A quantum field theroretical detector model for probing the Unruh effect

Franz Thoma

Elite Master Course Theoretical and Mathematical Physics
Ludwig-Maximilians-Universität München

Overview

- We analyze a quantum field theoretical detector model coupled to a massless scalar test field on 1+1-dimensional Minkowski space-time.
- The transition rate for the detector to become excited in the absence of test particles is a thermal distribution, proportional to the Rindler particle content of Minkowski vacuum.
- The process of ‘Rindler particle detection’ is better interpreted as spontaneous excitation of the detector sourced by the external accelerating field.

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  $\rightarrow$ We need to specify how the observer interacts with the field.

Probing the Unruh effect: Detector models

A detector model allows an *operational definition*:

$$\begin{aligned} \text{excitiation of the detector} &= \text{detection of a particle} \\ \text{excitiation rate} &\propto \text{particle content} \end{aligned}$$

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Two detector models proposed by Unruh:

- simple ‘particle in a box’ detector model:
  by construction detects modes with respect to a given time coordinate; relies on the Bogolyubov formalism
- full QFT detector:
  incorporates an external field as source for the external acceleration, calculations are done in Minkowski coordinates (no Bogolyubov transformations)

The QFT detector model

- Detector model consists of two fields Φ and Ψ of masses $M < \bar{M}$
- Test field: χ (massless)

$$
\begin{aligned}
S = \int d^2x \Big[\partial_\mu \Phi^\dagger \partial^\mu \Phi \quad & + M^2 \Phi^\dagger \Phi \\
+ \partial_\mu \Psi^\dagger \partial^\mu \Psi \quad & + \bar{M}^2 \Psi^\dagger \Psi \\
+ \partial_\mu \chi \partial^\mu \chi \quad & \\
& + \varepsilon \big(\Phi^\dagger \Psi + \Psi^\dagger \Phi \big) \chi \Big].
\end{aligned}
$$

The detection process

The process $\Phi\chi \to \Psi$ (excitation due to absorption of a test particle) corresponds to the detection of a test particle.

- Transition probability:

$$
\begin{aligned}
P_{\Phi\chi\to\Psi} &\propto \left|{}_{\text{out}}\langle 0,\Psi,0|\Phi,0,\chi\rangle_{\text{in}}\right|^2 \\
&\propto \left|\delta(\bar{\Omega}-\Omega-\omega)\delta(\bar{K}-K\pm\omega)\right|^2
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- Energy and momentum conservation yields detector energy gap:

$$\omega_{\text{d}} = \frac{\bar{M}^2 - M^2}{2M}$$

- Actual energy ω is redshifted (or blueshifted) by the relative motion of the detector:

$$\omega = \frac{M}{\Omega \mp K}\omega_{\text{d}}$$

Accelerated quantum fields

- Classical Hamiltonian function:

$$H = \sqrt{p^2 + m^2} + V(x), \qquad V(x) = -max$$

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- Quantization: $H \to i\partial_t$, $p \to -i\partial_x$

$$0 = (i\partial_t + max)^2\varphi + \partial_x^2\varphi - m^2\varphi =: (\mathcal{D}_\mu \mathcal{D}^\mu - m^2)\varphi$$

with the covariant derivatives $\mathcal{D}_x = \partial_x$, $i\mathcal{D}_t = i\partial_t + max$

Field expansion

- Stationary solutions are given in terms of *Parabolic cylinder functions* E_μ with $\mu = m/2a$:

$$\varphi_\omega(x) = (8ma)^{-1/4}\ \beta\ E^*_\mu\Big(\sqrt{2ma}\big(-x - \frac{\omega}{ma}\big)\Big)$$

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- "in-particle mode": $\varphi_\omega(x)$
  "out-particle mode": $\varphi^*_\omega(x)$

$$\varphi(x,t) = \begin{cases} \int_{-\infty}^{\infty} d\omega\ \left[a^{\text{in}}_\omega e^{-i\omega t}\varphi_\omega(x) + \hat{a}^{\text{in}\,\dagger}_\omega e^{-i\omega t}\varphi^*_{-\omega}(-x)\right] \\ \int_{-\infty}^{\infty} d\omega\ \left[a^{\text{out}}_\omega e^{-i\omega t}\varphi^*_\omega(x) + \hat{a}^{\text{out}\,\dagger}_\omega e^{-i\omega t}\varphi_{-\omega}(-x)\right] \end{cases}$$

The accelerated QFT detector

Replace the ordinary derivatives in the Φ and Ψ sectors by covariant ones, $\mathcal{D}_0 = \partial_0 - iMax$ and $\mathcal{D}_1 = \partial_1$:

$$\begin{aligned} S = \int d^2x \, \Big[& \mathcal{D}_\mu^* \Phi^\dagger \mathcal{D}^\mu \Phi && + M^2 \Phi^\dagger \Phi \\ & + \mathcal{D}_\mu^* \Psi^\dagger \mathcal{D}^\mu \Psi && + \bar{M}^2 \Psi^\dagger \Psi \\ & + \partial_\mu \chi \partial^\mu \chi && \\ & && + \varepsilon \big(\Phi^\dagger \Psi + \Psi^\dagger \Phi \big) \chi \Big] \end{aligned}$$

Field expansions

$$\Phi(x,t) = \begin{cases} \int_{-\infty}^{\infty} d\Omega \left[a_\Omega^{\text{in}} e^{-i\Omega t} \varphi_\Omega(x) + \hat{a}_\Omega^{\text{in}\dagger} e^{-i\Omega t} \varphi^*_{-\Omega}(-x) \right] \\ \int_{-\infty}^{\infty} d\Omega \left[a_\Omega^{\text{out}} e^{-i\Omega t} \varphi^*_\Omega(x) + \hat{a}_\Omega^{\text{out}\dagger} e^{-i\Omega t} \varphi_{-\Omega}(-x) \right] \end{cases}$$

$$\Psi(x,t) = \begin{cases} \int_{-\infty}^{\infty} d\bar{\Omega} \left[b_{\bar{\Omega}}^{\text{in}} e^{-i\bar{\Omega} t} \varphi_{\bar{\Omega}}(x) + \hat{b}_{\bar{\Omega}}^{\text{in}\dagger} e^{-i\bar{\Omega} t} \varphi^*_{-\bar{\Omega}}(-x) \right] \\ \int_{-\infty}^{\infty} d\bar{\Omega} \left[b_{\bar{\Omega}}^{\text{out}} e^{-i\bar{\Omega} t} \varphi^*_{\bar{\Omega}}(x) + \hat{b}_{\bar{\Omega}}^{\text{out}\dagger} e^{-i\bar{\Omega} t} \varphi_{-\bar{\Omega}}(-x) \right] \end{cases}$$

$$\chi(x,t) = \int_{-\infty}^{\infty} dk \left[c_k e^{-i\omega(k)t + ikx} + c_k^\dagger e^{i\omega(k)t - ikx} \right].$$

Spontaneous excitation

The $\Phi \to \Psi\chi$ amplitude gives the probability for the detector to switch to its excited state in the absence of test particles

- Transition probability in χ-vacuum:

$$
\begin{aligned}
P_{\Phi\to\Psi\chi} &\propto \left|{}_{\text{in}}\left\langle \Phi^{\text{in}}, 0^{\text{in}}, 0 \middle| 0^{\text{out}}, \Psi^{\text{out}}, \chi \right\rangle_{\text{out}}\right|^2 \\
&\propto \varepsilon^2 \left| \int_{-\infty}^{\infty} dx \; e^{-ikx} \varphi_\Omega(x) \varphi_{\bar{\Omega}}(x) \right|^2 \left| \delta(\Omega - \bar{\Omega} - \omega(k)) \right|^2 \\
&\propto \varepsilon^2 \frac{a}{\omega_{\text{d}}} \; \frac{|\alpha|^2}{e^{\frac{2\pi}{a}\omega_{\text{d}}} - 1} \; \left| \delta(\Omega - \bar{\Omega} - \omega(k)) \right|^2
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$$\propto \varepsilon^2 \left|\int_{-\infty}^{\infty} dx\; e^{-ikx} \varphi_\Omega(x)\varphi_{\bar{\Omega}}(x)\right|^2 \left|\delta(\Omega - \bar{\Omega} - \omega(k))\right|^2$$

$$\propto \varepsilon^2 \frac{a}{\omega_\text{d}} \; \frac{|\alpha|^2}{e^{\frac{2\pi}{a}\omega_\text{d}} - 1} \; \left|\delta(\Omega - \bar{\Omega} - \omega(k))\right|^2$$

- $\omega(k) = \Omega - \bar{\Omega} = \frac{M}{K(x) - (\Omega + Max)}\omega_\text{d}$

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- $\omega(k) = \Omega - \bar{\Omega} = \frac{M}{K(x) - (\Omega + Max)} \omega_{\text{d}}$
- The energy required to excite the detector and create the χ particle is supplied by the external potential $V = -Max$.

Summary

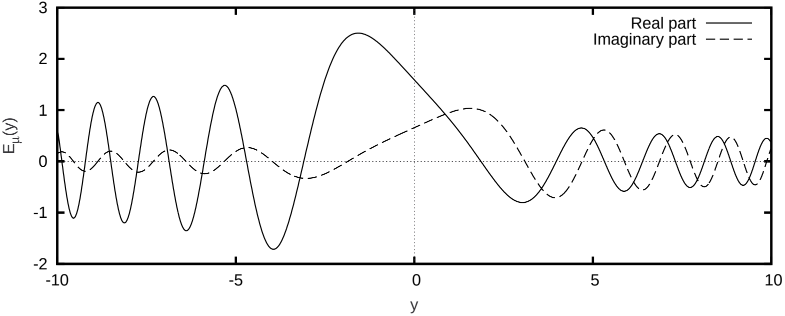
- An accelerated detector is excited even when there are no test particles present.
- This excitation is accompanied by the emission of a test particle.
- Conclusion: “Detection of a Rindler particle” is actually just a spontaneous reaction of the detector to the accelerating potential.

Further reading

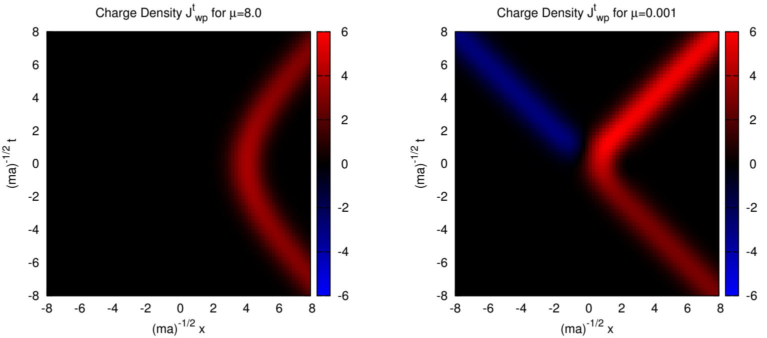
- Unruh detector: W. Unruh, *Notes on black-hole evaporation* (Phys.Rev. D)
- Accelerated quantum field: R. Brout et al., *A primer for Black Hole Quantum Physics* (arXiv)
- A similar conclusion was obtained from a non-relativistic detector model: T. Padmanabhan, *Why does an accelerated detector click?* (Class.Quant.Grav)
- F. Thoma, *A quantum field theoretical detector model for probing the Unruh effect* (arXiv)

Stationary solution (plot)

Plot of the parabolic cylinder function $E_\mu(y)$ for $\mu = 0.001$



Charge density (wave packet solution)



Particle creation: 1 particle $\to |\alpha|^2$ particles and $|\beta|^2 = |\alpha|^2 - 1$ anti-particles
$|\beta|^2 = e^{-2\pi\mu}$