Chern-Simons terms and the effective action of F-theory with $U(1)$ factors

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- 12D string theory (non-perturbative version of type IIB string theory)

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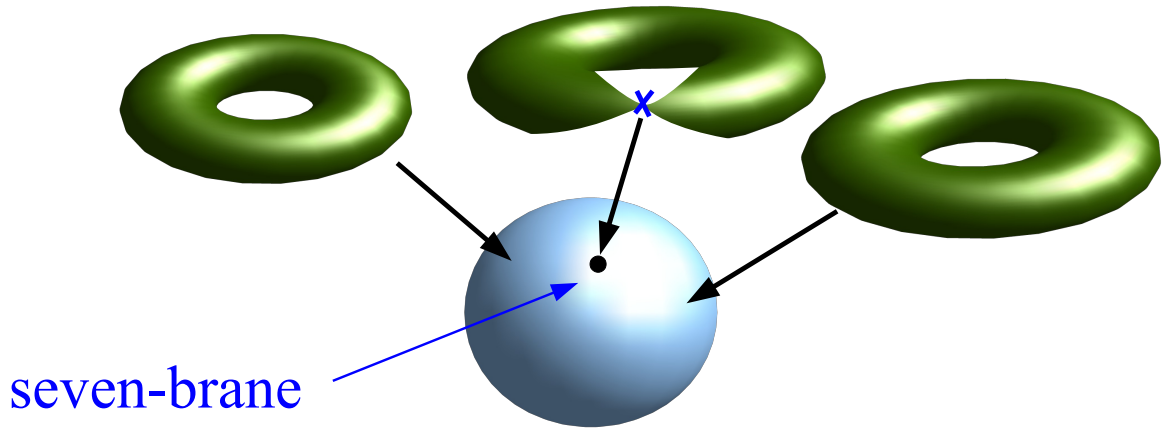

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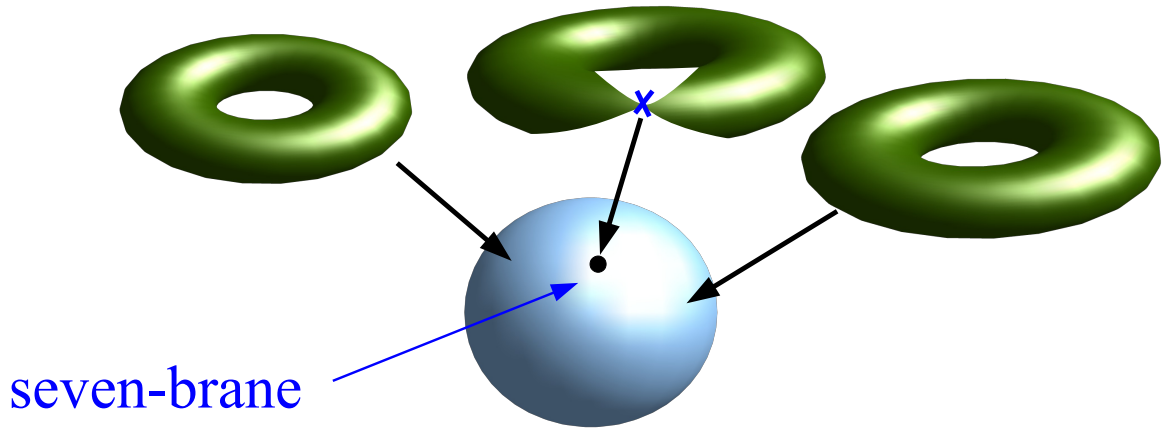

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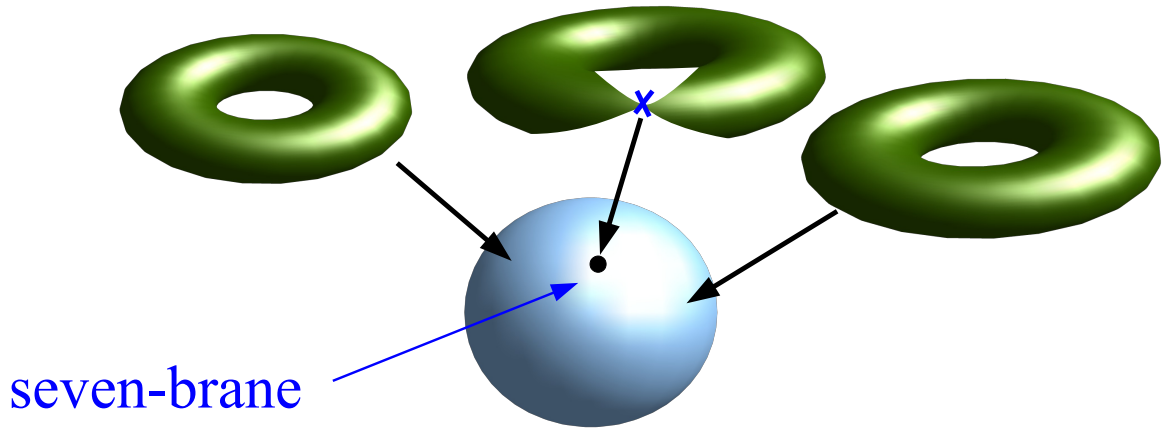

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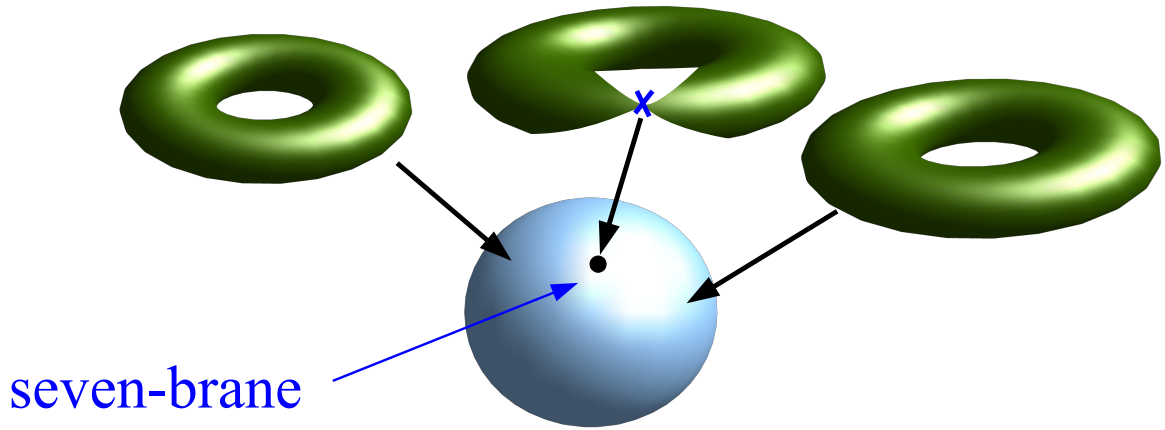

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- Effective action of F-theory compactifications via duality to M-theory
- M-theory: 11D theory, low-energy effective action described by 11D supergravity

6D F-theory effective action

Compactify 12D F-theory to 6D spacetime:

6D non-compact spacetime $\times$ 6D compact space (Calabi-Yau threefold, torus fibration)

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Why 6D spacetime?

- Strongly constrained by anomaly conditions
- Easier to handle than compactifications to 4D
- Already a lot of insights to 4D compactifications from 6D studies

Duality to M-theory

F-theory on a **singular** Calabi-Yau threefold:

6D non-Abelian theory (supergravity)

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dual

M-theory on the Calabi-Yau threefold with **singularities resolved**:

5D Abelian theory

Matching of Chern Simons terms

Chern-Simons terms in the 5D effective action:

$$S^{CS} = -\frac{1}{12}\int_{M_5} k_{\Lambda\Sigma\Theta} A^{\Lambda} \wedge F^{\Sigma} \wedge F^{\Theta}$$

k: Chern-Simons coefficient
A^{Λ}: Abelian gauge fields with field strength F^{Λ}

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Issue:

- Circle reduction of 6D F-theory **not quantum consistent** (massive modes)
- 5D M-theory compactification quantum consistent

$\Rightarrow$ Integrate out massive modes on the 5D F-theory side

Massive modes on the 5D F-theory side

Kaluza-Klein modes:

- Expand the 6D fields in Fourier modes along the circle direction:

$$\psi(x,y) = \sum_{n=-\infty}^{+\infty} \psi_n(x) e^{iny/r}$$

- Infinite tower of states $\psi_n(x)$ with mass $m_{KK} = n/r$

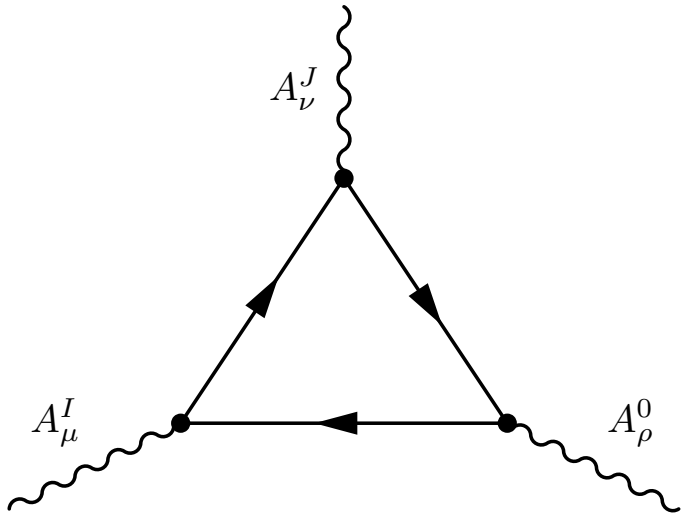
Coulomb branch modes:

- Charged matter becomes massive by gauge symmetry breaking
- Coulomb branch mass: m_{CB}

Inducing 5D Chern-Simons terms at one-loop

Integrate out:

- massive spin-1/2 fermions
- massive spin-3/2 fermions
- massive (anti-)self-dual tensors



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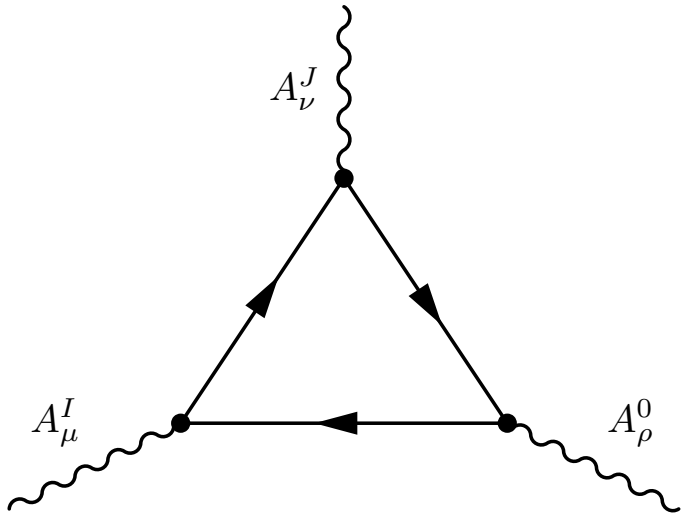

$\Rightarrow$ One-loop Chern-Simons terms (K. Intriligator, D. Morrison, N. Seiberg, F. Bonetti, T. Grimm, S. Hohenegger)

spin-1/2 fermion

$$k_{\Lambda\Sigma\Theta} = \tfrac{1}{2} q_\Lambda q_\Sigma q_\Theta \,\mathrm{sign}(m)$$

spin-3/2 fermion

$$k_{\Lambda\Sigma\Theta} = -\tfrac{5}{2} q_\Lambda q_\Sigma q_\Theta \,\mathrm{sign}(m)$$

tensor

$$k_{\Lambda\Sigma\Theta} = -2 q_\Lambda q_\Sigma q_\Theta \,\mathrm{sign}(m)$$

m: mass of the mode

q_Λ: charge under the Abelian gauge field A^Λ

One-loop matching

Matching of one-loop induced Chern-Simons terms on the F-theory side with corresponding terms on the M-theory side:

$$k^F_{loop} \stackrel{!}{=} k^M_{loop}$$

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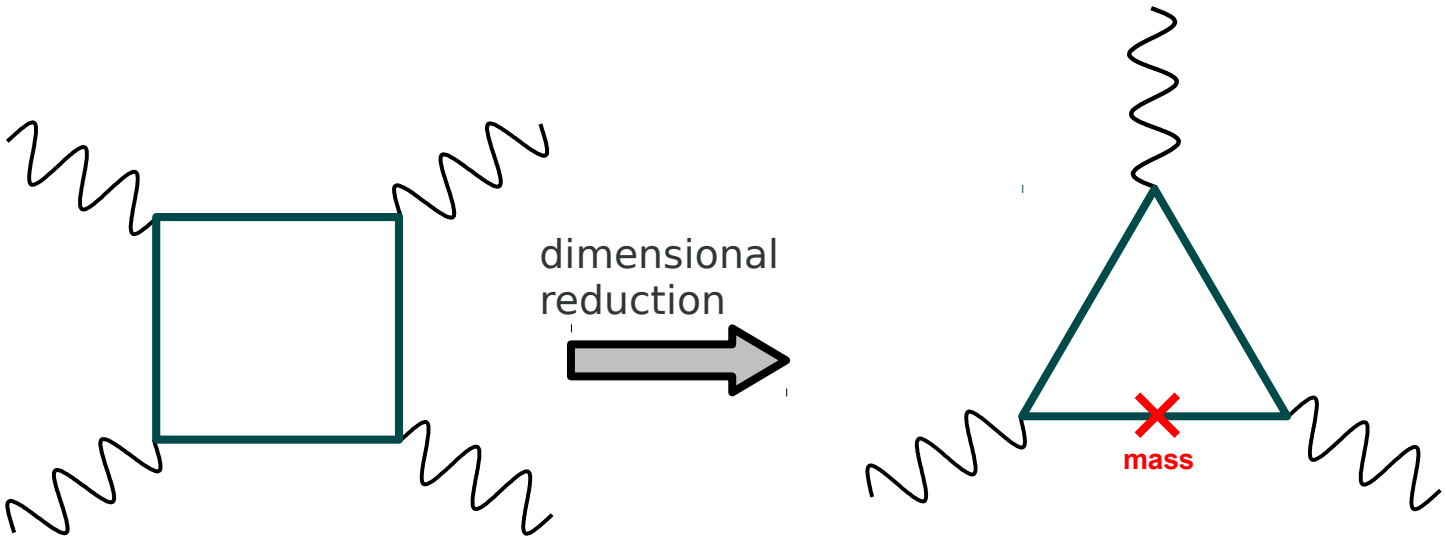
$\Rightarrow$ Information about the 6D F-theory **spectrum** and **anomalies** in terms of geometric data of the Calabi-Yau compactification space:

6D F-theory anomalies

Connection between 5D one-loop Chern-Simons terms and 6D anomalies from heuristic reduction of Feynman diagrams:



6D anomaly

5D loop inducing
Chern-Simons terms

Mass Hierarchies

Example of one-loop induced Chern-Simons coefficient:

$$k_{mnk} = \frac{1}{2}\sum_f \sum_{n=-\infty}^{+\infty} q_m q_n q_k \,\mathrm{sign}(m_{CB} + m_{KK})$$

$$= -\frac{1}{2}\sum_f \sum_{n=-\infty}^{+\infty} q_m q_n q_k \,\mathrm{sign}(m_{CB} + n/r)$$

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Conclusions and Future Directions

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- Matching of 5D one-loop Chern-Simons terms contains crucial information about 6D F-theory spectrum and anomalies
- Chern-Simons terms jump for a violated mass hierarchy $m_{CB} < 1/r$

Future directions:

- Apply results to 4D
- Better understanding of connection between anomalies, spectrum and Chern-Simons terms
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Thank You!