}{2}\mu^2\phi^\dagger\phi - \frac{\lambda}{4}(\phi^\dagger\phi)^2 \\
& - \bar{l}Y_l\phi e - \bar{q}Y_d\phi d - \bar{q}Y_u(i\sigma_2\phi^*)u + \text{h.c.}
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Spontaneous Breaking of Symmetries

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \vec{\phi}\right)\left(\partial^\mu \vec{\phi}\right) + \frac{1}{2}\mu^2 |\vec{\phi}|^2 - \frac{\lambda}{4}|\vec{\phi}|^4$$

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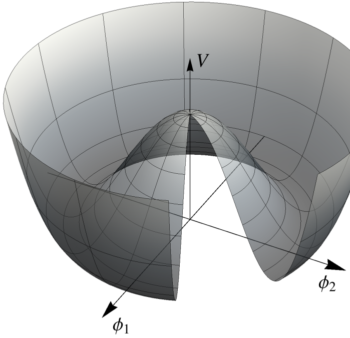

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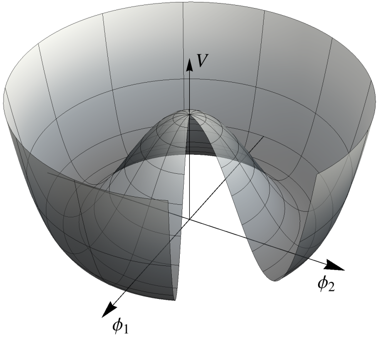

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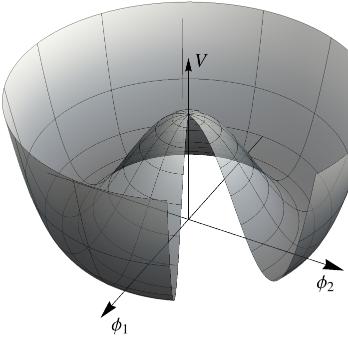

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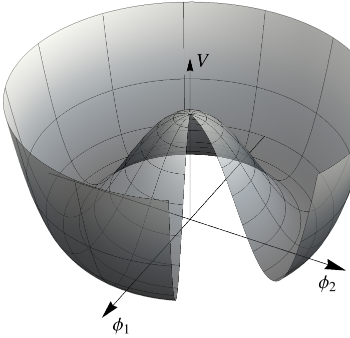

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \vec{\pi}\right)\left(\partial^\mu \vec{\pi}\right) + \frac{1}{2}\left(\partial_\mu \sigma\right)\left(\partial^\mu \sigma\right) - \frac{1}{2}(2\mu^2)\sigma^2 \\ - \frac{\lambda}{2}|\vec{\pi}|^2\sigma^2 - \sqrt{\lambda}\mu|\vec{\pi}|^2\sigma - \sqrt{\lambda}\mu\sigma^3 - \frac{\lambda}{4}|\vec{\pi}|^4 - \frac{\lambda}{4}\sigma^4$$

"The linear sigma model"

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After reparametrization we have:

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This was used in Chiral Perturbation Theory (χPT)

$$U \rightarrow lUr^\dagger, \qquad \text{where } l, r \in SU(2)_{L,R}$$

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- The Goldstones are described by $U = \exp\left\{ i \frac{2\, T_a \pi_a}{v} \right\}$ and become the longitudinal components of the gauge bosons. In unitary gauge: $\frac{v^2}{4} \langle (D_\mu U)(D^\mu U^\dagger) \rangle = \frac{g^2 v^2}{4} W_\mu^+ W^{\mu -} + \frac{(g^2 + g'^2) v^2}{8} Z_\mu Z^\mu$

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Effective Lagrangian at leading order

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\mathcal{L}_{\mathrm{LO}} =& \frac{1}{2}(\partial_\mu h)(\partial^\mu h) - \mathcal{V}\left(\tfrac{h}{v}\right) + \frac{v^2}{4}\langle (D_\mu U)(D^\mu U^\dagger)\rangle \, a_n \left(\tfrac{h}{v}\right)^n \\
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- We identify $\frac{v^2}{\Lambda^2} \simeq \frac{1}{16\pi^2}$.

Power-counting

$$\begin{aligned}\mathcal{L}_{\text{LO}} =&\frac{1}{2}(\partial_\mu h)(\partial^\mu h) - \mathcal{V}\left(\tfrac{h}{v}\right) + \frac{v^2}{4}\langle(D_\mu U)(D^\mu U^\dagger)\rangle\ a_n\left(\tfrac{h}{v}\right)^n\\ &+ i\bar{\Psi}_f \not{D}\Psi_f - v\left(\bar{\Psi}_f Y_{j,f} U\Psi_f + \text{h.c.}\right)\ e_{j,f}\left(\tfrac{h}{v}\right)^j\\ &- \frac{1}{2}\langle G_{\mu\nu}G^{\mu\nu}\rangle - \frac{1}{2}\langle W_{\mu\nu}W^{\mu\nu}\rangle - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}\end{aligned}$$

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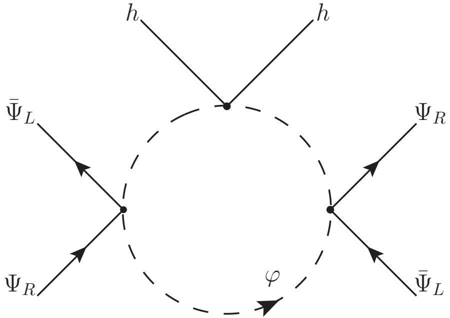

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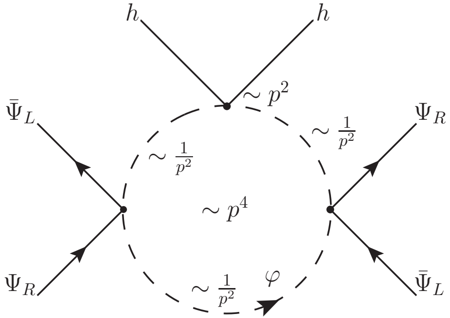

Power-counting

$$\mathcal{D} \sim p^{2L+2-X-\frac{1}{2}(F_L+F_R)-N_V} \left(\frac{\varphi}{v}\right)^B \left(\frac{h}{v}\right)^H \bar{\Psi}_L^{F_L^1} \Psi_L^{F_L^2} \bar{\Psi}_R^{F_R^1} \Psi_R^{F_R^2} \left(\frac{\mathcal{X}_{\mu\nu}}{v}\right)^X$$

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Classes of counterterms:

$$UD^2H, \quad UD^4H, \quad UHXD^2, \quad UHX^2,$$
$$UHD\Psi^2, \quad UHD^2\Psi^2, \quad UH\Psi^2X \text{ and } \Psi^4UH.$$

For convenience we define:
$L_\mu = iUD_\mu U^\dagger$, $\tau_L = UT_3U^\dagger$,
$P_\pm = \frac{1}{2} \pm T_3$, $P_{12} = T_1 + iT_2$, $P_{21} = T_1 - iT_2$,
$\eta = (\nu_R, e_R)^T$ and $r = (u_R, d_R)^T$.

Example for operators

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$$\mathcal{O}_{\beta_1} = v^2\langle\tau_L L_\mu\rangle\langle\tau_L L^\mu\rangle\,\mathcal{F}$$

$\sum$: 1 operator

$\mathcal{F}\left(\frac{h}{v}\right) = 1 + a_1\left(\frac{h}{v}\right) + a_2\left(\frac{h}{v}\right)^2 + \dots$

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$$\mathcal{O}_{D0,1} = \langle L_\mu L^\mu \rangle \langle L_\nu L^\nu \rangle \, \mathcal{F}$$

$$\mathcal{O}_{D1,4} = \langle L_\mu L_\nu \rangle \langle \tau_L L^\mu \rangle \left(\partial^\nu \frac{h}{v}\right) \, \mathcal{F}$$

$\sum$: 15 operators

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$$\mathcal{O}_{XU9} = B_{\mu\nu}\langle\tau_L W^{\mu\nu}\rangle\,\mathcal{F}$$

$\sum$: 10 operators

$$\mathcal{F}\left(\frac{h}{v}\right) = 1 + a_1\left(\frac{h}{v}\right) + a_2\left(\frac{h}{v}\right)^2 + \dots \qquad \widetilde{\mathcal{F}}\left(\frac{h}{v}\right) = \widetilde{a}_1\left(\frac{h}{v}\right) + \widetilde{a}_2\left(\frac{h}{v}\right)^2 + \dots$$

Example for operators

Classes of counterterms:

$$UD^2H, \quad UD^4H, \quad UHXD^2, \quad UHX^2,$$
$$UHD\Psi^2, \quad UHD^2\Psi^2, \quad UH\Psi^2X \text{ and } \Psi^4UH.$$

$$\mathcal{O}_{\Psi V1} = (\bar{q}\gamma^\mu q)\langle \tau_L L_\mu \rangle \, \mathcal{F}$$

$\sum$: 13 operators

$\mathcal{F}\left(\frac{h}{v}\right) = 1 + a_1\left(\frac{h}{v}\right) + a_2\left(\frac{h}{v}\right)^2 + \dots$

$\widetilde{\mathcal{F}}\left(\frac{h}{v}\right) = \widetilde{a}_1\left(\frac{h}{v}\right) + \widetilde{a}_2\left(\frac{h}{v}\right)^2 + \dots$

Example for operators

Classes of counterterms:

$$UD^2H, \quad UD^4H, \quad UHXD^2, \quad UHX^2,$$
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$$\mathcal{O}_{\Psi S1/2} = \bar{q}UP_{\pm}r\langle L_\mu L^\mu\rangle \, \mathcal{F}$$

$$\mathcal{O}_{\Psi T1/2} = \bar{q}\sigma_{\mu\nu}UP_{\pm}r\langle \tau_L L_\mu L_\nu\rangle \, \mathcal{F}$$

$\sum$: 28 operators + h.c.

$\mathcal{F}\left(\frac{h}{v}\right) = 1 + a_1\left(\frac{h}{v}\right) + a_2\left(\frac{h}{v}\right)^2 + \dots$ $\qquad$ $\widetilde{\mathcal{F}}\left(\frac{h}{v}\right) = \widetilde{a}_1\left(\frac{h}{v}\right) + \widetilde{a}_2\left(\frac{h}{v}\right)^2 + \dots$

Example for operators

Classes of counterterms:

$$UD^2H, \quad UD^4H, \quad UHXD^2, \quad UHX^2,$$
$$UHD\Psi^2, \quad UHD^2\Psi^2, \quad UH\Psi^2X \text{ and } \Psi^4UH.$$

$$\mathcal{O}_{\Psi X1/2} = \bar{q}\sigma_{\mu\nu}UP_{\pm}rB^{\mu\nu}\,\mathcal{F}$$

$\sum$: 11 operators + h.c.

$$\mathcal{F}\left(\frac{h}{v}\right) = 1 + a_1\left(\frac{h}{v}\right) + a_2\left(\frac{h}{v}\right)^2 + \ldots \qquad \widetilde{\mathcal{F}}\left(\frac{h}{v}\right) = \widetilde{a}_1\left(\frac{h}{v}\right) + \widetilde{a}_2\left(\frac{h}{v}\right)^2 + \ldots$$

Example for operators

Classes of counterterms:

$$UD^2H,\quad UD^4H,\quad UHXD^2,\quad UHX^2,$$
$$UHD\Psi^2,\quad UHD^2\Psi^2,\quad UH\Psi^2X \text{ and } \Psi^4 UH.$$

$$\mathcal{O}_{LL1} = (\bar{q}\gamma^{\mu} q)(\bar{q}\gamma_{\mu} q)\,\mathcal{F}$$

$\sum$: 64 operators (+ h.c.)

$$\mathcal{F}\left(\frac{h}{v}\right) = 1 + a_1\left(\frac{h}{v}\right) + a_2\left(\frac{h}{v}\right)^2 + \ldots \qquad \widetilde{\mathcal{F}}\left(\frac{h}{v}\right) = \widetilde{a}_1\left(\frac{h}{v}\right) + \widetilde{a}_2\left(\frac{h}{v}\right)^2 + \ldots$$

Conclusions

- A full set of operators, including also the CP-odd terms was constructed.

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Conclusions

- A full set of operators, including also the CP-odd terms was constructed.
- It was explained in detail what systematics defines next-to-leading order of the effective expansion with the use of a power-counting formula.
- Several tests have been performed in order to check whether the presented operator basis contains redundancies.
- The presented basis of operators extends the basis of (Buchalla and Catà, arXiv:1203.6510, JHEP), where no light scalar was included.

Thank you very much for your attention!
Feel free to ask questions

Backup

Counterterms without fermions

UD^2H

$$\mathcal{O}_{\beta_1} = v^2 \langle \tau_L L_\mu \rangle \langle \tau_L L^\mu \rangle \, \mathcal{F}$$

UD^4H

$$\mathcal{O}_{D0,1} = \langle L_\mu L^\mu \rangle \langle L_\nu L^\nu \rangle \, \mathcal{F} \qquad \mathcal{O}_{D0,2} = \langle L_\mu L_\nu \rangle \langle L^\mu L^\nu \rangle \, \mathcal{F}$$

$$\mathcal{O}_{D0,3} = \langle \tau_L L_\mu \rangle \langle \tau_L L^\mu \rangle \langle \tau_L L_\nu \rangle \langle \tau_L L^\nu \rangle \, \mathcal{F} \qquad \mathcal{O}_{D0,4} = \langle \tau_L L_\mu \rangle \langle \tau_L L^\mu \rangle \langle L_\nu L^\nu \rangle \, \mathcal{F}$$

$$\mathcal{O}_{D0,5} = \langle \tau_L L_\mu \rangle \langle \tau_L L_\nu \rangle \langle L^\mu L^\nu \rangle \, \mathcal{F}$$

$$\mathcal{O}_{D1,1} = \langle \tau_L L_\mu \rangle \langle \tau_L L^\mu \rangle \langle \tau_L L_\nu \rangle \left(\partial^\nu \tfrac{h}{v}\right) \mathcal{F} \qquad \mathcal{O}_{D1,2} = \langle L_\mu L^\mu \rangle \langle \tau_L L_\nu \rangle \left(\partial^\nu \tfrac{h}{v}\right) \mathcal{F}$$

$$\mathcal{O}_{D1,3} = \langle \tau_L L_\mu L_\nu \rangle \langle \tau_L L^\mu \rangle \left(\partial^\nu \tfrac{h}{v}\right) \mathcal{F} \qquad \mathcal{O}_{D1,4} = \langle L_\mu L_\nu \rangle \langle \tau_L L^\mu \rangle \left(\partial^\nu \tfrac{h}{v}\right) \mathcal{F}$$

$$\mathcal{O}_{D2,1} = \langle \tau_L L_\mu \rangle \langle \tau_L L_\nu \rangle \left(\partial^\nu \tfrac{h}{v}\right) \left(\partial^\mu \tfrac{h}{v}\right) \mathcal{F} \qquad \mathcal{O}_{D2,3} = \langle L_\mu L_\nu \rangle \left(\partial^\nu \tfrac{h}{v}\right) \left(\partial^\mu \tfrac{h}{v}\right) \mathcal{F}$$

$$\mathcal{O}_{D2,2} = \langle \tau_L L_\mu \rangle \langle \tau_L L^\mu \rangle \left(\partial_\nu \tfrac{h}{v}\right) \left(\partial^\nu \tfrac{h}{v}\right) \mathcal{F} \qquad \mathcal{O}_{D2,4} = \langle L_\mu L^\mu \rangle \left(\partial_\nu \tfrac{h}{v}\right) \left(\partial^\nu \tfrac{h}{v}\right) \mathcal{F}$$

$$\mathcal{O}_{D3,1} = \langle \tau_L L_\mu \rangle \left(\partial^\mu \tfrac{h}{v}\right) \left(\partial_\nu \tfrac{h}{v}\right) \left(\partial^\nu \tfrac{h}{v}\right) \mathcal{F} \qquad \mathcal{O}_{D4,1} = \left(\partial^\mu \tfrac{h}{v}\right) \left(\partial^\nu \tfrac{h}{v}\right) \left(\partial_\mu \tfrac{h}{v}\right) \left(\partial_\nu \tfrac{h}{v}\right) \mathcal{F}$$

$$\mathcal{F}\left(\tfrac{h}{v}\right) = 1 + a_1 \left(\tfrac{h}{v}\right) + a_2 \left(\tfrac{h}{v}\right)^2 + \ldots$$

Counterterms without fermions

UD^2H

$$\mathcal{O}_{\beta_1} = v^2 \langle \tau_L L_\mu \rangle \langle \tau_L L^\mu \rangle \, \mathcal{F}$$

UD^4H

$$\mathcal{O}_{D0,1} = \langle L_\mu L^\mu \rangle \langle L_\nu L^\nu \rangle \, \mathcal{F} \qquad \mathcal{O}_{D0,2} = \langle L_\mu L_\nu \rangle \langle L^\mu L^\nu \rangle \, \mathcal{F}$$

$$\mathcal{O}_{D0,3} = \langle \tau_L L_\mu \rangle \langle \tau_L L^\mu \rangle \langle \tau_L L_\nu \rangle \langle \tau_L L^\nu \rangle \, \mathcal{F} \qquad \mathcal{O}_{D0,4} = \langle \tau_L L_\mu \rangle \langle \tau_L L^\mu \rangle \langle L_\nu L^\nu \rangle \, \mathcal{F}$$

$$\mathcal{O}_{D0,5} = \langle \tau_L L_\mu \rangle \langle \tau_L L_\nu \rangle \langle L^\mu L^\nu \rangle \, \mathcal{F}$$

$$\mathcal{O}_{D1,1} = \langle \tau_L L_\mu \rangle \langle \tau_L L^\mu \rangle \langle \tau_L L_\nu \rangle \left(\partial^\nu \tfrac{h}{v}\right) \mathcal{F} \qquad \mathcal{O}_{D1,2} = \langle L_\mu L^\mu \rangle \langle \tau_L L_\nu \rangle \left(\partial^\nu \tfrac{h}{v}\right) \mathcal{F}$$

$$\mathcal{O}_{D1,3} = \langle \tau_L L_\mu L_\nu \rangle \langle \tau_L L^\mu \rangle \left(\partial^\nu \tfrac{h}{v}\right) \mathcal{F} \qquad \mathcal{O}_{D1,4} = \langle L_\mu L_\nu \rangle \langle \tau_L L^\mu \rangle \left(\partial^\nu \tfrac{h}{v}\right) \mathcal{F}$$

$$\mathcal{O}_{D2,1} = \langle \tau_L L_\mu \rangle \langle \tau_L L_\nu \rangle \left(\partial^\nu \tfrac{h}{v}\right) \left(\partial^\mu \tfrac{h}{v}\right) \mathcal{F} \qquad \mathcal{O}_{D2,3} = \langle L_\mu L_\nu \rangle \left(\partial^\nu \tfrac{h}{v}\right) \left(\partial^\mu \tfrac{h}{v}\right) \mathcal{F}$$

$$\mathcal{O}_{D2,2} = \langle \tau_L L_\mu \rangle \langle \tau_L L^\mu \rangle \left(\partial_\nu \tfrac{h}{v}\right) \left(\partial^\nu \tfrac{h}{v}\right) \mathcal{F} \qquad \mathcal{O}_{D2,4} = \langle L_\mu L^\mu \rangle \left(\partial_\nu \tfrac{h}{v}\right) \left(\partial^\nu \tfrac{h}{v}\right) \mathcal{F}$$

$$\mathcal{O}_{D3,1} = \langle \tau_L L_\mu \rangle \left(\partial^\mu \tfrac{h}{v}\right) \left(\partial_\nu \tfrac{h}{v}\right) \left(\partial^\nu \tfrac{h}{v}\right) \mathcal{F} \qquad \mathcal{O}_{D4,1} = \left(\partial^\mu \tfrac{h}{v}\right) \left(\partial^\nu \tfrac{h}{v}\right) \left(\partial_\mu \tfrac{h}{v}\right) \left(\partial_\nu \tfrac{h}{v}\right) \mathcal{F}$$

$$\mathcal{F}\left(\tfrac{h}{v}\right) = 1 + a_1 \left(\tfrac{h}{v}\right) + a_2 \left(\tfrac{h}{v}\right)^2 + \dots$$

ξ ξ^2 ξ^3 ξ^4

Counterterms without fermions II

$UHXD^2$

$$\mathcal{O}_{XUD1} = \langle \tau_L L_\mu L_\nu \rangle B^{\mu\nu}\, \widetilde{\mathcal{F}}$$
$$\mathcal{O}_{XUD2} = \langle \tau_L L_\mu L_\nu \rangle \widetilde{B}^{\mu\nu}\, \widetilde{\mathcal{F}}$$
$$\mathcal{O}_{XUD3} = \langle \tau_L L_\mu L_\nu \rangle \langle \tau_L W^{\mu\nu} \rangle\, \widetilde{\mathcal{F}}$$
$$\mathcal{O}_{XUD4} = \langle \tau_L L_\mu L_\nu \rangle \langle \tau_L \widetilde{W}^{\mu\nu} \rangle\, \widetilde{\mathcal{F}}$$
$$\mathcal{O}_{XUD5} = \langle L_\mu L_\nu W^{\mu\nu} \rangle\, \widetilde{\mathcal{F}}$$
$$\mathcal{O}_{XUD6} = \langle L_\mu L_\nu \widetilde{W}^{\mu\nu} \rangle\, \widetilde{\mathcal{F}}$$
$$\mathcal{O}_{XUD7} = \langle \tau_L L_\mu \rangle \langle L_\nu W^{\mu\nu} \rangle\, \mathcal{F}$$
$$\mathcal{O}_{XUD8} = \langle \tau_L L_\mu \rangle \langle L_\nu \widetilde{W}^{\mu\nu} \rangle\, \mathcal{F}$$

UHX^2

$$\mathcal{O}_{XU1} = B_{\mu\nu} B^{\mu\nu}\, \widetilde{\mathcal{F}}$$
$$\mathcal{O}_{XU2} = B_{\mu\nu} \widetilde{B}^{\mu\nu}\, \widetilde{\mathcal{F}}$$
$$\mathcal{O}_{XU3} = W_{\mu\nu} W^{\mu\nu}\, \widetilde{\mathcal{F}}$$
$$\mathcal{O}_{XU4} = W_{\mu\nu} \widetilde{W}^{\mu\nu}\, \widetilde{\mathcal{F}}$$
$$\mathcal{O}_{XU5} = G_{\mu\nu} G^{\mu\nu}\, \widetilde{\mathcal{F}}$$
$$\mathcal{O}_{XU6} = G_{\mu\nu} \widetilde{G}^{\mu\nu}\, \widetilde{\mathcal{F}}$$
$$\mathcal{O}_{XU7} = \langle \tau_L W_{\mu\nu} \rangle \langle \tau_L W^{\mu\nu} \rangle\, \mathcal{F}$$
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$$\mathcal{O}_{XU9} = B_{\mu\nu} \langle \tau_L W^{\mu\nu} \rangle\, \mathcal{F}$$
$$\mathcal{O}_{XU10} = B_{\mu\nu} \langle \tau_L \widetilde{W}^{\mu\nu} \rangle\, \mathcal{F}$$

$$\mathcal{F}\left(\tfrac{h}{v}\right) = 1 + a_1 \left(\tfrac{h}{v}\right) + a_2 \left(\tfrac{h}{v}\right)^2 + \ldots$$

$$\widetilde{\mathcal{F}}\left(\tfrac{h}{v}\right) = \widetilde{a}_1 \left(\tfrac{h}{v}\right) + \widetilde{a}_2 \left(\tfrac{h}{v}\right)^2 + \ldots$$

Counterterms without fermions II

$UHXD^2$

$$\mathcal{O}_{XUD1} = \langle \tau_L L_\mu L_\nu \rangle B^{\mu\nu}\, \widetilde{\mathcal{F}}$$
$$\mathcal{O}_{XUD2} = \langle \tau_L L_\mu L_\nu \rangle \widetilde{B}^{\mu\nu}\, \widetilde{\mathcal{F}}$$
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$$\mathcal{O}_{XUD7} = \langle \tau_L L_\mu \rangle \langle L_\nu W^{\mu\nu} \rangle\, \mathcal{F}$$
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UHX^2

$$\mathcal{O}_{XU1} = B_{\mu\nu} B^{\mu\nu}\, \widetilde{\mathcal{F}}$$
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$$\mathcal{F}\left(\frac{h}{v}\right) = 1 + a_1 \left(\frac{h}{v}\right) + a_2 \left(\frac{h}{v}\right)^2 + \dots \qquad \widetilde{\mathcal{F}}\left(\frac{h}{v}\right) = \widetilde{a}_1 \left(\frac{h}{v}\right) + \widetilde{a}_2 \left(\frac{h}{v}\right)^2 + \dots$$

ξ $\quad \xi^2 \quad \xi^3 \quad \xi^4$

Counterterms with two fermions I

$UHD\Psi^2$

$$\mathcal{O}_{\Psi V1} = (\bar{q}\gamma^\mu q)\langle \tau_L L_\mu\rangle\,\mathcal{F}$$
$$\mathcal{O}_{\Psi V2} = (\bar{q}\gamma^\mu \tau_L q)\langle \tau_L L_\mu\rangle\,\mathcal{F}$$
$$\mathcal{O}_{\Psi V3} = (\bar{q}\gamma^\mu UP_{12}U^\dagger q)\langle UP_{21}U^\dagger L_\mu\rangle\,\mathcal{F}$$
$$\mathcal{O}_{\Psi V4} = (\bar{u}\gamma^\mu u)\langle \tau_L L_\mu\rangle\,\mathcal{F}$$
$$\mathcal{O}_{\Psi V5} = (\bar{d}\gamma^\mu d)\langle \tau_L L_\mu\rangle\,\mathcal{F}$$
$$\mathcal{O}_{\Psi V6} = (\bar{u}\gamma^\mu d)\langle UP_{21}U^\dagger L_\mu\rangle\,\mathcal{F}$$
$$\mathcal{O}_{\Psi V7} = (\bar{l}\gamma^\mu l)\langle \tau_L L_\mu\rangle\,\mathcal{F}$$
$$\mathcal{O}_{\Psi V8} = (\bar{l}\gamma^\mu \tau_L l)\langle \tau_L L_\mu\rangle\,\mathcal{F}$$
$$\mathcal{O}_{\Psi V9} = (\bar{l}\gamma^\mu UP_{12}U^\dagger l)\langle UP_{21}U^\dagger L_\mu\rangle\,\mathcal{F}$$
$$\mathcal{O}_{\Psi V10} = (\bar{e}\gamma^\mu e)\langle \tau_L L_\mu\rangle\,\mathcal{F}$$
$$\mathcal{O}_{\Psi V3^\dagger}, \mathcal{O}_{\Psi V6^\dagger}, \mathcal{O}_{\Psi V9^\dagger}$$

$UH\Psi^2 X$

$$\mathcal{O}_{\Psi X1} = \bar{q}\sigma_{\mu\nu}UP_+ rB^{\mu\nu}\,\mathcal{F}$$
$$\mathcal{O}_{\Psi X2} = \bar{q}\sigma_{\mu\nu}UP_- rB^{\mu\nu}\,\mathcal{F}$$
$$\mathcal{O}_{\Psi X3} = \bar{q}\sigma_{\mu\nu}UP_+ r\langle \tau_L W^{\mu\nu}\rangle\,\mathcal{F}$$
$$\mathcal{O}_{\Psi X4} = \bar{q}\sigma_{\mu\nu}UP_- r\langle \tau_L W^{\mu\nu}\rangle\,\mathcal{F}$$
$$\mathcal{O}_{\Psi X5} = \bar{q}\sigma_{\mu\nu}UP_{12} r\langle UP_{21}U^\dagger W^{\mu\nu}\rangle\,\mathcal{F}$$
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$$\mathcal{O}_{\Psi X7} = \bar{q}\sigma_{\mu\nu}G^{\mu\nu}UP_+ r\,\mathcal{F}$$
$$\mathcal{O}_{\Psi X8} = \bar{q}\sigma_{\mu\nu}G^{\mu\nu}UP_- r\,\mathcal{F}$$
$$\mathcal{O}_{\Psi X9} = \bar{l}\sigma_{\mu\nu}UP_-\eta B^{\mu\nu}\,\mathcal{F}$$
$$\mathcal{O}_{\Psi X10} = \bar{l}\sigma_{\mu\nu}UP_-\eta\langle \tau_L W^{\mu\nu}\rangle\,\mathcal{F}$$
$$\mathcal{O}_{\Psi X11} = \bar{l}\sigma_{\mu\nu}UP_{12}\eta\langle UP_{21}U^\dagger W^{\mu\nu}\rangle\,\mathcal{F}$$

$$\mathcal{F}\left(\tfrac{h}{v}\right) = 1 + a_1\left(\tfrac{h}{v}\right) + a_2\left(\tfrac{h}{v}\right)^2 + \dots$$

Counterterms with two fermions I

$UHD\Psi^2$

$$\mathcal{O}_{\Psi V1} = (\bar{q}\gamma^\mu q)\langle\tau_L L_\mu\rangle\,\mathcal{F}$$
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$$\mathcal{O}_{\Psi V9} = (\bar{l}\gamma^\mu UP_{12}U^\dagger l)\langle UP_{21}U^\dagger L_\mu\rangle\,\mathcal{F}$$
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$$\mathcal{O}_{\Psi V3^\dagger}, \mathcal{O}_{\Psi V6^\dagger}, \mathcal{O}_{\Psi V9^\dagger}$$

$UH\Psi^2 X$

$$\mathcal{O}_{\Psi X1} = \bar{q}\sigma_{\mu\nu}UP_+ rB^{\mu\nu}\,\mathcal{F}$$
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$$\mathcal{O}_{\Psi X3} = \bar{q}\sigma_{\mu\nu}UP_+ r\langle\tau_L W^{\mu\nu}\rangle\,\mathcal{F}$$
$$\mathcal{O}_{\Psi X4} = \bar{q}\sigma_{\mu\nu}UP_- r\langle\tau_L W^{\mu\nu}\rangle\,\mathcal{F}$$
$$\mathcal{O}_{\Psi X5} = \bar{q}\sigma_{\mu\nu}UP_{12} r\langle UP_{21}U^\dagger W^{\mu\nu}\rangle\,\mathcal{F}$$
$$\mathcal{O}_{\Psi X6} = \bar{q}\sigma_{\mu\nu}UP_{21} r\langle UP_{12}U^\dagger W^{\mu\nu}\rangle\,\mathcal{F}$$
$$\mathcal{O}_{\Psi X7} = \bar{q}\sigma_{\mu\nu}G^{\mu\nu}UP_+ r\,\mathcal{F}$$
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$$\mathcal{O}_{\Psi X9} = \bar{l}\sigma_{\mu\nu}UP_- \eta B^{\mu\nu}\,\mathcal{F}$$
$$\mathcal{O}_{\Psi X10} = \bar{l}\sigma_{\mu\nu}UP_- \eta\langle\tau_L W^{\mu\nu}\rangle\,\mathcal{F}$$
$$\mathcal{O}_{\Psi X11} = \bar{l}\sigma_{\mu\nu}UP_{12} \eta\langle UP_{21}U^\dagger W^{\mu\nu}\rangle\,\mathcal{F}$$

$$\mathcal{F}\left(\tfrac{h}{v}\right) = 1 + a_1\left(\tfrac{h}{v}\right) + a_2\left(\tfrac{h}{v}\right)^2 + \dots$$

ξ $\quad\xi^2$ $\quad\xi^3$ $\quad\xi^4$

Counterterms with two fermions II

$UHD^2\Psi^2$ scalar currents

$$\mathcal{O}_{\Psi S1} = \bar{q} U P_+ r \langle L_\mu L^\mu \rangle \, \mathcal{F} \qquad \mathcal{O}_{\Psi S2} = \bar{q} U P_- r \langle L_\mu L^\mu \rangle \, \mathcal{F}$$

$$\mathcal{O}_{\Psi S3} = \bar{q} U P_+ r \langle \tau_L L_\mu \rangle \langle \tau_L L^\mu \rangle \, \mathcal{F} \qquad \mathcal{O}_{\Psi S4} = \bar{q} U P_- r \langle \tau_L L_\mu \rangle \langle \tau_L L^\mu \rangle \, \mathcal{F}$$

$$\mathcal{O}_{\Psi S5} = \bar{q} U P_{12} r \langle \tau_L L_\mu \rangle \langle U P_{21} U^\dagger L^\mu \rangle \, \mathcal{F} \qquad \mathcal{O}_{\Psi S6} = \bar{q} U P_{21} r \langle \tau_L L_\mu \rangle \langle U P_{12} U^\dagger L^\mu \rangle \, \mathcal{F}$$

$$\mathcal{O}_{\Psi S7} = \bar{l} U P_- \eta \langle L_\mu L^\mu \rangle \, \mathcal{F} \qquad \mathcal{O}_{\Psi S8} = \bar{l} U P_- \eta \langle \tau_L L_\mu \rangle \langle \tau_L L^\mu \rangle \, \mathcal{F}$$

$$\mathcal{O}_{\Psi S9} = \bar{l} U P_{12} \eta \langle \tau_L L_\mu \rangle \langle U P_{21} U^\dagger L^\mu \rangle \, \mathcal{F}$$

$$\mathcal{O}_{\Psi S10} = \bar{q} U P_+ r \langle \tau_L L_\mu \rangle \left(\partial^\mu \tfrac{h}{v}\right) \mathcal{F} \qquad \mathcal{O}_{\Psi S11} = \bar{q} U P_- r \langle \tau_L L_\mu \rangle \left(\partial^\mu \tfrac{h}{v}\right) \mathcal{F}$$

$$\mathcal{O}_{\Psi S12} = \bar{q} U P_{12} r \langle U P_{21} U^\dagger L_\mu \rangle \left(\partial^\mu \tfrac{h}{v}\right) \mathcal{F} \qquad \mathcal{O}_{\Psi S13} = \bar{q} U P_{21} r \langle U P_{12} U^\dagger L_\mu \rangle \left(\partial^\mu \tfrac{h}{v}\right) \mathcal{F}$$

$$\mathcal{O}_{\Psi S14} = \bar{l} U P_- \eta \langle \tau_L L_\mu \rangle \left(\partial^\mu \tfrac{h}{v}\right) \mathcal{F} \qquad \mathcal{O}_{\Psi S15} = \bar{l} U P_{12} \eta \langle U P_{21} U^\dagger L_\mu \rangle \left(\partial^\mu \tfrac{h}{v}\right) \mathcal{F}$$

$$\mathcal{O}_{\Psi S16} = \bar{q} U P_+ r \left(\partial_\mu \tfrac{h}{v}\right) \left(\partial^\mu \tfrac{h}{v}\right) \mathcal{F} \qquad \mathcal{O}_{\Psi S17} = \bar{q} U P_- r \left(\partial_\mu \tfrac{h}{v}\right) \left(\partial^\mu \tfrac{h}{v}\right) \mathcal{F}$$

$$\mathcal{O}_{\Psi S18} = \bar{l} U P_- \eta \left(\partial_\mu \tfrac{h}{v}\right) \left(\partial^\mu \tfrac{h}{v}\right) \mathcal{F}$$

$$\mathcal{F}\left(\tfrac{h}{v}\right) = 1 + a_1 \left(\tfrac{h}{v}\right) + a_2 \left(\tfrac{h}{v}\right)^2 + \ldots$$

Counterterms with two fermions II

$UHD^2\Psi^2$ scalar currents

$\mathcal{O}_{\Psi S1} = \bar{q} U P_+ r \langle L_\mu L^\mu \rangle \mathcal{F}$

$\mathcal{O}_{\Psi S2} = \bar{q} U P_- r \langle L_\mu L^\mu \rangle \mathcal{F}$

$\mathcal{O}_{\Psi S3} = \bar{q} U P_+ r \langle \tau_L L_\mu \rangle \langle \tau_L L^\mu \rangle \mathcal{F}$

$\mathcal{O}_{\Psi S4} = \bar{q} U P_- r \langle \tau_L L_\mu \rangle \langle \tau_L L^\mu \rangle \mathcal{F}$

$\mathcal{O}_{\Psi S5} = \bar{q} U P_{12} r \langle \tau_L L_\mu \rangle \langle U P_{21} U^\dagger L^\mu \rangle \mathcal{F}$

$\mathcal{O}_{\Psi S6} = \bar{q} U P_{21} r \langle \tau_L L_\mu \rangle \langle U P_{12} U^\dagger L^\mu \rangle \mathcal{F}$

$\mathcal{O}_{\Psi S7} = \bar{l} U P_- \eta \langle L_\mu L^\mu \rangle \mathcal{F}$

$\mathcal{O}_{\Psi S8} = \bar{l} U P_- \eta \langle \tau_L L_\mu \rangle \langle \tau_L L^\mu \rangle \mathcal{F}$

$\mathcal{O}_{\Psi S9} = \bar{l} U P_{12} \eta \langle \tau_L L_\mu \rangle \langle U P_{21} U^\dagger L^\mu \rangle \mathcal{F}$

$\mathcal{O}_{\Psi S10} = \bar{q} U P_+ r \langle \tau_L L_\mu \rangle \left(\partial^\mu \frac{h}{v}\right) \mathcal{F}$

$\mathcal{O}_{\Psi S11} = \bar{q} U P_- r \langle \tau_L L_\mu \rangle \left(\partial^\mu \frac{h}{v}\right) \mathcal{F}$

$\mathcal{O}_{\Psi S12} = \bar{q} U P_{12} r \langle U P_{21} U^\dagger L_\mu \rangle \left(\partial^\mu \frac{h}{v}\right) \mathcal{F}$

$\mathcal{O}_{\Psi S13} = \bar{q} U P_{21} r \langle U P_{12} U^\dagger L_\mu \rangle \left(\partial^\mu \frac{h}{v}\right) \mathcal{F}$

$\mathcal{O}_{\Psi S14} = \bar{l} U P_- \eta \langle \tau_L L_\mu \rangle \left(\partial^\mu \frac{h}{v}\right) \mathcal{F}$

$\mathcal{O}_{\Psi S15} = \bar{l} U P_{12} \eta \langle U P_{21} U^\dagger L_\mu \rangle \left(\partial^\mu \frac{h}{v}\right) \mathcal{F}$

$\mathcal{O}_{\Psi S16} = \bar{q} U P_+ r \left(\partial_\mu \frac{h}{v}\right) \left(\partial^\mu \frac{h}{v}\right) \mathcal{F}$

$\mathcal{O}_{\Psi S17} = \bar{q} U P_- r \left(\partial_\mu \frac{h}{v}\right) \left(\partial^\mu \frac{h}{v}\right) \mathcal{F}$

$\mathcal{O}_{\Psi S18} = \bar{l} U P_- \eta \left(\partial_\mu \frac{h}{v}\right) \left(\partial^\mu \frac{h}{v}\right) \mathcal{F}$

$\mathcal{F}\left(\frac{h}{v}\right) = 1 + a_1 \left(\frac{h}{v}\right) + a_2 \left(\frac{h}{v}\right)^2 + \dots$

ξ $\quad \xi^2$ $\quad \xi^3$ $\quad \xi^4$

Counterterms with two fermions III

$UHD^2\Psi^2$ tensor currents

$$\mathcal{O}_{\Psi T1} = \bar{q}\sigma_{\mu\nu} UP_+ r\langle\tau_L L_\mu L_\nu\rangle\,\mathcal{F}$$
$$\mathcal{O}_{\Psi T2} = \bar{q}\sigma_{\mu\nu} UP_- r\langle\tau_L L_\mu L_\nu\rangle\,\mathcal{F}$$
$$\mathcal{O}_{\Psi T3} = \bar{q}\sigma_{\mu\nu} UP_{12} r\langle\tau_L L^\mu\rangle\langle UP_{21}U^\dagger L^\nu\rangle\,\mathcal{F}$$
$$\mathcal{O}_{\Psi T4} = \bar{q}\sigma_{\mu\nu} UP_{21} r\langle\tau_L L^\mu\rangle\langle UP_{12}U^\dagger L^\nu\rangle\,\mathcal{F}$$
$$\mathcal{O}_{\Psi T5} = \bar{l}\sigma_{\mu\nu} UP_{12}\eta\langle\tau_L L^\mu\rangle\langle UP_{21}U^\dagger L^\nu\rangle\,\mathcal{F}$$
$$\mathcal{O}_{\Psi T6} = \bar{l}\sigma_{\mu\nu} UP_-\eta\langle\tau_L L_\mu L_\nu\rangle\,\mathcal{F}$$
$$\mathcal{O}_{\Psi T7} = \bar{q}\sigma_{\mu\nu} UP_+ r\langle\tau_L L^\mu\rangle\left(\partial^\nu\tfrac{h}{v}\right)\,\mathcal{F}$$
$$\mathcal{O}_{\Psi T8} = \bar{q}\sigma_{\mu\nu} UP_- r\langle\tau_L L^\mu\rangle\left(\partial^\nu\tfrac{h}{v}\right)\,\mathcal{F}$$
$$\mathcal{O}_{\Psi T9} = \bar{q}\sigma_{\mu\nu} UP_{21} r\langle UP_{12}U^\dagger L^\mu\rangle\left(\partial^\nu\tfrac{h}{v}\right)\,\mathcal{F}$$
$$\mathcal{O}_{\Psi T10} = \bar{q}\sigma_{\mu\nu} UP_{12} r\langle UP_{21}U^\dagger L^\mu\rangle\left(\partial^\nu\tfrac{h}{v}\right)\,\mathcal{F}$$
$$\mathcal{O}_{\Psi T11} = \bar{l}\sigma_{\mu\nu} UP_-\eta\langle\tau_L L^\mu\rangle\left(\partial^\nu\tfrac{h}{v}\right)\,\mathcal{F}$$
$$\mathcal{O}_{\Psi T12} = \bar{l}\sigma_{\mu\nu} UP_{12}\eta\langle UP_{21}U^\dagger L^\mu\rangle\left(\partial^\nu\tfrac{h}{v}\right)\,\mathcal{F}$$

$$\mathcal{F}\left(\tfrac{h}{v}\right) = 1 + a_1\left(\tfrac{h}{v}\right) + a_2\left(\tfrac{h}{v}\right)^2 + \ldots$$

Counterterms with two fermions III

$UHD^2\Psi^2$ tensor currents

$$\mathcal{O}_{\Psi T1} = \bar{q}\sigma_{\mu\nu} UP_{+} r \langle \tau_L L_\mu L_\nu \rangle \, \mathcal{F}$$
$$\mathcal{O}_{\Psi T2} = \bar{q}\sigma_{\mu\nu} UP_{-} r \langle \tau_L L_\mu L_\nu \rangle \, \mathcal{F}$$
$$\mathcal{O}_{\Psi T3} = \bar{q}\sigma_{\mu\nu} UP_{12} r \langle \tau_L L^\mu \rangle \langle UP_{21} U^\dagger L^\nu \rangle \, \mathcal{F}$$
$$\mathcal{O}_{\Psi T4} = \bar{q}\sigma_{\mu\nu} UP_{21} r \langle \tau_L L^\mu \rangle \langle UP_{12} U^\dagger L^\nu \rangle \, \mathcal{F}$$
$$\mathcal{O}_{\Psi T5} = \bar{l}\sigma_{\mu\nu} UP_{12} \eta \langle \tau_L L^\mu \rangle \langle UP_{21} U^\dagger L^\nu \rangle \, \mathcal{F}$$
$$\mathcal{O}_{\Psi T6} = \bar{l}\sigma_{\mu\nu} UP_{-} \eta \langle \tau_L L_\mu L_\nu \rangle \, \mathcal{F}$$

$$\mathcal{O}_{\Psi T7} = \bar{q}\sigma_{\mu\nu} UP_{+} r \langle \tau_L L^\mu \rangle \left(\partial^\nu \tfrac{h}{v}\right) \mathcal{F}$$
$$\mathcal{O}_{\Psi T8} = \bar{q}\sigma_{\mu\nu} UP_{-} r \langle \tau_L L^\mu \rangle \left(\partial^\nu \tfrac{h}{v}\right) \mathcal{F}$$
$$\mathcal{O}_{\Psi T9} = \bar{q}\sigma_{\mu\nu} UP_{21} r \langle UP_{12} U^\dagger L^\mu \rangle \left(\partial^\nu \tfrac{h}{v}\right) \mathcal{F}$$
$$\mathcal{O}_{\Psi T10} = \bar{q}\sigma_{\mu\nu} UP_{12} r \langle UP_{21} U^\dagger L^\mu \rangle \left(\partial^\nu \tfrac{h}{v}\right) \mathcal{F}$$
$$\mathcal{O}_{\Psi T11} = \bar{l}\sigma_{\mu\nu} UP_{-} \eta \langle \tau_L L^\mu \rangle \left(\partial^\nu \tfrac{h}{v}\right) \mathcal{F}$$
$$\mathcal{O}_{\Psi T12} = \bar{l}\sigma_{\mu\nu} UP_{12} \eta \langle UP_{21} U^\dagger L^\mu \rangle \left(\partial^\nu \tfrac{h}{v}\right) \mathcal{F}$$

$$\mathcal{F}\left(\tfrac{h}{v}\right) = 1 + a_1 \left(\tfrac{h}{v}\right) + a_2 \left(\tfrac{h}{v}\right)^2 + \dots$$

ξ ξ^2 ξ^3 ξ^4

Counterterms with four fermions

$\Psi^4 UH : \bar{L}L\bar{L}L$

$$\begin{aligned}
\mathcal{O}_{LL1} &= (\bar{q}\gamma^\mu q)(\bar{q}\gamma_\mu q)\,\mathcal{F}\\
\mathcal{O}_{LL2} &= (\bar{q}\gamma^\mu T^a q)(\bar{q}\gamma_\mu T^a q)\,\mathcal{F}\\
\mathcal{O}_{LL3} &= (\bar{q}\gamma^\mu q)(\bar{l}\gamma_\mu l)\,\mathcal{F}\\
\mathcal{O}_{LL4} &= (\bar{q}\gamma^\mu T^a q)(\bar{l}\gamma_\mu T^a l)\,\mathcal{F}\\
\mathcal{O}_{LL5} &= (\bar{l}\gamma^\mu l)(\bar{l}\gamma_\mu l)\,\mathcal{F}\\
\mathcal{O}_{LL6} &= (\bar{q}\gamma^\mu \tau_L q)(\bar{q}\gamma_\mu \tau_L q)\,\mathcal{F}\\
\mathcal{O}_{LL7} &= (\bar{q}\gamma^\mu \tau_L q)(\bar{q}\gamma_\mu q)\,\mathcal{F}\\
\mathcal{O}_{LL8} &= (\bar{q}_\alpha\gamma^\mu \tau_L q_\beta)(\bar{q}_\beta\gamma_\mu \tau_L q_\alpha)\,\mathcal{F}\\
\mathcal{O}_{LL9} &= (\bar{q}_\alpha\gamma^\mu \tau_L q_\beta)(\bar{q}_\beta\gamma_\mu q_\alpha)\,\mathcal{F}\\
\mathcal{O}_{LL10} &= (\bar{q}\gamma^\mu \tau_L q)(\bar{l}\gamma_\mu \tau_L l)\,\mathcal{F}\\
\mathcal{O}_{LL11} &= (\bar{q}\gamma^\mu \tau_L q)(\bar{l}\gamma_\mu l)\,\mathcal{F}\\
\mathcal{O}_{LL12} &= (\bar{q}\gamma^\mu q)(\bar{l}\gamma_\mu \tau_L l)\,\mathcal{F}\\
\mathcal{O}_{LL13} &= (\bar{q}\gamma^\mu \tau_L l)(\bar{l}\gamma_\mu \tau_L q)\,\mathcal{F}\\
\mathcal{O}_{LL14} &= (\bar{q}\gamma^\mu \tau_L l)(\bar{l}\gamma_\mu q)\,\mathcal{F}\\
\mathcal{O}_{LL15} &= (\bar{l}\gamma^\mu \tau_L l)(\bar{l}\gamma_\mu \tau_L l)\,\mathcal{F}\\
\mathcal{O}_{LL16} &= (\bar{l}\gamma^\mu \tau_L l)(\bar{l}\gamma_\mu l)\,\mathcal{F}
\end{aligned}$$

$\Psi^4 UH : \bar{R}R\bar{R}R$

$$\begin{aligned}
\mathcal{O}_{RR1} &= (\bar{u}\gamma^\mu u)(\bar{u}\gamma_\mu u)\,\mathcal{F}\\
\mathcal{O}_{RR2} &= (\bar{d}\gamma^\mu d)(\bar{d}\gamma_\mu d)\,\mathcal{F}\\
\mathcal{O}_{RR3} &= (\bar{u}\gamma^\mu u)(\bar{d}\gamma_\mu d)\,\mathcal{F}\\
\mathcal{O}_{RR4} &= (\bar{u}\gamma^\mu T^A u)(\bar{d}\gamma_\mu T^A d)\,\mathcal{F}\\
\mathcal{O}_{RR5} &= (\bar{u}\gamma^\mu u)(\bar{e}\gamma_\mu e)\,\mathcal{F}\\
\mathcal{O}_{RR6} &= (\bar{d}\gamma^\mu d)(\bar{e}\gamma_\mu e)\,\mathcal{F}\\
\mathcal{O}_{RR7} &= (\bar{e}\gamma^\mu e)(\bar{e}\gamma_\mu e)\,\mathcal{F}
\end{aligned}$$

$$\mathcal{F}\left(\frac{h}{v}\right) = 1 + a_1\left(\frac{h}{v}\right) + a_2\left(\frac{h}{v}\right)^2 + \dots$$

Counterterms with four fermions

$\Psi^4 UH : \bar{L}L\bar{L}L$

$$\mathcal{O}_{LL1} = (\bar{q}\gamma^\mu q)(\bar{q}\gamma_\mu q)\,\mathcal{F}$$
$$\mathcal{O}_{LL2} = (\bar{q}\gamma^\mu T^a q)(\bar{q}\gamma_\mu T^a q)\,\mathcal{F}$$
$$\mathcal{O}_{LL3} = (\bar{q}\gamma^\mu q)(\bar{l}\gamma_\mu l)\,\mathcal{F}$$
$$\mathcal{O}_{LL4} = (\bar{q}\gamma^\mu T^a q)(\bar{l}\gamma_\mu T^a l)\,\mathcal{F}$$
$$\mathcal{O}_{LL5} = (\bar{l}\gamma^\mu l)(\bar{l}\gamma_\mu l)\,\mathcal{F}$$
$$\mathcal{O}_{LL6} = (\bar{q}\gamma^\mu \tau_L q)(\bar{q}\gamma_\mu \tau_L q)\,\mathcal{F}$$
$$\mathcal{O}_{LL7} = (\bar{q}\gamma^\mu \tau_L q)(\bar{q}\gamma_\mu q)\,\mathcal{F}$$
$$\mathcal{O}_{LL8} = (\bar{q}_\alpha\gamma^\mu \tau_L q_\beta)(\bar{q}_\beta\gamma_\mu \tau_L q_\alpha)\,\mathcal{F}$$
$$\mathcal{O}_{LL9} = (\bar{q}_\alpha\gamma^\mu \tau_L q_\beta)(\bar{q}_\beta\gamma_\mu q_\alpha)\,\mathcal{F}$$
$$\mathcal{O}_{LL10} = (\bar{q}\gamma^\mu \tau_L q)(\bar{l}\gamma_\mu \tau_L l)\,\mathcal{F}$$
$$\mathcal{O}_{LL11} = (\bar{q}\gamma^\mu \tau_L q)(\bar{l}\gamma_\mu l)\,\mathcal{F}$$
$$\mathcal{O}_{LL12} = (\bar{q}\gamma^\mu q)(\bar{l}\gamma_\mu \tau_L l)\,\mathcal{F}$$
$$\mathcal{O}_{LL13} = (\bar{q}\gamma^\mu \tau_L l)(\bar{l}\gamma_\mu \tau_L q)\,\mathcal{F}$$
$$\mathcal{O}_{LL14} = (\bar{q}\gamma^\mu \tau_L l)(\bar{l}\gamma_\mu q)\,\mathcal{F}$$
$$\mathcal{O}_{LL15} = (\bar{l}\gamma^\mu \tau_L l)(\bar{l}\gamma_\mu \tau_L l)\,\mathcal{F}$$
$$\mathcal{O}_{LL16} = (\bar{l}\gamma^\mu \tau_L l)(\bar{l}\gamma_\mu l)\,\mathcal{F}$$

$\Psi^4 UH : \bar{R}R\bar{R}R$

$$\mathcal{O}_{RR1} = (\bar{u}\gamma^\mu u)(\bar{u}\gamma_\mu u)\,\mathcal{F}$$
$$\mathcal{O}_{RR2} = (\bar{d}\gamma^\mu d)(\bar{d}\gamma_\mu d)\,\mathcal{F}$$
$$\mathcal{O}_{RR3} = (\bar{u}\gamma^\mu u)(\bar{d}\gamma_\mu d)\,\mathcal{F}$$
$$\mathcal{O}_{RR4} = (\bar{u}\gamma^\mu T^A u)(\bar{d}\gamma_\mu T^A d)\,\mathcal{F}$$
$$\mathcal{O}_{RR5} = (\bar{u}\gamma^\mu u)(\bar{e}\gamma_\mu e)\,\mathcal{F}$$
$$\mathcal{O}_{RR6} = (\bar{d}\gamma^\mu d)(\bar{e}\gamma_\mu e)\,\mathcal{F}$$
$$\mathcal{O}_{RR7} = (\bar{e}\gamma^\mu e)(\bar{e}\gamma_\mu e)\,\mathcal{F}$$

$$\mathcal{F}\left(\frac{h}{v}\right) = 1 + a_1\left(\frac{h}{v}\right) + a_2\left(\frac{h}{v}\right)^2 + \dots$$

ξ ξ^2 ξ^3 ξ^4

Counterterms with four fermions II

$\Psi^4 UH : \bar{L}L\bar{R}R$

$$
\begin{aligned}
\mathcal{O}_{LR1} &= (\bar{q}\gamma^{\mu} q)(\bar{u}\gamma_{\mu} u)\,\mathcal{F}\\
\mathcal{O}_{LR2} &= (\bar{q}\gamma^{\mu} T^A q)(\bar{u}\gamma_{\mu} T^A u)\,\mathcal{F}\\
\mathcal{O}_{LR3} &= (\bar{q}\gamma^{\mu} q)(\bar{d}\gamma_{\mu} d)\,\mathcal{F}\\
\mathcal{O}_{LR4} &= (\bar{q}\gamma^{\mu} T^A q)(\bar{d}\gamma_{\mu} T^A d)\,\mathcal{F}\\
\mathcal{O}_{LR5} &= (\bar{u}\gamma^{\mu} u)(\bar{l}\gamma_{\mu} l)\,\mathcal{F}\\
\mathcal{O}_{LR6} &= (\bar{d}\gamma^{\mu} d)(\bar{l}\gamma_{\mu} l)\,\mathcal{F}\\
\mathcal{O}_{LR7} &= (\bar{q}\gamma^{\mu} q)(\bar{e}\gamma_{\mu} e)\,\mathcal{F}\\
\mathcal{O}_{LR8} &= (\bar{l}\gamma^{\mu} l)(\bar{e}\gamma_{\mu} e)\,\mathcal{F}\\
\mathcal{O}_{LR9} &= (\bar{q}\gamma^{\mu} l)(\bar{e}\gamma_{\mu} d)\,\mathcal{F}\\
\mathcal{O}_{LR10} &= (\bar{q}\gamma^{\mu}\tau_L q)(\bar{u}\gamma_{\mu} u)\,\mathcal{F}\\
\mathcal{O}_{LR11} &= (\bar{q}\gamma^{\mu} T^A\tau_L q)(\bar{u}\gamma_{\mu} T^A u)\,\mathcal{F}\\
\mathcal{O}_{LR12} &= (\bar{q}\gamma^{\mu}\tau_L q)(\bar{d}\gamma_{\mu} d)\,\mathcal{F}\\
\mathcal{O}_{LR13} &= (\bar{q}\gamma^{\mu} T^A\tau_L q)(\bar{d}\gamma_{\mu} T^A d)\,\mathcal{F}\\
\mathcal{O}_{LR14} &= (\bar{l}\gamma^{\mu}\tau_L l)(\bar{u}\gamma_{\mu} u)\,\mathcal{F}\\
\mathcal{O}_{LR15} &= (\bar{l}\gamma^{\mu}\tau_L l)(\bar{d}\gamma_{\mu} d)\,\mathcal{F}\\
\mathcal{O}_{LR16} &= (\bar{q}\gamma^{\mu}\tau_L q)(\bar{e}\gamma_{\mu} e)\,\mathcal{F}\\
\mathcal{O}_{LR17} &= (\bar{l}\gamma^{\mu}\tau_L l)(\bar{e}\gamma_{\mu} e)\,\mathcal{F}\\
\mathcal{O}_{LR18} &= (\bar{q}\gamma^{\mu}\tau_L l)(\bar{e}\gamma_{\mu} d)\,\mathcal{F}
\end{aligned}
$$

$\Psi^4 UH : \bar{L}R\bar{L}R$

$$
\begin{aligned}
\mathcal{O}_{ST1} &= \epsilon_{ij}(\bar{q}^i u)(\bar{q}^j d)\,\mathcal{F}\\
\mathcal{O}_{ST2} &= \epsilon_{ij}(\bar{q}^i T^A u)(\bar{q}^j T^A d)\,\mathcal{F}\\
\mathcal{O}_{ST3} &= \epsilon_{ij}(\bar{q}^i u)(\bar{l}^j e)\,\mathcal{F}\\
\mathcal{O}_{ST4} &= \epsilon_{ij}(\bar{q}^i \sigma^{\mu\nu} u)(\bar{l}^j \sigma_{\mu\nu} e)\,\mathcal{F}\\
\mathcal{O}_{ST5} &= (\bar{q}UP_+ r)(\bar{q}UP_- r)\,\mathcal{F}\\
\mathcal{O}_{ST6} &= (\bar{q}UP_{21} r)(\bar{q}UP_{12} r)\,\mathcal{F}\\
\mathcal{O}_{ST7} &= (\bar{q}UP_+ T^A r)(\bar{q}UP_- T^A r)\,\mathcal{F}\\
\mathcal{O}_{ST8} &= (\bar{q}UP_{21} T^A r)(\bar{q}UP_{12} T^A r)\,\mathcal{F}\\
\mathcal{O}_{ST9} &= (\bar{q}UP_+ r)(\bar{l}UP_- \eta)\,\mathcal{F}\\
\mathcal{O}_{ST10} &= (\bar{q}UP_{21} r)(\bar{l}UP_{12} \eta)\,\mathcal{F}\\
\mathcal{O}_{ST11} &= (\bar{q}\sigma^{\mu\nu} UP_+ r)(\bar{l}\sigma_{\mu\nu} UP_- \eta)\,\mathcal{F}\\
\mathcal{O}_{ST12} &= (\bar{q}\sigma^{\mu\nu} UP_{21} r)(\bar{l}\sigma_{\mu\nu} UP_{12} \eta)\,\mathcal{F}
\end{aligned}
$$

$$\mathcal{F}\left(\tfrac{h}{v}\right) = 1 + a_1\left(\tfrac{h}{v}\right) + a_2\left(\tfrac{h}{v}\right)^2 + \dots$$

Counterterms with four fermions II

$\Psi^4 UH : \bar{L}L\bar{R}R$

$$\mathcal{O}_{LR1} = (\bar{q}\gamma^\mu q)(\bar{u}\gamma_\mu u)\,\mathcal{F}$$
$$\mathcal{O}_{LR2} = (\bar{q}\gamma^\mu T^A q)(\bar{u}\gamma_\mu T^A u)\,\mathcal{F}$$
$$\mathcal{O}_{LR3} = (\bar{q}\gamma^\mu q)(\bar{d}\gamma_\mu d)\,\mathcal{F}$$
$$\mathcal{O}_{LR4} = (\bar{q}\gamma^\mu T^A q)(\bar{d}\gamma_\mu T^A d)\,\mathcal{F}$$
$$\mathcal{O}_{LR5} = (\bar{u}\gamma^\mu u)(\bar{l}\gamma_\mu l)\,\mathcal{F}$$
$$\mathcal{O}_{LR6} = (\bar{d}\gamma^\mu d)(\bar{l}\gamma_\mu l)\,\mathcal{F}$$
$$\mathcal{O}_{LR7} = (\bar{q}\gamma^\mu q)(\bar{e}\gamma_\mu e)\,\mathcal{F}$$
$$\mathcal{O}_{LR8} = (\bar{l}\gamma^\mu l)(\bar{e}\gamma_\mu e)\,\mathcal{F}$$
$$\mathcal{O}_{LR9} = (\bar{q}\gamma^\mu l)(\bar{e}\gamma_\mu d)\,\mathcal{F}$$
$$\mathcal{O}_{LR10} = (\bar{q}\gamma^\mu \tau_L q)(\bar{u}\gamma_\mu u)\,\mathcal{F}$$
$$\mathcal{O}_{LR11} = (\bar{q}\gamma^\mu T^A \tau_L q)(\bar{u}\gamma_\mu T^A u)\,\mathcal{F}$$
$$\mathcal{O}_{LR12} = (\bar{q}\gamma^\mu \tau_L q)(\bar{d}\gamma_\mu d)\,\mathcal{F}$$
$$\mathcal{O}_{LR13} = (\bar{q}\gamma^\mu T^A \tau_L q)(\bar{d}\gamma_\mu T^A d)\,\mathcal{F}$$
$$\mathcal{O}_{LR14} = (\bar{l}\gamma^\mu \tau_L l)(\bar{u}\gamma_\mu u)\,\mathcal{F}$$
$$\mathcal{O}_{LR15} = (\bar{l}\gamma^\mu \tau_L l)(\bar{d}\gamma_\mu d)\,\mathcal{F}$$
$$\mathcal{O}_{LR16} = (\bar{q}\gamma^\mu \tau_L q)(\bar{e}\gamma_\mu e)\,\mathcal{F}$$
$$\mathcal{O}_{LR17} = (\bar{l}\gamma^\mu \tau_L l)(\bar{e}\gamma_\mu e)\,\mathcal{F}$$
$$\mathcal{O}_{LR18} = (\bar{q}\gamma^\mu \tau_L l)(\bar{e}\gamma_\mu d)\,\mathcal{F}$$

$\Psi^4 UH : \bar{L}R\bar{L}R$

$$\mathcal{O}_{ST1} = \epsilon_{ij}(\bar{q}^i u)(\bar{q}^j d)\,\mathcal{F}$$
$$\mathcal{O}_{ST2} = \epsilon_{ij}(\bar{q}^i T^A u)(\bar{q}^j T^A d)\,\mathcal{F}$$
$$\mathcal{O}_{ST3} = \epsilon_{ij}(\bar{q}^i u)(\bar{l}^j e)\,\mathcal{F}$$
$$\mathcal{O}_{ST4} = \epsilon_{ij}(\bar{q}^i \sigma^{\mu\nu} u)(\bar{l}^j \sigma_{\mu\nu} e)\,\mathcal{F}$$
$$\mathcal{O}_{ST5} = (\bar{q}UP_+ r)(\bar{q}UP_- r)\,\mathcal{F}$$
$$\mathcal{O}_{ST6} = (\bar{q}UP_{21} r)(\bar{q}UP_{12} r)\,\mathcal{F}$$
$$\mathcal{O}_{ST7} = (\bar{q}UP_+ T^A r)(\bar{q}UP_- T^A r)\,\mathcal{F}$$
$$\mathcal{O}_{ST8} = (\bar{q}UP_{21} T^A r)(\bar{q}UP_{12} T^A r)\,\mathcal{F}$$
$$\mathcal{O}_{ST9} = (\bar{q}UP_+ r)(\bar{l}UP_- \eta)\,\mathcal{F}$$
$$\mathcal{O}_{ST10} = (\bar{q}UP_{21} r)(\bar{l}UP_{12} \eta)\,\mathcal{F}$$
$$\mathcal{O}_{ST11} = (\bar{q}\sigma^{\mu\nu} UP_+ r)(\bar{l}\sigma_{\mu\nu} UP_- \eta)\,\mathcal{F}$$
$$\mathcal{O}_{ST12} = (\bar{q}\sigma^{\mu\nu} UP_{21} r)(\bar{l}\sigma_{\mu\nu} UP_{12} \eta)\,\mathcal{F}$$

$$\mathcal{F}\left(\frac{h}{v}\right) = 1 + a_1\left(\frac{h}{v}\right) + a_2\left(\frac{h}{v}\right)^2 + \ldots$$

ξ ξ^2 ξ^3 ξ^4

Counterterms with four fermions III

$\Psi^4 UH : \bar{L}R\bar{L}R$

$$
\begin{aligned}
\mathcal{O}_{FY1} &= (\bar{q}UP_+ r)(\bar{q}UP_+ r)\,\mathcal{F} \\
\mathcal{O}_{FY2} &= (\bar{q}UP_+ T^A r)(\bar{q}UP_+ T^A r)\,\mathcal{F} \\
\mathcal{O}_{FY3} &= (\bar{q}UP_- r)(\bar{q}UP_- r)\,\mathcal{F} \\
\mathcal{O}_{FY4} &= (\bar{q}UP_- T^A r)(\bar{q}UP_- T^A r)\,\mathcal{F} \\
\mathcal{O}_{FY5} &= (\bar{q}UP_- r)(\bar{r}P_+ U^\dagger q)\,\mathcal{F} \\
\mathcal{O}_{FY6} &= (\bar{q}UP_- T^A r)(\bar{r}P_+ U^\dagger T^A q)\,\mathcal{F} \\
\mathcal{O}_{FY7} &= (\bar{q}UP_- r)(\bar{l}UP_- \eta)\,\mathcal{F} \\
\mathcal{O}_{FY8} &= (\bar{q}\sigma^{\mu\nu} UP_- r)(\bar{l}\sigma_{\mu\nu} UP_- \eta)\,\mathcal{F} \\
\mathcal{O}_{FY9} &= (\bar{l}UP_- \eta)(\bar{r}P_+ U^\dagger q)\,\mathcal{F} \\
\mathcal{O}_{FY10} &= (\bar{l}UP_- \eta)(\bar{l}UP_- \eta)\,\mathcal{F} \\
\mathcal{O}_{FY11} &= (\bar{l}UP_- r)(\bar{r}P_+ U^\dagger l)\,\mathcal{F}
\end{aligned}
$$

$$\mathcal{F}\left(\frac{h}{v}\right) = 1 + a_1\left(\frac{h}{v}\right) + a_2\left(\frac{h}{v}\right)^2 + \dots$$

The covariant derivative of U reads:

$$D_\mu U = \partial_\mu U + igW_\mu U - ig' B_\mu U T_3$$

Counterterms with four fermions III

$\Psi^4 UH : \bar{L}R\bar{L}R$

$$\begin{aligned}
\mathcal{O}_{FY1} &= (\bar{q}UP_+r)(\bar{q}UP_+r)\,\mathcal{F} \\
\mathcal{O}_{FY2} &= (\bar{q}UP_+T^Ar)(\bar{q}UP_+T^Ar)\,\mathcal{F} \\
\mathcal{O}_{FY3} &= (\bar{q}UP_-r)(\bar{q}UP_-r)\,\mathcal{F} \\
\mathcal{O}_{FY4} &= (\bar{q}UP_-T^Ar)(\bar{q}UP_-T^Ar)\,\mathcal{F} \\
\mathcal{O}_{FY5} &= (\bar{q}UP_-r)(\bar{r}P_+U^\dagger q)\,\mathcal{F} \\
\mathcal{O}_{FY6} &= (\bar{q}UP_-T^Ar)(\bar{r}P_+U^\dagger T^Aq)\,\mathcal{F} \\
\mathcal{O}_{FY7} &= (\bar{q}UP_-r)(\bar{l}UP_-\eta)\,\mathcal{F} \\
\mathcal{O}_{FY8} &= (\bar{q}\sigma^{\mu\nu}UP_-r)(\bar{l}\sigma_{\mu\nu}UP_-\eta)\,\mathcal{F} \\
\mathcal{O}_{FY9} &= (\bar{l}UP_-\eta)(\bar{r}P_+U^\dagger q)\,\mathcal{F} \\
\mathcal{O}_{FY10} &= (\bar{l}UP_-\eta)(\bar{l}UP_-\eta)\,\mathcal{F} \\
\mathcal{O}_{FY11} &= (\bar{l}UP_-r)(\bar{r}P_+U^\dagger l)\,\mathcal{F}
\end{aligned}$$

$$\mathcal{F}\left(\tfrac{h}{v}\right) = 1 + a_1\left(\tfrac{h}{v}\right) + a_2\left(\tfrac{h}{v}\right)^2 + \dots$$

ξ ξ^2 ξ^3 ξ^4

The covariant derivative of U reads:

$$D_\mu U = \partial_\mu U + igW_\mu U - ig'B_\mu UT_3$$

Equations of motion I

$$\partial_\mu \partial^\mu h = a_n\, n \tfrac{v}{4} \langle (D_\mu U)(D^\mu U^\dagger) \rangle \left(\tfrac{h}{v}\right)^{n-1} - b_m\, m\, v^3 \left(\tfrac{h}{v}\right)^{m-1}$$
$$- c_{f,l}\, l (\bar{\Psi}_f Y_{\Psi_f} U P_\pm \Psi_f + \text{h.c.}) \left(\tfrac{h}{v}\right)^{l-1}$$

$$i \not{D} \Psi_L = v Y^{(l)}_{\Psi_R} U P_+ \Psi_R\, c_{f,l} \left(\tfrac{h}{v}\right)^l + v Y^{(l)}_{\Psi_R} U P_- \Psi_R\, c_{f,l} \left(\tfrac{h}{v}\right)^l$$
$$i \not{D} \Psi_R = v P_+ U^\dagger Y^{(l)\dagger}_{\Psi_R} \Psi_L\, c_{f,l} \left(\tfrac{h}{v}\right)^l + v P_- U^\dagger Y^{(l)\dagger}_{\Psi_R} \Psi_L\, c_{f,l} \left(\tfrac{h}{v}\right)^l$$
$$i \bar{\Psi}_L \overleftarrow{\not{D}} = -v \bar{\Psi}_R P_+ U^\dagger Y^{(l)\dagger}_{\Psi_R}\, c_{f,l} \left(\tfrac{h}{v}\right)^l - v \bar{\Psi}_R P_- U^\dagger Y^{(l)\dagger}_{\Psi_R}\, c_{f,l} \left(\tfrac{h}{v}\right)^l$$
$$i \bar{\Psi}_R \overleftarrow{\not{D}} = -v \bar{\Psi}_L Y^{(l)}_{\Psi_R} U P_+\, c_{f,l} \left(\tfrac{h}{v}\right)^l - v \bar{\Psi}_L Y^{(l)}_{\Psi_R} U P_-\, c_{f,l} \left(\tfrac{h}{v}\right)^l$$

$$\partial_\mu B^{\mu\nu} = g' \bar{\Psi}_f \gamma^\nu \mathrm{Y}_f \Psi_f + g' \tfrac{v^2}{2} \langle L^\nu \tau_L \rangle\, a_n \left(\tfrac{h}{v}\right)^n$$
$$[D_\mu W^{\mu\nu}]_i = g \bar{\Psi}_L \gamma^\nu T^i \Psi_L - g \tfrac{v^2}{2} \langle T^i L^\nu \rangle\, a_n \left(\tfrac{h}{v}\right)^n$$
$$[D_\mu G^{\mu\nu}]_A = g_s \bar{\Psi} \gamma^\nu T^A \Psi$$

Equations of motion II

From

$$S = \int d^4x \ \tfrac{v^2}{4}(D_\mu U)_{ab}(D^\mu U^\dagger)_{ba}\ a_n \left(\frac{h}{v}\right)^n + \lambda(\det U - 1)$$
$$- v\left(\bar\Psi_{L,a} Y^{(l)} U_{ab} \Psi_{R,b} + \bar\Psi_{R,a} Y^{(l)\dagger} U^\dagger_{ab} \Psi_{L,b}\right)\ c_{f,l}\left(\frac{h}{v}\right)^l$$

and

$$U_{ij}\delta U^\dagger_{jk} = -\delta U_{ij} U^\dagger_{jk}$$
$$\delta(\det U) = \langle(\delta U)U^\dagger\rangle \cdot \det U$$
$$(D_\mu D^\mu U)U^\dagger - U(D_\mu D^\mu U^\dagger) = 2iD_\mu L^\mu$$

we find:

$$\tfrac{v^2}{2} a_n \left(iD_\mu L^\mu_{ab}\left(\frac{h}{v}\right)^n + i\, nL^\mu_{ab}(\partial_\mu \tfrac{h}{v})\left(\frac{h}{v}\right)^{n-1}\right)$$
$$- v\left(\bar\Psi_{L,b} Y^{(l)} (U\Psi_R)_a - (\bar\Psi_R U^\dagger)_b Y^{(l)\dagger}\Psi_{L,a}\right) c_{f,l}\left(\frac{h}{v}\right)^l$$
$$+ \tfrac{v}{2}\left(\bar\Psi_{L,i} Y^{(l)} U_{ij}\Psi_{R,j} - \bar\Psi_{R,i} U^\dagger_{ij} Y^{(l)\dagger}\Psi_{L,j}\right) c_{f,l}\left(\frac{h}{v}\right)^l \delta_{ab} = 0$$

Useful relations for the reduction of operators

$$D_\mu L_\nu - D_\nu L_\mu = gW_{\mu\nu} - g' B_{\mu\nu}\tau_L + i[L_\mu, L_\nu]$$
$$D_\mu \tau_L = i[L_\mu, \tau_L]$$
$$[D_\mu, D_\nu]L_\rho = ig[W_{\mu\nu}, L_\rho]$$

$$\langle \tau_L AB\rangle\langle \tau_L C\rangle = \frac{1}{2}\langle ABC\rangle - \langle \tau_L BC\rangle\langle \tau_L A\rangle + \langle \tau_L AC\rangle\langle \tau_L B\rangle$$

$$\begin{aligned} i\bar{\Psi} UP(D^\mu \Psi) = \frac{1}{2}\Big(&- i\bar{\Psi}\overleftarrow{\not{D}}\gamma^\mu UP\Psi + \bar{\Psi}\sigma^{\mu\nu}(D_\nu U)\Psi + \bar{\Psi}\gamma^\mu UP(i\not{D}\Psi) \\ &- i\bar{\Psi}(D^\mu U)P\Psi + iD^\mu(\bar{\Psi}UP\Psi) - D_\nu(\bar{\Psi}\sigma^{\mu\nu}UP\Psi)\Big) \\ i(D^\mu\bar{\Psi})UP\Psi = \frac{1}{2}\Big(&D_\nu(\bar{\Psi}\sigma^{\mu\nu}UP\Psi) + iD^\mu(\bar{\Psi}UP\Psi) + i\bar{\Psi}\overleftarrow{\not{D}}\gamma^\mu UP\Psi \\ &- \bar{\Psi}\sigma^{\mu\nu}(D_\nu U)\Psi - \bar{\Psi}\gamma^\mu UP(i\not{D}\Psi) - i\bar{\Psi}(D^\mu U)P\Psi\Big) \end{aligned}$$

Comparison – “A light dynamical ‘Higgs Particle’ ”

by Alonso, Gavela, Merlo, Rigolin and Yepes
(arXiv:1212.3305, PLB; arXiv:1212.3307, PRD; arXiv:1304.5937)

Similar to our analysis, but with crucial differences:

- NLO-operators are of dimension less than or equal to 5.

→ Their basis, excluding fermions, is almost the same as ours. The only operator that was not present in their basis is $\mathcal{O}_{D4,1}$.

→ Operators with 4 fermions are not considered at all.

→ They list all operators of class $UH\Psi^2X$, but without h.

→ In the class $UHD^2\Psi^2$, we find that they list 15 of 30 operators.

- The equations of motion are wrong.

→ 8 of their operators are redundant.

- The presented counting of ξ is inconsistent
- They only consider CP-even operators
- They exclude operators with right-handed fermions in the class $UHD\Psi^2$ because of MFV.

The "Strongly-Interacting Light Higgs" (SILH)

$$\mathcal{L}_{\text{SILH}} =$$

$$\frac{c_H}{2f^2}\partial^\mu\left(H^\dagger H\right)\partial_\mu\left(H^\dagger H\right) + \frac{c_T}{2f^2}\left(H^\dagger \overleftrightarrow{D^\mu} H\right)\left(H^\dagger \overleftrightarrow{D}_\mu H\right) - \frac{c_6\lambda}{f^2}\left(H^\dagger H\right)^3$$

$$+\left(\frac{c_y y_f}{f^2}H^\dagger H \bar{\Psi}_L H \Psi_R + \text{h.c.}\right) + \frac{ic_W g}{2m_\rho^2}\left(H^\dagger \sigma^i \overleftrightarrow{D^\mu} H\right)(D^\nu W_{\mu\nu})^i + \frac{ic_B g'}{2m_\rho^2}\left(H^\dagger \overleftrightarrow{D^\mu} H\right)\partial^\nu B_{\mu\nu}$$

$$+\frac{ic_{HW} g}{16\pi^2 f^2}(D^\mu H)^\dagger \sigma^i (D^\nu H) W^i_{\mu\nu} + \frac{ic_{HB} g'}{16\pi^2 f^2}(D^\mu H)^\dagger (D^\nu H) B_{\mu\nu}$$

$$+\frac{c_\gamma g'^2}{16\pi^2 f^2}\frac{g^2}{g_\rho^2}H^\dagger H B_{\mu\nu}B^{\mu\nu} + \frac{c_g g_S^2}{16\pi^2 f^2}\frac{y_t^2}{g_\rho^2}H^\dagger H G^a_{\mu\nu}G^{a\mu\nu}$$

$$-\frac{c_{2W} g^2}{2g_\rho^2 m_\rho^2}(D^\mu W_{\mu\nu})^i (D_\rho W^{\rho\nu})^i - \frac{c_{2B} g'^2}{2g_\rho^2 m_\rho^2}(\partial^\mu B_{\mu\nu})(\partial_\rho B^{\rho\nu}) - \frac{c_{2g} g_s^2}{2g_\rho^2 m_\rho^2}(D^\mu G_{\mu\nu})^a (D_\rho G^{\rho\nu})^a$$

$$+\frac{c_{3W} g^3}{16\pi^2 m_\rho^2}\epsilon_{ijk} W^{i\,\nu}_{\mu} W^j_{\nu\rho} W^{k\,\rho\mu} + \frac{c_{3g} g_s^3}{16\pi^2 m_\rho^2} f_{abc} G^{a\,\nu}_{\mu} G^b_{\nu\rho} G^{c\,\rho\mu}$$

Relation to our conventions:

$$H = \frac{(v+h)}{\sqrt{2}}\, U \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

The "Strongly-Interacting Light Higgs" (SILH)

By comparing the operators, we find:

- $\mathcal{O}_{\beta 1}, \mathcal{O}_{XU1}, \mathcal{O}_{XU3}, \mathcal{O}_{XU5}$ and $\mathcal{O}_{XU9}$ are generated with independent coefficients.
- Corrections to the leading order Lagrangian are given for the terms $\Delta\mathcal{L}_{\text{kin},h}$, $\Delta\mathcal{V}(h)$ and $\Delta\mathcal{L}_{\text{Yukawa}}$ with independent coefficients. The term $\Delta\mathcal{L}_{\text{kin, GB}}$ is generated, but not with independent coefficient.
- Only two operators with a fermion vector current, $\mathcal{O}_{\Psi Vi}$, are generated independently. The two linear combinations that arise are:

$$\sum_f (\mathrm{Y}_f \mathcal{O}^{(0)}_{\Psi Vf})(v+h)^2 \quad \text{and} \quad \left(2\mathcal{O}^{(0)}_{\Psi V2,8} + \mathcal{O}^{(0)}_{\Psi V3,9} + \mathcal{O}^{(0)\dagger}_{\Psi V3,9}\right)(v+h)^2$$

- In the four fermion sector, they only consider three independent terms:

$$(\bar{\Psi}_L \gamma_\mu T^i \Psi_L)(\bar{\Psi}'_L \gamma^\mu T^i \Psi'_L), \quad (\bar{\Psi}_f \gamma_\mu \mathrm{Y}_f \Psi_f)(\bar{\Psi}_{f'} \gamma^\mu \mathrm{Y}_{f'} \Psi_{f'})$$
$$\text{and} \quad (\bar{\Psi}_q \gamma_\mu T^A \Psi_q)(\bar{\Psi}_{q'} \gamma^\mu T^A \Psi_{q'}).$$

- The operators of the class $UH\Psi^2 X$ are not generated.
- They consider two additional operators of the class X^3.

Example – The Higgs portal[1]

$$\mathcal{V} = -\frac{\mu_s^2}{2}|\phi_s|^2 + \frac{\lambda_s}{4}|\phi_s|^4 - \frac{\mu_h^2}{2}|\phi_h|^2 + \frac{\lambda_h}{4}|\phi_h|^4 + \frac{\eta}{2}|\phi_s|^2|\phi_h|^2$$

$$\frac{v_s}{\sqrt{2}} = \sqrt{\frac{\eta\mu_h^2 - \lambda_h\mu_s^2}{\eta^2 - \lambda_s\lambda_h}}, \qquad \frac{v_h}{\sqrt{2}} = \sqrt{\frac{\eta\mu_s^2 - \lambda_s\mu_h^2}{\eta^2 - \lambda_s\lambda_h}}$$

$$\mathcal{V} = \frac{v_s^2\lambda_s}{4}h_s^2 + \frac{v_h^2\lambda_h}{4}h_h^2 + \frac{\eta}{2}v_s v_h h_s h_h + \mathcal{O}(h_i^3)$$

$$\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \begin{pmatrix} \cos\chi & -\sin\chi \\ \sin\chi & \cos\chi \end{pmatrix} \begin{pmatrix} h_s \\ h_h \end{pmatrix}$$

$$\tan(2\chi) = \frac{2\eta v_s v_h}{v_h^2\lambda_h - v_s^2\lambda_s}$$

[1] e.g. Englert, Plehn, Zerwas and Zerwas, arXiv:1106.3097, PLB
Patt and Wilczek, arXiv:hep-ph/0605188

Example – The Higgs portal

$$
\begin{aligned}
\mathcal{L}_H = & \frac{1}{2}(\partial_\mu H_1)(\partial^\mu H_1) + \frac{1}{2}(\partial_\mu H_2)(\partial^\mu H_2) - \frac{1}{2}M_1^2 H_1^2 - \frac{1}{2}M_2^2 H_2^2 + \lambda_1 H_1^3 \\
& + \lambda_2 H_1^2 H_2 + \lambda_3 H_1 H_2^2 + \lambda_4 H_2^3 + z_1 H_1^4 + z_2 H_1^3 H_2 + z_3 H_1^2 H_2^2 + z_4 H_1 H_2^3 + z_5 H_2^4 \\
& - v\left(\bar{q} Y_u U P_+ r + \bar{q} Y_d U P_- r + \bar{l} Y_e U P_- \eta + \text{ h.c.}\right)\left(1 + \frac{\cos\chi}{v} H_1 + \frac{\sin\chi}{v} H_2\right) \\
& + \frac{v^2}{4}\langle D_\mu U D^\mu U^\dagger \rangle \Big(1 + \frac{2\cos\chi}{v} H_1 + \frac{2\sin\chi}{v} H_2 + \frac{\cos^2\chi}{v^2} H_1^2 \\
& \qquad + \frac{2\sin\chi\cos\chi}{v^2} H_1 H_2 + \frac{\sin^2\chi}{v^2} H_2^2\Big)
\end{aligned}
$$

Example – The Higgs portal

$$
\begin{aligned}
\mathcal{L}_H = {} & \frac{1}{2}(\partial_\mu H_1)(\partial^\mu H_1) + \frac{1}{2}(\partial_\mu H_2)(\partial^\mu H_2) - \frac{1}{2}M_1^2 H_1^2 - \frac{1}{2}M_2^2 H_2^2 + \lambda_1 H_1^3 \\
& + \lambda_2 H_1^2 H_2 + \lambda_3 H_1 H_2^2 + \lambda_4 H_2^3 + z_1 H_1^4 + z_2 H_1^3 H_2 + z_3 H_1^2 H_2^2 + z_4 H_1 H_2^3 + z_5 H_2^4 \\
& - v\left(\bar{q} Y_u U P_+ r + \bar{q} Y_d U P_- r + \bar{l} Y_e U P_- \eta + \text{ h.c.}\right)\left(1 + \frac{\cos\chi}{v} H_1 + \frac{\sin\chi}{v} H_2\right) \\
& + \frac{v^2}{4}\langle D_\mu U D^\mu U^\dagger\rangle \Big(1 + \frac{2\cos\chi}{v} H_1 + \frac{2\sin\chi}{v} H_2 + \frac{\cos^2\chi}{v^2} H_1^2 \\
& \qquad\qquad + \frac{2\sin\chi\cos\chi}{v^2} H_1 H_2 + \frac{\sin^2\chi}{v^2} H_2^2\Big)
\end{aligned}
$$

Example – The Higgs portal

$$
\begin{aligned}
\mathcal{L}_H = & \frac{1}{2}(\partial_\mu H_1)(\partial^\mu H_1) + \frac{1}{2}(\partial_\mu H_2)(\partial^\mu H_2) - \frac{1}{2}M_1^2 H_1^2 - \frac{1}{2}M_2^2 H_2^2 + \lambda_1 H_1^3 \\
& + \lambda_2 H_1^2 H_2 + \lambda_3 H_1 H_2^2 + \lambda_4 H_2^3 + z_1 H_1^4 + z_2 H_1^3 H_2 + z_3 H_1^2 H_2^2 + z_4 H_1 H_2^3 + z_5 H_2^4 \\
& - v\left(\bar{q} Y_u U P_+ r + \bar{q} Y_d U P_- r + \bar{l} Y_e U P_- \eta + \text{ h.c.}\right)\left(1 + \frac{\cos\chi}{v} H_1 + \frac{\sin\chi}{v} H_2\right) \\
& + \frac{v^2}{4}\langle D_\mu U D^\mu U^\dagger \rangle \Big(1 + \frac{2\cos\chi}{v} H_1 + \frac{2\sin\chi}{v} H_2 + \frac{\cos^2\chi}{v^2} H_1^2 \\
& \qquad + \frac{2\sin\chi\cos\chi}{v^2} H_1 H_2 + \frac{\sin^2\chi}{v^2} H_2^2\Big)
\end{aligned}
$$

Example – The Higgs portal

$$\begin{aligned}\mathcal{L}_H = &\frac{1}{2}(\partial_\mu H_1)(\partial^\mu H_1) + \frac{1}{2}(\partial_\mu H_2)(\partial^\mu H_2) - \frac{1}{2}M_1^2 H_1^2 - \frac{1}{2}M_2^2 H_2^2 + \lambda_1 H_1^3 \\ &+ \lambda_2 H_1^2 H_2 + \lambda_3 H_1 H_2^2 + \lambda_4 H_2^3 + z_1 H_1^4 + z_2 H_1^3 H_2 + z_3 H_1^2 H_2^2 + z_4 H_1 H_2^3 + z_5 H_2^4 \\ &- v\left(\bar{q} Y_u U P_+ r + \bar{q} Y_d U P_- r + \bar{l} Y_e U P_- \eta + \text{ h.c.}\right)\left(1 + \frac{\cos\chi}{v} H_1 + \frac{\sin\chi}{v} H_2\right) \\ &+ \frac{v^2}{4}\langle D_\mu U D^\mu U^\dagger\rangle\Big(1 + \frac{2\cos\chi}{v} H_1 + \frac{2\sin\chi}{v} H_2 + \frac{\cos^2\chi}{v^2} H_1^2 \\ &\qquad + \frac{2\sin\chi\cos\chi}{v^2} H_1 H_2 + \frac{\sin^2\chi}{v^2} H_2^2\Big)\end{aligned}$$

Example – The Higgs portal

$$\mathcal{L}_H = \frac{1}{2}(\partial_\mu H_1)(\partial^\mu H_1) + \frac{1}{2}(\partial_\mu H_2)(\partial^\mu H_2) - \frac{1}{2}M_1^2 H_1^2 - \frac{1}{2}M_2^2 H_2^2 + \lambda_1 H_1^3$$
$$+ \lambda_2 H_1^2 H_2 + \lambda_3 H_1 H_2^2 + \lambda_4 H_2^3 + z_1 H_1^4 + z_2 H_1^3 H_2 + z_3 H_1^2 H_2^2 + z_4 H_1 H_2^3 + z_5 H_2^4$$
$$- v\left(\bar{q} Y_u U P_+ r + \bar{q} Y_d U P_- r + \bar{l} Y_e U P_- \eta + \text{ h.c.}\right)\left(1 + \frac{\cos\chi}{v} H_1 + \frac{\sin\chi}{v} H_2\right)$$
$$+ \frac{v^2}{4}\langle D_\mu U D^\mu U^\dagger \rangle \Big(1 + \frac{2\cos\chi}{v} H_1 + \frac{2\sin\chi}{v} H_2 + \frac{\cos^2\chi}{v^2} H_1^2$$
$$+ \frac{2\sin\chi\cos\chi}{v^2} H_1 H_2 + \frac{\sin^2\chi}{v^2} H_2^2\Big)$$

Example – The Higgs portal

$$\mathcal{L}_H = \frac{1}{2}(\partial_\mu H_1)(\partial^\mu H_1) + \frac{1}{2}(\partial_\mu H_2)(\partial^\mu H_2) - \frac{1}{2}M_1^2 H_1^2 - \frac{1}{2}M_2^2 H_2^2 + \lambda_1 H_1^3$$
$$+ \lambda_2 H_1^2 H_2 + \lambda_3 H_1 H_2^2 + \lambda_4 H_2^3 + z_1 H_1^4 + z_2 H_1^3 H_2 + z_3 H_1^2 H_2^2 + z_4 H_1 H_2^3 + z_5 H_2^4$$
$$- v\left(\bar{q} Y_u U P_+ r + \bar{q} Y_d U P_- r + \bar{l} Y_e U P_- \eta + \text{ h.c.}\right)\left(1 + \frac{\cos\chi}{v} H_1 + \frac{\sin\chi}{v} H_2\right)$$
$$+ \frac{v^2}{4}\langle D_\mu U D^\mu U^\dagger \rangle \Big(1 + \frac{2\cos\chi}{v} H_1 + \frac{2\sin\chi}{v} H_2 + \frac{\cos^2\chi}{v^2} H_1^2$$
$$+ \frac{2\sin\chi\cos\chi}{v^2} H_1 H_2 + \frac{\sin^2\chi}{v^2} H_2^2\Big)$$

Example – The Higgs portal

$$\mathcal{L}_H = \frac{1}{2}(\partial_\mu H_1)(\partial^\mu H_1) + \frac{1}{2}(\partial_\mu H_2)(\partial^\mu H_2) - \frac{1}{2}M_1^2 H_1^2 - \frac{1}{2}M_2^2 H_2^2 + \lambda_1 H_1^3$$
$$+ \lambda_2 H_1^2 H_2 + \lambda_3 H_1 H_2^2 + \lambda_4 H_2^3 + z_1 H_1^4 + z_2 H_1^3 H_2 + z_3 H_1^2 H_2^2 + z_4 H_1 H_2^3 + z_5 H_2^4$$
$$- v\left(\bar{q} Y_u U P_+ r + \bar{q} Y_d U P_- r + \bar{l} Y_e U P_- \eta + \text{ h.c.}\right)\left(1 + \frac{\cos\chi}{v} H_1 + \frac{\sin\chi}{v} H_2\right)$$
$$+ \frac{v^2}{4}\langle D_\mu U D^\mu U^\dagger\rangle \Big(1 + \frac{2\cos\chi}{v} H_1 + \frac{2\sin\chi}{v} H_2 + \frac{\cos^2\chi}{v^2} H_1^2$$
$$+ \frac{2\sin\chi\cos\chi}{v^2} H_1 H_2 + \frac{\sin^2\chi}{v^2} H_2^2\Big)$$

$$(\Box + M_2^2) H_2 = A_0 + A_1 H_2 + A_2 H_2^2 + A_3 H_2^3$$

$$H_2 \approx \tfrac{A_0}{M_2^2} + A_1 \tfrac{A_0}{M_2^4} + \mathcal{O}(\tfrac{1}{M_2^4})$$

$$A_0 = \lambda_2 H_1^2 + z_2 H_1^3 + \frac{v^2}{4}\langle D_\mu U D^\mu U^\dagger\rangle \left(\frac{2\sin\chi}{v} + \frac{2\sin\chi\cos\chi}{v^2} H_1\right)$$
$$- \sin\chi\left(\bar{q} Y_u U P_+ r + \bar{q} Y_d U P_- r + \bar{l} Y_e U P_- \eta + \text{ h.c.}\right)$$

Example – The Higgs portal

$$\mathcal{L}_H = \frac{1}{2}(\partial_\mu H_1)(\partial^\mu H_1) + \frac{1}{2}(\partial_\mu H_2)(\partial^\mu H_2) - \frac{1}{2}M_1^2 H_1^2 - \frac{1}{2}M_2^2 H_2^2 + \lambda_1 H_1^3$$
$$+ \lambda_2 H_1^2 H_2 + \lambda_3 H_1 H_2^2 + \lambda_4 H_2^3 + z_1 H_1^4 + z_2 H_1^3 H_2 + z_3 H_1^2 H_2^2 + z_4 H_1 H_2^3 + z_5 H_2^4$$
$$- v\left(\bar{q} Y_u U P_+ r + \bar{q} Y_d U P_- r + \bar{l} Y_e U P_- \eta + \text{ h.c.}\right)\left(1 + \frac{\cos\chi}{v} H_1 + \frac{\sin\chi}{v} H_2\right)$$
$$+ \frac{v^2}{4}\langle D_\mu U D^\mu U^\dagger \rangle \Big(1 + \frac{2\cos\chi}{v} H_1 + \frac{2\sin\chi}{v} H_2 + \frac{\cos^2\chi}{v^2} H_1^2$$
$$+ \frac{2\sin\chi\cos\chi}{v^2} H_1 H_2 + \frac{\sin^2\chi}{v^2} H_2^2\Big)$$

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$$H_2 \approx \frac{A_0}{M_2^2} + A_1 \frac{A_0}{M_2^4} + \mathcal{O}\left(\frac{1}{M_2^4}\right)$$

$$A_0 = \lambda_2 H_1^2 + z_2 H_1^3 + \frac{v^2}{4}\langle D_\mu U D^\mu U^\dagger \rangle \left(\frac{2\sin\chi}{v} + \frac{2\sin\chi\cos\chi}{v^2} H_1\right)$$
$$- \sin\chi\left(\bar{q} Y_u U P_+ r + \bar{q} Y_d U P_- r + \bar{l} Y_e U P_- \eta + \text{ h.c.}\right)$$

The Higgs portal

We find:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{without } H_2} + \frac{A_0^2}{2M_2^2} + \mathcal{O}(\frac{1}{M_2^4})$$

This contains:

$\mathcal{O}_{D0,1}, \mathcal{O}_{\Psi S1}, \mathcal{O}_{\Psi S2}, \mathcal{O}_{\Psi S7}$ and operators with 4 fermions