Holographic Superconductors with Broken Translational Symmetry

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Outline

Introduction

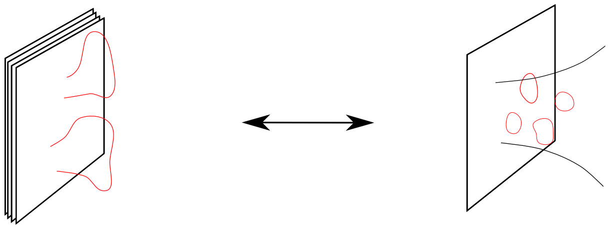
- Holography:
  QFT in d dimensions $\Leftrightarrow$ gravity in $d+1$ dimensions
- Strongly coupled field theory $\Leftrightarrow$ classical gravity theory
- Heavy ion physics [Kovtun, Son, Starinets, 2003]

$$\frac{\eta}{s} = \frac{1}{4\pi}\frac{\hbar}{k_B}$$

- Condensed matter physics [Homes et al., 2004]

$$\rho_s = C \cdot \sigma_{DC}(T_c)T_c$$

Original AdS/CFT Correspondence [Maldacena, 1997]



$\mathcal{N} = 4$ Super-Yang-Mills theory with gauge group $SU(N)$

SUGRA on $AdS_5 \times S^5$

$$\mathrm{d}s^2 = \frac{\mathrm{d}r^2}{r^2} + r^2(-\,\mathrm{d}t^2 + \,\mathrm{d}\vec{x}^2)$$

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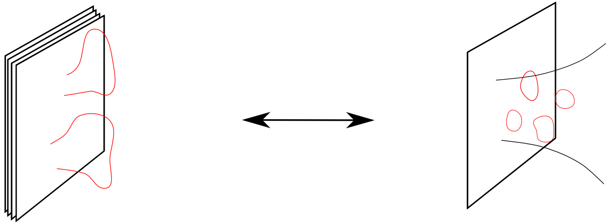

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$N \to \infty$,
strong 't Hooft coupling $\lambda \gg 1$

No string loop correction, classical gravity

Gauge/Gravity Dictionary

Gravity		Field theory
Extra dim. r	$\longleftrightarrow$	Energy scale μ
Asymptotically AdS space		UV fixed point
Black hole with temp. T		Field theory at finite T
Scalar field ϕ		Scalar operator $\mathcal{O}_\phi$
Vector A_μ		Conserved current j_μ
Metric $g_{\mu\nu}$		Stress-energy $T_{\mu\nu}$

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$$\phi \sim \frac{\phi^{(s)}}{r^{\Delta_-}} + \frac{\phi^{(v)}}{r^{\Delta_+}}$$

Partition Function

$$Z_{\text{QFT}}[\phi^{(s)}] = e^{-S_{\text{gravity}}[\phi^{(s)}]}$$

Holographic Setup

Einstein-Maxwell-Scalar action:

$$S = \int \mathrm{d}^{4+1}x\sqrt{-g}\left[R + 12 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}W^{\mu\nu}W_{\mu\nu} - |\partial\rho - iqA\rho|^2\right]$$

[Without scalar: Donos, Hartnoll, 2012]

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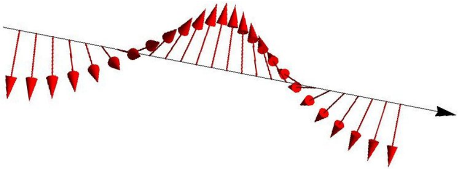
$$\begin{aligned} \omega_1 &= dx \\ \omega_2 &= \cos(px)\,dy - \sin(px)\,dz \\ \omega_3 &= \sin(px)\,dy + \cos(px)\,dz \end{aligned}$$



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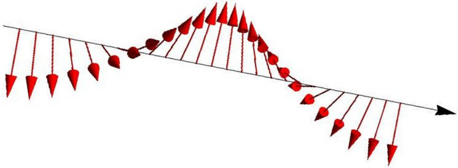

Metric and gauge field ansatz:

$$\mathrm{d}s^2 = -U(r)\,\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{U(r)} + e^{2v_1(r)}\omega_1^2 + e^{2v_2(r)}\omega_2^2 + e^{2v_3(r)}\omega_3^2$$

$A = a(r)dt \quad \longleftrightarrow \quad$ finite charge density

$B = w(r)\omega_2 \quad \longleftrightarrow \quad$ helical lattice

$\rho \neq 0 \quad \longleftrightarrow \quad$ broken global U(1)

Calculational Recipe

1. Equations of motion

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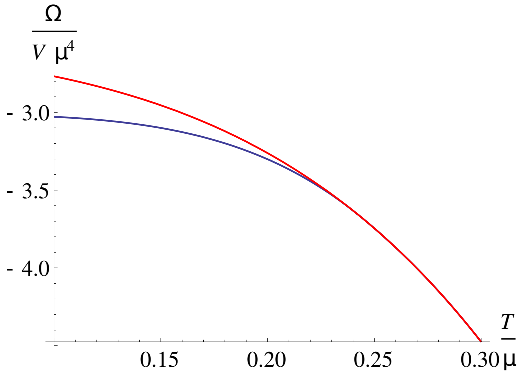
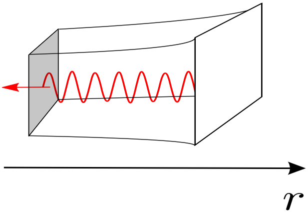
4. On-shell action, thermodynamics
   - Hawking temperature, Bekenstein-Hawking entropy
   - $Z = e^{-S_{\text{gravity}}} \Longrightarrow \Omega = TS_{\text{gravity}}$

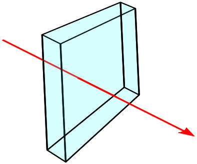
Second Order Phase Transition



$$q = 6, \frac{p}{\mu} = 2.5, \frac{\lambda}{\mu} = 1$$

Linear Response [Son, Starinets, 2002]





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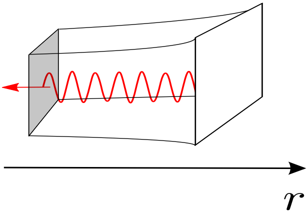

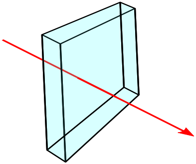

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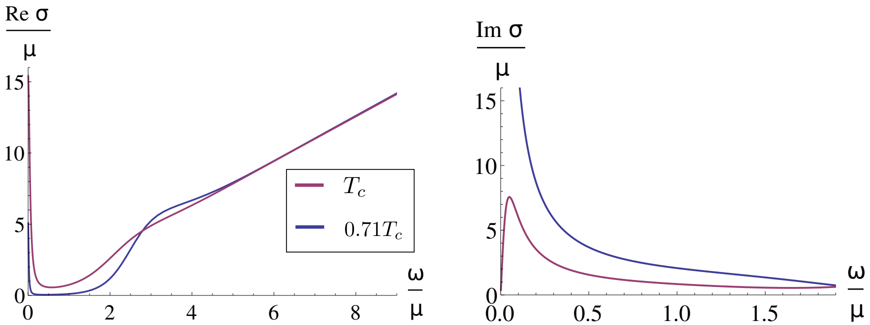
Gravity:

$$\delta A = A(t,r)\,\mathrm{d}x$$

Electrical Conductivity



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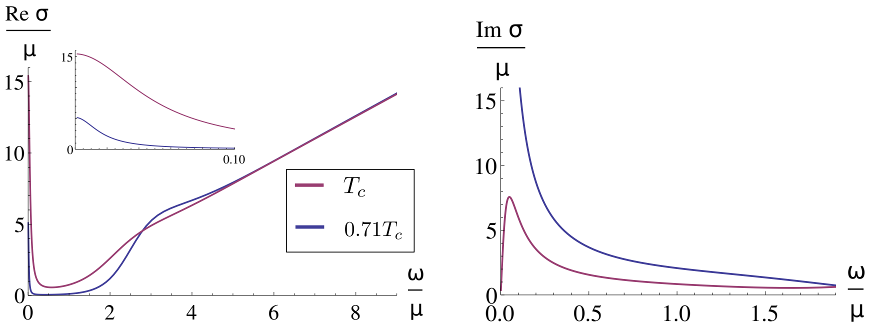

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Summary

- Holography: useful tool to study strongly coupled field theories
- Holographic model of superconductors:
  - $\rightarrow$ Break translational invariance
  - $\rightarrow$ Infinite DC conductivity only in broken phase
  - $\rightarrow$ Important step towards holographic realization of Homes' law
- Further steps:
  - $\rightarrow$ Explore the parameter space
  - $\rightarrow$ Apply to Homes' law

$$\rho_s = C \cdot \sigma_{DC}(T_c) T_c$$

- Forthcoming publication:
  Homes' law in helical holographic superconductors,
  J. Erdmenger, B. Herwerth, R. Meyer, S. Müller, K. Schalm