Gauge/Gravity Duality & Superfluidity

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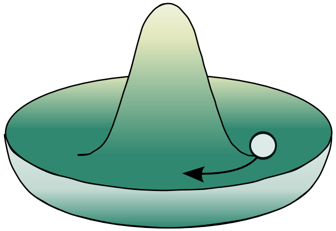
Holographic Superfluidity

Superfluidity in Strongly Coupled Quantum Systems

. Quark-Gluon-Plasma
For Example: $\langle \rho \rangle$

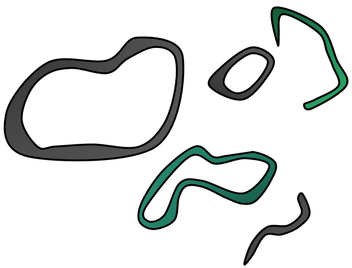


. Condensed Matter Systems

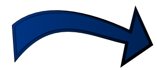
Holography: A Powerful Tool to describe these Sytems

AdS/CFT Correspondence (Maldacena '97)

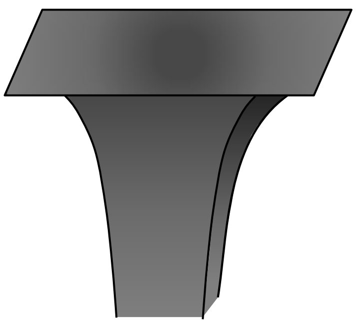
The original form of the duality



2 Interpretations of D-Branes



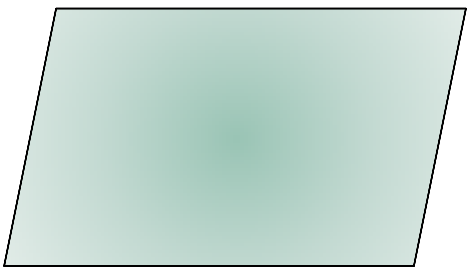
Special Limit



IIB Supergravity on $AdS_5 \times S^5$

Weakly coupled, 4+1 dim

$\widehat{=}$

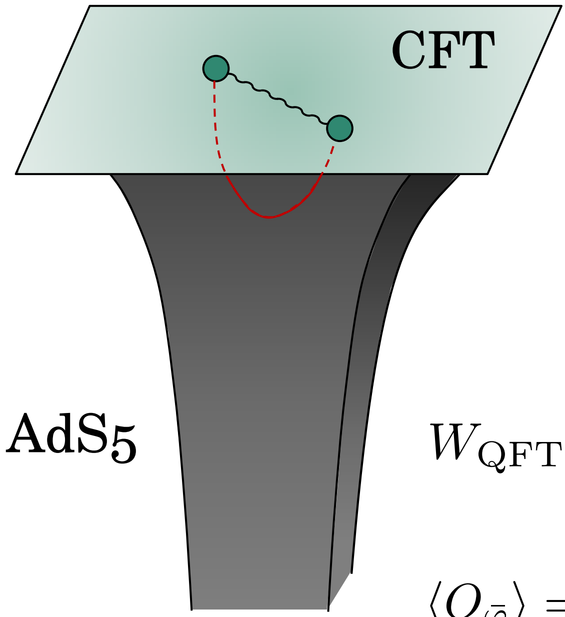


$\mathcal{N} = 4$ SYM (CFT)

Strongly coupled, 4 dim

AdS/CFT Correspondence (Maldacena '97)

The original form of the duality



$$W_{\mathrm{QFT}}\left[\bar{\varphi}\right] = S_{\mathrm{Grav}}\left[\varphi\right]\Big|_{\varphi_{\mathrm{BDY}}=\bar{\varphi}}$$

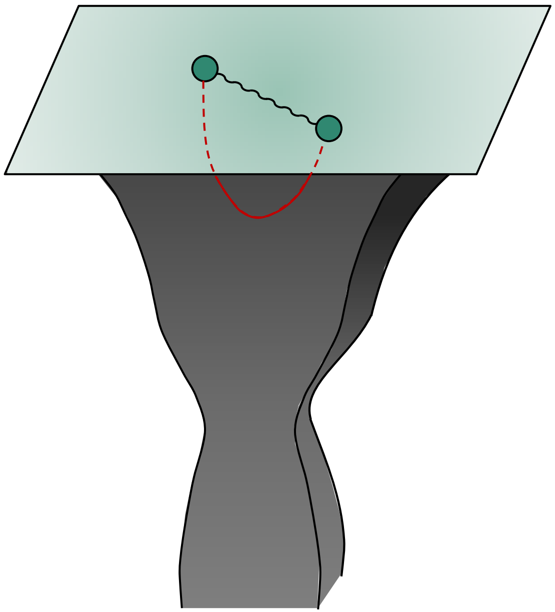
$$\langle O_{\bar{\varphi}}\rangle = \frac{1}{\sqrt{-g_0}}\frac{\delta S_{\mathrm{Grav}}}{\delta\bar{\varphi}}\Bigg|_{\mathrm{BDY}}$$

Gauge/Gravity Duality

Gravity Side

Field Theory Side

Deformation



Less Symmetry

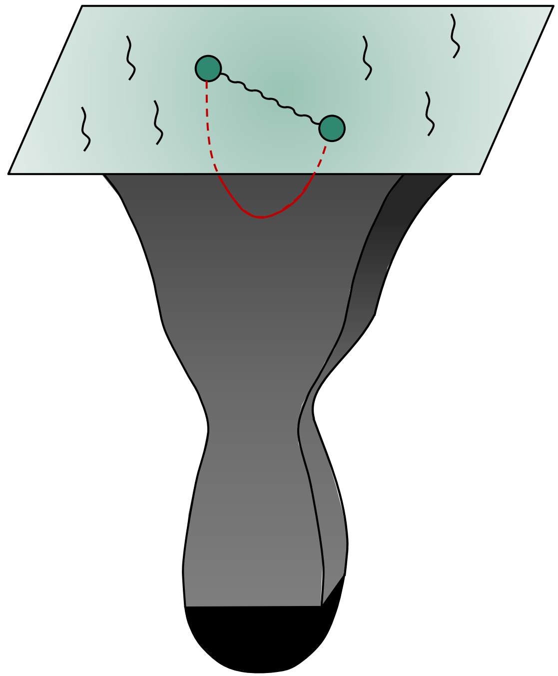
Gauge/Gravity Duality

Gravity Side

Field Theory Side

Deformation

Black Hole



Less Symmetry

Nonzero Temperature

Gauge/Gravity Duality

Gravity Side

Field Theory Side

Deformation

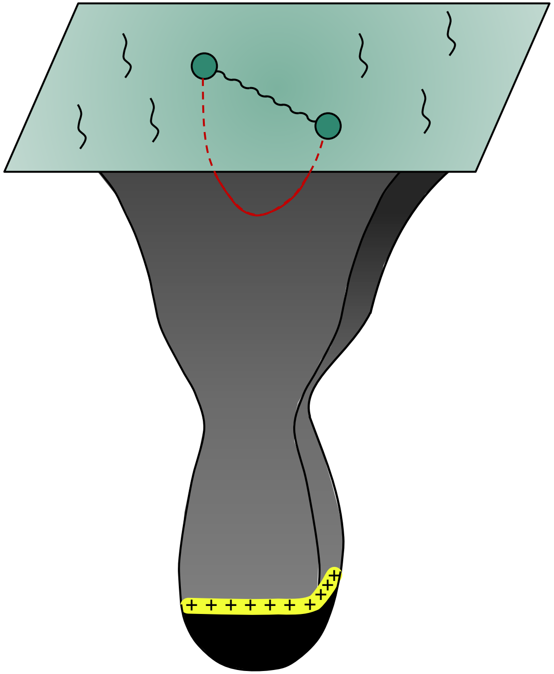
Less Symmetry

Black Hole

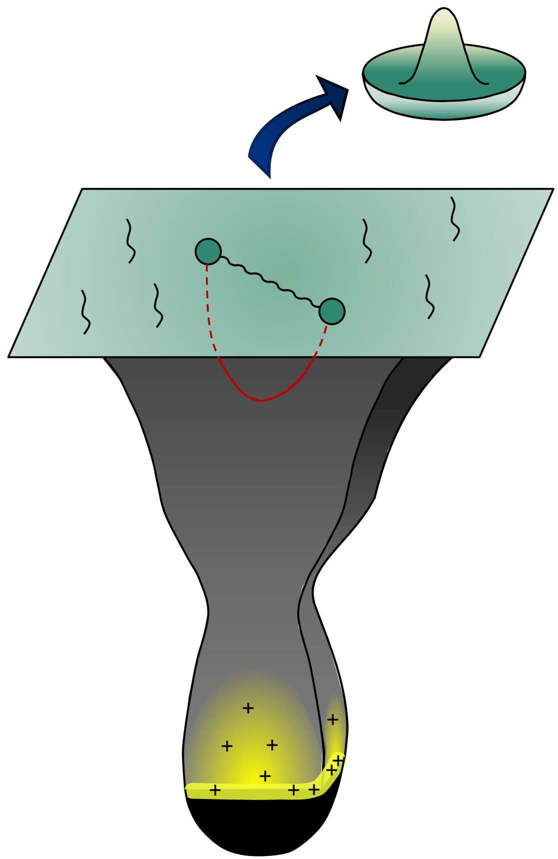
Nonzero Temperature

Charged Black Hole

Finite Density



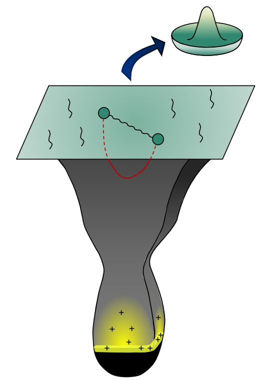
Gauge/Gravity Duality



Gravity Side	Field Theory Side
Deformation	Less Symmetry
Black Hole	Nonzero Temperature
Charged Black Hole	Finite Density
"Hair"	Spontaneous Symmetry Breaking

Holographic p-wave Model (Gubser, Pufu '08)

SU(2) Einstein-Yang-Mills Action



$$S = \int \mathrm{d}^5x\sqrt{-g}\frac{1}{\kappa_5^2}\left[\frac{1}{2}(R-2\Lambda) - \alpha^2 F^a_{\mu\nu}F^{a\mu\nu}\right]$$

Ansatz

$$A = \phi(u)\,\tau^3\mathrm{d}t + w(u)\,\tau^1\mathrm{d}x$$

SU(2)

Chemical Potential

$$\phi(u) = \mu + \langle\rho\rangle\, u^2 + \mathcal{O}(u^3)$$

→ SU(2) ⇓ $U(1)_3$ SO(3) — explicit

Condensate

$$w(u) = 0 + \langle J_x\rangle\, u^2 + \mathcal{O}(u^3)$$

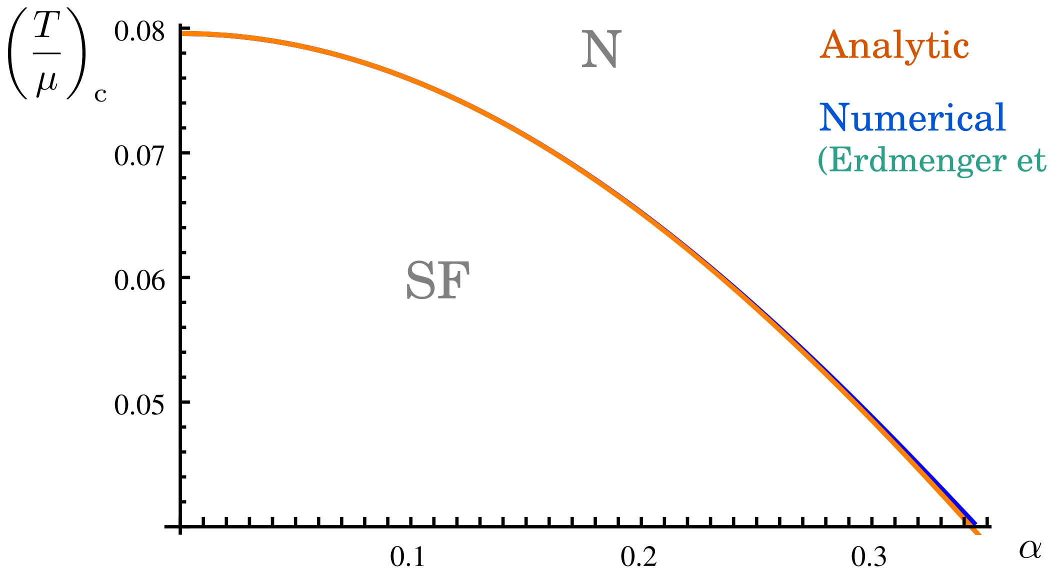
→ $U(1)_3$ ⇓ $\mathbb{Z}_2$, SO(3) ⇓ SO(2) — spontaneous

An Analytic Approach (based on Herzog, Pufu '09)

Double Expansion

$$\varphi(u) = \sum_{i=0}^{n\epsilon} \sum_{j=0}^{n\alpha} \varphi_{i,j}(u) \langle J_x \rangle^i \alpha^{2j}$$

Phase Diagram

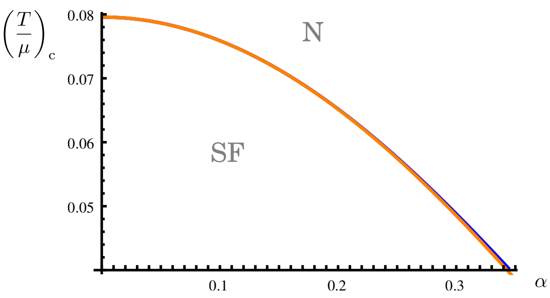


Thermodynamics

Mean-Field Results for T < T$_c$

$$\Delta F = F_{\rm N} - F_{\rm SF} \propto \left(1 - \frac{T}{T_{\rm c}}\right)^2$$

$$\langle J_x \rangle \propto \left(1 - \frac{T}{T_{\rm c}}\right)^{\frac{1}{2}}$$



Isotropy of Energy-Momentum Tensor in Equilibrium

$$\langle T_{ab} \rangle = \lim_{u\to 0} \frac{-2}{\sqrt{-g_0}} \frac{\delta S}{\delta g_0^{ab}}$$

$$\langle T_{xx} \rangle - \langle T_{yy} \rangle = 0 + \mathcal{O}\left(\langle J_x \rangle^6\right)\alpha^2 + \mathcal{O}\left(\langle J_x \rangle^3\right)\alpha^4 + \mathcal{O}\left(\alpha^6\right)$$

$$= 0$$

(Donos, Gauntlett '13)

Optical Conductivity

Fluctuate about the equilibrium

$$\varphi(u) \to \varphi(u) + \delta\varphi(u,t)$$

Calculate Green functions (Son, Starinets '02)

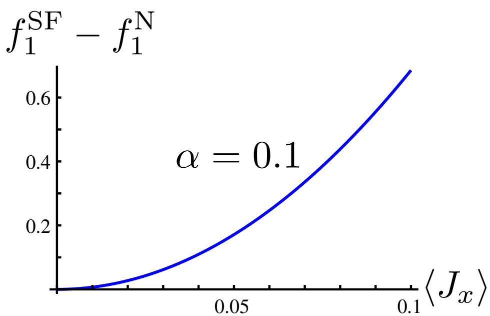
$$S_{\mathrm{on-shell}} = \mathrm{Vol}_3 \int \frac{\mathrm{d}\omega}{2\pi} \delta\bar{\varphi}(-\omega)\mathcal{F}(u,\omega)\delta\bar{\varphi}(\omega)\Big|_{u=0}^{u=u_{\mathrm{H}}}$$

$$G^{\mathrm{R}}(\omega) = 2 \lim_{u\to 0} \mathcal{F}(u,\omega)$$

Optical Conductivity $\langle J_x \rangle = \sigma_{x,x} E_x$

$$\sigma(\omega) = \lim_{\vec{k}\to 0} \frac{1}{i\omega} G^{\mathrm{R}}\left(\omega, \vec{k}\right)$$

$$\sigma_{x,x}^{3,3}(\omega) = f_1\left(\langle J_x \rangle, \alpha^2\right)\delta(\omega) + f_2\left(\langle J_x \rangle, \alpha^2\right) T + \mathcal{O}(\omega)$$



Single Scaling of $\sigma(0)$

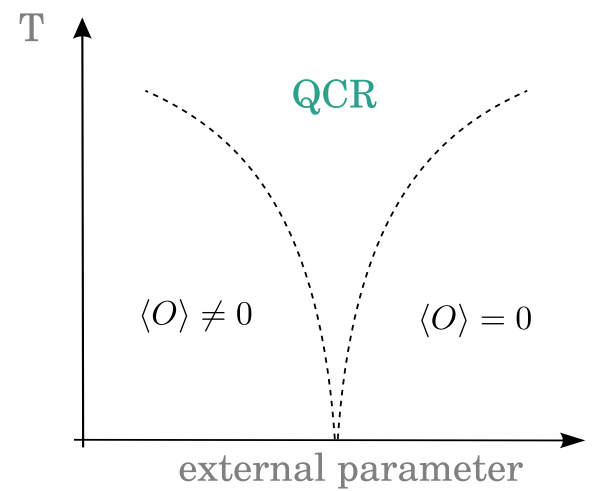
Single scaling in quantum critical region (QCR)

$$\sigma(\omega = 0) = \frac{Q^2}{\hbar}\Sigma(0)\left(\frac{k_{\mathrm{B}}T}{\hbar c}\right)^{(d-2)/z}$$

(Phillips '05)

$z = 1,\ d = 3$

$$\sigma(\omega = 0) = \Sigma(0)\,T$$



Analytic Calculation of $\Sigma(0)$

$$f_2\left(\langle J_x\rangle, \alpha^2\right) = \Sigma(0)$$

Conclusion & Outlook

. Analytic confirmation of previous numerical results

. Mean field exponents

. Isotropic energy-momentum tensor

. Scaling behaviour of optical conductivity

. Compare $\Sigma\,(0)$ with experiments

. Forthcoming Publication:
"Scaling Behaviour of optical conductivities from holography"
(J. Erdmenger, S. Klug, H. Zeller, A. S.)

. Learn more about the phenomenological p-wave model

Thank You.