The Effective Field Theory of Large Scale Structures

David Ciupke

Supervisors: Jens Niemeyer and Thorsten Battefeld

University of Göttingen

July 8th 2013

Motivation/Overview

- Large Scale Structure: Probe for dark energy, modified gravity and primordial perturbations
- LSS - large scales: linear problem - small scales: non-linear problem
- Usual approach: Numerics, i.e. N-body simulations
- Analytic models: Many different approaches, not clear which one "does the job best"

Goals

Towards the **non-linear scales**, LSS with **modified gravity**

Eulerian Standard Approach

- "Microscopic" system of collisionless DM particles, that interact only gravitationally in some flat FLRW background
- Approximation: Ideal fluid

Continuity and Euler equation

$$\frac{\partial}{\partial\tau}\delta(\mathbf{k},\tau)+\theta(\mathbf{k},\tau)=-\int \mathrm{d}^3\mathbf{q}\,\alpha(\mathbf{q},\mathbf{k}-\mathbf{q})\theta(\mathbf{q},\tau)\delta(\mathbf{k}-\mathbf{q},\tau),$$

$$\frac{\partial}{\partial\tau}\theta(\mathbf{k},\tau)+\mathcal{H}(\tau)\theta(\mathbf{k},\tau)+\frac{3}{2}\Omega_m\mathcal{H}^2(\tau)\delta(\mathbf{k},\tau)$$

$$=-\int \mathrm{d}^3\mathbf{q}\,\beta(\mathbf{q},\mathbf{k}-\mathbf{q})\theta(\mathbf{q},\tau)\theta(\mathbf{k}-\mathbf{q},\tau)$$

Eulerian Standard Approach

- "Microscopic" system of collisionless DM particles, that interact only gravitationally in some flat FLRW background
- Approximation: Ideal fluid

Continuity and Euler equation

$$\frac{\partial}{\partial\tau}\delta(\mathbf{k},\tau)+\theta(\mathbf{k},\tau)=-\int \mathrm{d}^3\mathbf{q}\,\alpha(\mathbf{q},\mathbf{k}-\mathbf{q})\theta(\mathbf{q},\tau)\delta(\mathbf{k}-\mathbf{q},\tau),$$

$$\frac{\partial}{\partial\tau}\theta(\mathbf{k},\tau)+\mathcal{H}(\tau)\theta(\mathbf{k},\tau)+\frac{3}{2}\Omega_m\mathcal{H}^2(\tau)\delta(\mathbf{k},\tau)$$

$$=-\int \mathrm{d}^3\mathbf{q}\,\beta(\mathbf{q},\mathbf{k}-\mathbf{q})\theta(\mathbf{q},\tau)\theta(\mathbf{k}-\mathbf{q},\tau)$$

Standard Perturbation Theory

- Instructive example: Einstein-de Sitter ($\Omega_m = 1$)
- Linear system solution: $\delta_L(\mathbf{k}, \tau) = a(\tau)\, \delta_1(\mathbf{k}) + a^{-3/2}(\tau)\, \widetilde{\delta_1}(\mathbf{k})$ freely propagating

Standard Perturbation Theory

- Instructive example: Einstein-de Sitter ($\Omega_m = 1$)
- Linear system solution: $\delta_L(\mathbf{k},\tau) = a(\tau)\,\delta_1(\mathbf{k}) + a^{-3/2}(\tau)\,\widetilde{\delta_1}(\mathbf{k})$ freely propagating
- Perturbative ansatz in initial conditions:

$$\delta(\mathbf{k},\tau) = \sum_{n=1}^{\infty} a^n(\tau)\delta_n(\mathbf{k})$$

$$\delta_n(\mathbf{k}) = \int \mathrm{d}^3\mathbf{k}_1 \ldots \mathrm{d}^3\mathbf{k}_n\, \delta^{(3)}(\mathbf{k} - \mathbf{k}_{1\ldots n}) F_n(\mathbf{k}_1, \ldots, \mathbf{k}_n)\delta_1(\mathbf{k}_1)\ldots\delta_1(\mathbf{k}_n)$$

Standard Perturbation Theory

- Instructive example: Einstein-de Sitter ($\Omega_m = 1$)
- Linear system solution: $\delta_L(\mathbf{k},\tau) = a(\tau)\,\delta_1(\mathbf{k}) + a^{-3/2}(\tau)\,\widetilde{\delta_1}(\mathbf{k})$ freely propagating
- Perturbative ansatz in initial conditions:

$$\delta(\mathbf{k},\tau) = \sum_{n=1}^{\infty} a^n(\tau)\delta_n(\mathbf{k})$$

$$\delta_n(\mathbf{k}) = \int \mathrm{d}^3\mathbf{k}_1 \ldots \mathrm{d}^3\mathbf{k}_n\, \delta^{(3)}(\mathbf{k} - \mathbf{k}_{1\ldots n})F_n(\mathbf{k}_1,\ldots,\mathbf{k}_n)\delta_1(\mathbf{k}_1)\ldots\delta_1(\mathbf{k}_n)$$

- Fluid equations translate into recursion relations for kernels F_n
- $\Rightarrow$ Standard Perturbation Theory

Standard Perturbation Theory

- Instructive example: Einstein-de Sitter ($\Omega_m = 1$)
- Linear system solution: $\delta_L(\mathbf{k},\tau) = a(\tau)\,\delta_1(\mathbf{k}) + a^{-3/2}(\tau)\,\widetilde{\delta_1}(\mathbf{k})$ freely propagating
- Perturbative ansatz in initial conditions:

$$\delta(\mathbf{k},\tau) = \sum_{n=1}^{\infty} a^n(\tau)\delta_n(\mathbf{k})$$

$$\delta_n(\mathbf{k}) = \int \mathrm{d}^3\mathbf{k}_1 \ldots \mathrm{d}^3\mathbf{k}_n\, \delta^{(3)}(\mathbf{k} - \mathbf{k}_{1\ldots n}) F_n(\mathbf{k}_1, \ldots, \mathbf{k}_n)\delta_1(\mathbf{k}_1)\ldots\delta_1(\mathbf{k}_n)$$

- Fluid equations translate into recursion relations for kernels F_n
- $\Rightarrow$ Standard Perturbation Theory

Problems of Standard Perturbation Theory

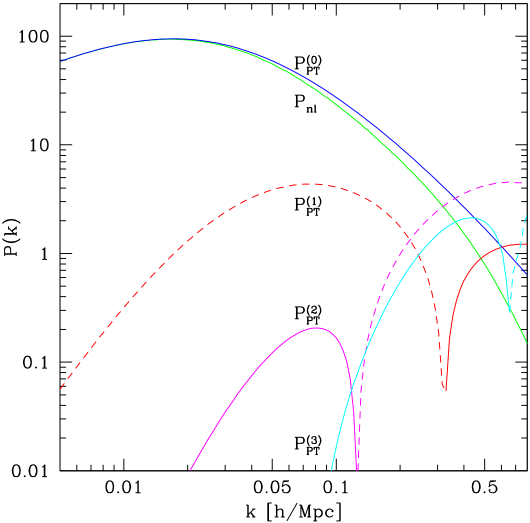
- **Perturbative solution not well defined on small scales**
- How large are the theoretical errors?
- How good is the approximation of an ideal fluid?
- **UV-divergences** in loop contributions to correlation functions, e.g. for Gaussian initial conditions

$$\langle \delta_1(\mathbf{k})\delta_3(-\mathbf{k})\rangle \propto \int \mathrm{d}^3\mathbf{q}\, F_3(\mathbf{k},\mathbf{q},-\mathbf{q})P_0(k)P_0(q)$$

$$\text{leading order divergence} : \sim k^2 P_0(k) \int \mathrm{d}q\, P_0(q)$$

- Typically $P_0(q) \propto q^n$
- Expect renormalization, but no counter-terms

Loop corrections to power spectrum



Effective Field Theory Approach

- Treatment only valid up to a scale $k_{NL} \Rightarrow$ should be understood in the sense of an effective field theory (Baumann et al '10, Carrasco et al '12, Hertzberg '12)
- Rough analogy: Mass of a heavy field

Effective Field Theory Approach

- Treatment only valid up to a scale k_{NL} $\Rightarrow$ should be understood in the sense of an effective field theory (Baumann et al '10, Carrasco et al '12, Hertzberg '12)
- Rough analogy: Mass of a heavy field
- EFT logic: Write down most general (local) field equations that respect the symmetries (Galilei)

Effective Field Theory Approach

- Treatment only valid up to a scale k_{NL} $\Rightarrow$ should be understood in the sense of an effective field theory (Baumann et al '10, Carrasco et al '12, Hertzberg '12)
- Rough analogy: Mass of a heavy field
- EFT logic: Write down most general (local) field equations that respect the symmetries (Galilei)
- (Baumann et al '10, Pietroni et al '11): Separate into short and long modes via coarse-graining and integrate out short modes (include stress-tensor)

Effective Field Theory Approach

- Treatment only valid up to a scale k_{NL} $\Rightarrow$ should be understood in the sense of an effective field theory (Baumann et al '10, Carrasco et al '12, Hertzberg '12)
- Rough analogy: Mass of a heavy field
- EFT logic: Write down most general (local) field equations that respect the symmetries (Galilei)
- (Baumann et al '10, Pietroni et al '11): Separate into short and long modes via coarse-graining and integrate out short modes (include stress-tensor)

Effective Field Theory of LSS

Effective Field Equations (relevant terms)

$$\frac{\partial \delta_l}{\partial \tau} + \theta_l = -\int \mathrm{d}^3\mathbf{q}\, \alpha(\mathbf{q}, \mathbf{k}-\mathbf{q}) \theta_l(\mathbf{q}, \tau) \delta_l(\mathbf{k}-\mathbf{q}, \tau),$$

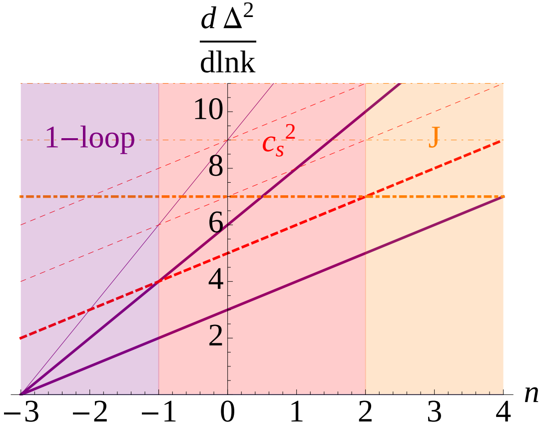
$$\frac{\partial \theta_l}{\partial \tau} + \mathcal{H}\theta_l + \frac{3}{2}\Omega_m \mathcal{H}^2 \delta_l - \mathbf{c_s^2 k^2} \delta_\mathbf{l} + \frac{\mathbf{c_v^2 k^2}}{\mathcal{H}} \theta_\mathbf{l} + \mathbf{\Delta J} + \dots$$

$$= -\int \mathrm{d}^3\mathbf{q}\, \beta(\mathbf{q}, \mathbf{k}-\mathbf{q}) \theta_l(\mathbf{q}, \tau) \theta_l(\mathbf{k}-\mathbf{q}, \tau)$$

- New operators at lowest order: Pressure, viscosity and stochastic noise
- Solution to problems of SPT: Small parameter (k/k_{NL}), viscous fluid with pressure, coarse-graining $\sim$ cutoff regularization, new operators act as counter-terms

Renormalization in EdS [Pajer, Zaldarriaga '13]

- "Threshold corrections", e.g. for the power spectrum $P = P_0 + P_{1-loop} + P_{c^2} + P_{\Delta J} + \dots$
- In our universe: $n = -2.5$, so:
  - c^2-terms $>$ 2-loop
  - $\Delta J <$ 3-loop



Modified Gravity

- Typical example: Scalar-tensor theories:

$$S = \int \mathrm{d}^4x \sqrt{-g} \left[\frac{M_{pl}^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi - V(\phi) \right] + S_m$$

- Assume: Models for dark energy. Typically very constrained on small scales $\Rightarrow$ not viable
- Screened models can evade these constraints, e.g. chameleon, dilaton

Modified Gravity and Perturbation Theory

- Field equations: Fluid equations (modified couplings), Poisson and Klein-Gordon equation
- General state: Barely any non-linear results known

Modified Gravity and Perturbation Theory

- Field equations: Fluid equations (modified couplings), Poisson and Klein-Gordon equation
- General state: Barely any non-linear results known
- At linear level, decoupling:

$$\frac{3}{2}\Omega_m \mathcal{H}^2 \to \frac{3}{2}\Omega_m \mathcal{H}^2(1+\epsilon(k,\tau)) \text{ where } \epsilon(k,t) = \frac{2\beta^2}{1+m^2a^2/k^2}$$

Modified Gravity and Perturbation Theory

- Field equations: Fluid equations (modified couplings), Poisson and Klein-Gordon equation
- General state: Barely any non-linear results known
- At linear level, decoupling:

$$\frac{3}{2}\Omega_m \mathcal{H}^2 \to \frac{3}{2}\Omega_m \mathcal{H}^2 (1+\epsilon(k,\tau)) \text{ where } \epsilon(k,t) = \frac{2\beta^2}{1+m^2a^2/k^2}$$

- (Bernardeau '11, Brax et al '12): Model-dependent parametrization with $\epsilon(k,\tau)$. k dependence contradicts EFT philosophy at linear level

Modified Gravity and Perturbation Theory

- Field equations: Fluid equations (modified couplings), Poisson and Klein-Gordon equation
- General state: Barely any non-linear results known
- At linear level, decoupling:

$$\frac{3}{2}\Omega_m \mathcal{H}^2 \to \frac{3}{2}\Omega_m \mathcal{H}^2(1+\epsilon(k,\tau)) \text{ where } \epsilon(k,t) = \frac{2\beta^2}{1+m^2a^2/k^2}$$

- (Bernardeau '11, Brax et al '12): Model-dependent parametrization with $\epsilon(k,\tau)$. k dependence contradicts EFT philosophy at linear level
- $\Rightarrow$ include only $\epsilon(\tau)$, e.g. via γ-model. Result: Changes not significant (Form of divergences stable)

Modified Gravity and Perturbation Theory

- Field equations: Fluid equations (modified couplings), Poisson and Klein-Gordon equation
- General state: Barely any non-linear results known
- At linear level, decoupling:

$$\frac{3}{2}\Omega_m \mathcal{H}^2 \to \frac{3}{2}\Omega_m \mathcal{H}^2 (1 + \epsilon(k, \tau)) \text{ where } \epsilon(k, t) = \frac{2\beta^2}{1 + m^2 a^2 / k^2}$$

- (Bernardeau '11, Brax et al '12): Model-dependent parametrization with $\epsilon(k, \tau)$. k dependence contradicts EFT philosophy at linear level
- $\Rightarrow$ include only $\epsilon(\tau)$, e.g. via γ-model. Result: Changes not significant (Form of divergences stable)
- Threshold corrections with modified gravity?

Modified Gravity and Perturbation Theory

- Field equations: Fluid equations (modified couplings), Poisson and Klein-Gordon equation
- General state: Barely any non-linear results known
- At linear level, decoupling:

$$\frac{3}{2}\Omega_m\mathcal{H}^2 \to \frac{3}{2}\Omega_m\mathcal{H}^2(1+\epsilon(k,\tau)) \text{ where } \epsilon(k,t) = \frac{2\beta^2}{1+m^2a^2/k^2}$$

- (Bernardeau '11, Brax et al '12): Model-dependent parametrization with $\epsilon(k,\tau)$. k dependence contradicts EFT philosophy at linear level
- $\Rightarrow$ include only $\epsilon(\tau)$, e.g. via γ-model. Result: Changes not significant (Form of divergences stable)
- Threshold corrections with modified gravity?

Modified Gravity and Perturbation Theory 2

- Linear solutions for some models (chameleon, dilaton) are very inaccurate (Brax et al '12,'13)
- $\Rightarrow$ loop corrections must be important
- Very recently (Brax, Valageas '13): General approach via tomography. Roughly: Expand mass and coupling in perturbations of ϕ and solve Klein-Gordon equation. Result: Field equations with vertices at all orders

Modified Gravity and Perturbation Theory 2

- Linear solutions for some models (chameleon, dilaton) are very inaccurate (Brax et al '12,'13)
- $\Rightarrow$ loop corrections must be important
- Very recently (Brax, Valageas '13): General approach via tomography. Roughly: Expand mass and coupling in perturbations of ϕ and solve Klein-Gordon equation. Result: Field equations with vertices at all orders
- Idea: Expand new operators in powers of $(k/m_\phi)^2$, this should be understood as integrating out $\delta\phi \Rightarrow$ validity of perturbation theory depends on (dynamical) scale m_ϕ. Feasible as propagator is the same as in GR. Degeneracy with EFT operators?

Modified Gravity and Perturbation Theory 2

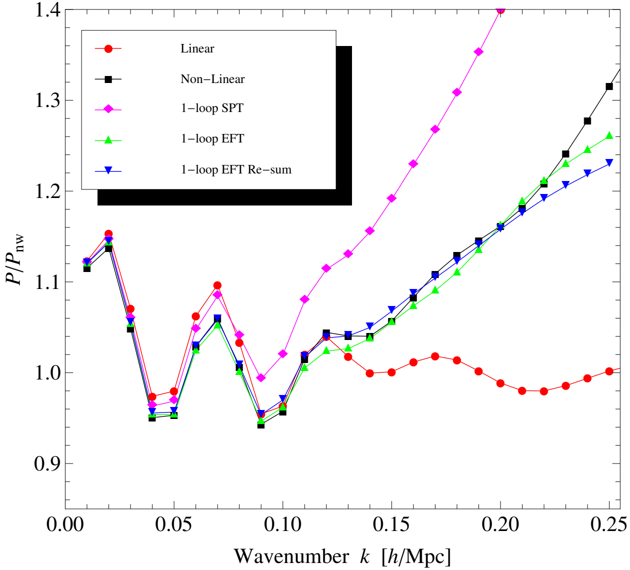
- Linear solutions for some models (chameleon, dilaton) are very inaccurate (Brax et al '12,'13)
- $\Rightarrow$ loop corrections must be important
- Very recently (Brax, Valageas '13): General approach via tomography. Roughly: Expand mass and coupling in perturbations of ϕ and solve Klein-Gordon equation. Result: Field equations with vertices at all orders
- Idea: Expand new operators in powers of $(k/m_\phi)^2$, this should be understood as integrating out $\delta\phi$ $\Rightarrow$ validity of perturbation theory depends on (dynamical) scale m_ϕ. Feasible as propagator is the same as in GR. Degeneracy with EFT operators?

Conclusions

- SPT comes with problems: breakdown at small scales, UV-divergences
- The EFT approach addresses these problems. It is a well-defined framework for studying LSS.
- New operators yield contributions that are more important than 2-loop terms
- Including modified gravity is well-motivated but challenging, so far γ-model, linear results
- Based on (Brax, Valageas '13) a deeper understanding might be possible
- Computation of the bispectrum in the EFT

The End

Backup: EFToLSS: Results



Backup: γ-model

- Assume $f = \Omega_m^\gamma$ and a ΛCDM background (for GR: $\gamma = 6/11$)
- Field equations modified only by $\epsilon(\tau) \Rightarrow$ obtain $\epsilon_\gamma(\tau)$
- Perturbative solution at late times:

$$F_2(\mathbf{k},\mathbf{q}) = \left(\frac{3\,\nu_2}{4} - \frac{1}{2}\right) + \frac{1}{2}\mathbf{k}\cdot\mathbf{q}\left(\frac{1}{k^2} + \frac{1}{q^2}\right) + \left(\frac{3}{2} - \frac{3\nu_2}{4}\right)\frac{(\mathbf{k}\cdot\mathbf{q})^2}{k^2 q^2}$$

- Find approximate solution for ν_2 and repeat regularization of loop integrals (similarly for F_3)