Localization and Diffusion in Polymer Quantum Field Theory

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Outline:

- Localization in QFT;
- Polymer Quantum Field Theory;
- Localization in Polymer Quantum Field Theory;
- Diffusion and Spectral Dimension.

Localization in QFT

- Landau-Peierls procedure *L. Landau and R. Peierls, Z. Phys. 69 (1931) 56*

  measurement of the position of a particle at rest involving the collision with a photon

  (precision $\delta x \to E_{ph} \geq \frac{1}{\delta x}$ *) + (Pair creation limit* $E_{ph} \lesssim M$*)*

  $$\Rightarrow E \geq 1/\delta x$$

- Tuning the boundary conditions *D. Colosi and C. Rovelli, "What is a particle?" Class. Quant. Grav. 26, 025002 (2009)*

  field in a box of size L : $\varphi(-L/2) = \varphi(L/2)$

  $$\Rightarrow \ \omega_k^2 = \left(\frac{2\pi n}{L}\right)^2 + m^2$$

Localization in QFT

- Smeared states

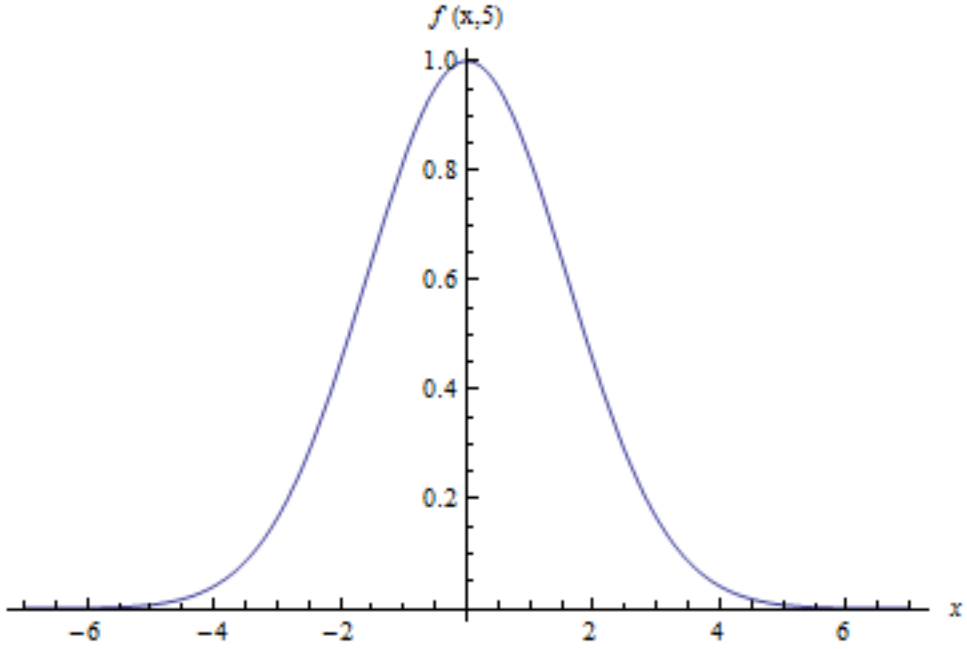
R. Brustein and J. Kupferman, "Black hole entropy divergence and the uncertainty principle." Phys. Rev. D 83, 124014 (2011), [arXiv:hep-th/1010.4157]

$$|\varphi\rangle = \int dx\, f(x - x_0, \sigma)\varphi(x)|0\rangle$$

$f(x - x_0, \sigma)$ *window function*

$\rightarrow$ *localizing the state without changing the boundary conditions*

$$f(x - x_0, \sigma) \propto e^{\frac{-(x-x_0)^2}{2\sigma}} \rightarrow \langle E \rangle = \frac{1}{\sqrt{\pi}\sigma}$$



Polymer Quantum Field Theory

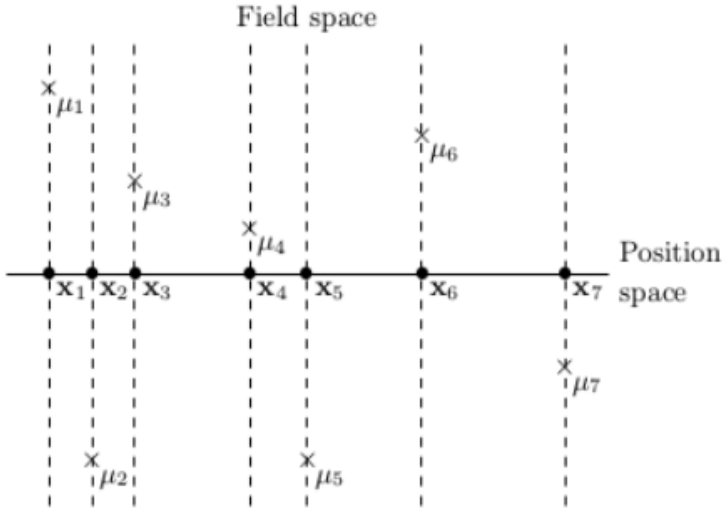
V. Husain and A. Kreienbuehl, “Ultraviolet behavior in background independent quantum field theory", Phys. Rev. D 81, 084043 (2010).

Set of spatial points

$\{x_i, i = 1, \dots, N\}$ with no characteristic scale associated

Foundamental variables $\rightarrow$ $\phi_i = \phi(x_i)$, $U_i^{\ell} = e^{i\ell\pi(x_i)}$.

ℓ is a scale in *field space* $\Rightarrow$ *the momentum operator is ill defined*



Polymer Quantum Field Theory

$\pi_i^{\ell} = \frac{1}{2i\ell}[U_i^{\ell} - U_i^{\ell\dagger}]$ *Polymerized momentum operator*

Hilbert space

$$\phi_i|\mu_1, \mu_2, \dots, \mu_N\rangle = \mu_i|\mu_1, \mu_2, \dots, \mu_N\rangle$$
$$U_i^{\ell}|\mu_1, \mu_2, \dots \mu_N\rangle = |\mu_1, \mu_2, \dots, \mu_i - \ell, \dots, \mu_N\rangle$$

→ *Fock operators and vacuum state (coll. of harmonic oscillators)*

$\langle 0|H|0\rangle \quad \Rightarrow \quad$ *MDR* $\quad k^2 = E_k^2 - m^2 + \frac{\ell E_k^3}{2} + \mathcal{O}\left(\ell^2 E_k^5\right)$

Localization in Polymer Quantum Field Theory

- Smeared states in PQFT

in momentum space $\rightarrow |\phi\rangle \propto \int dp\, G(p,\sigma) a_p^\dagger |0\rangle, \quad G(p,\sigma) = e^{-\frac{p^2\sigma^2}{2}}$

Energy expectation value $\langle E\rangle_{poly} = \left(\frac{\sigma}{\sqrt{\pi}}\right) \int dp\, G^2(p,\sigma)\omega_k \Delta_\ell(p)$,

$\Delta_\ell(p)$ polymer correction to one particle energy

Localization in Polymer Quantum Field Theory

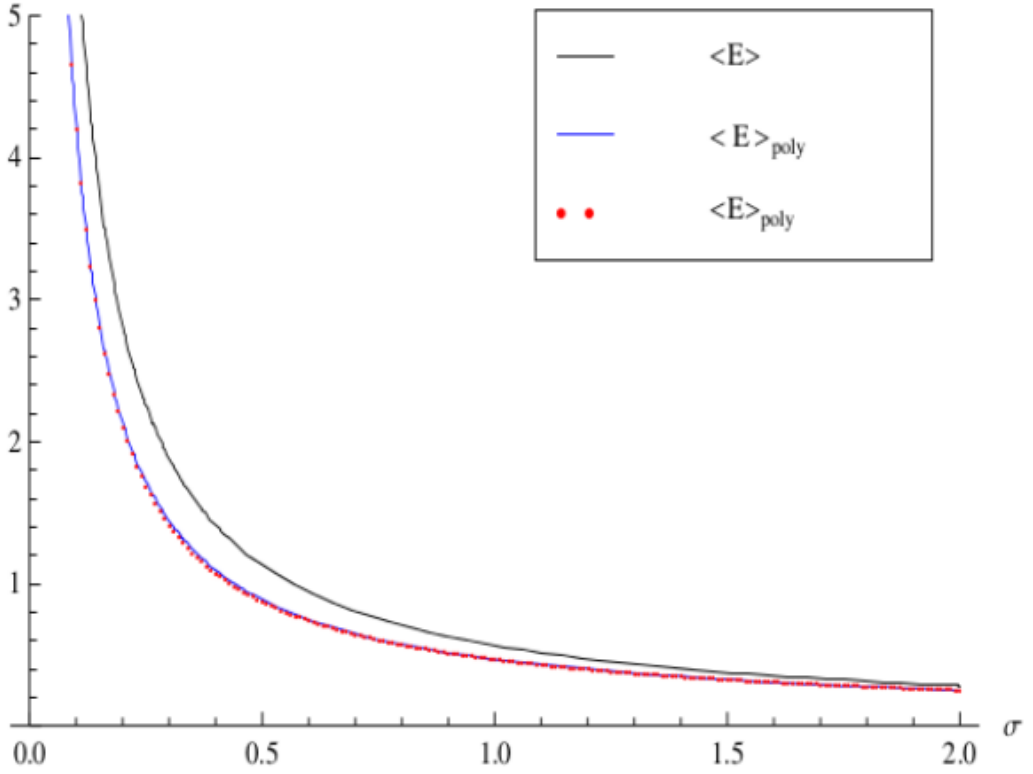
- Analytical & Numerical estimations

Polymer corrections for $\sigma \sim \ell$

$$\langle E \rangle_{poly} \simeq \frac{1}{\sqrt{\pi}\sigma} - \frac{1}{\ell\sigma^2}, \quad \ell \ll \sigma$$

$$\langle E \rangle_{poly} \simeq \frac{3}{4\sqrt{\pi}\sigma} + \mathcal{O}(\sigma), \quad \sigma \ll \ell$$

Still divergent for $\sigma \to 0$



Diffusion and Spectral Dimension

- Heat kernel and spectral dimension

  spectral dimension d_s → effective dimensions of spacetime seen by random walker

  (lattices, spacetime foam, fractal properties,…)

Heat equation for a (generalized) Laplacian operator (flat spacetime)

$\partial_S K(x,x';s) + \Delta_x K(x,x';s) = 0$ → Heat kernel $k(x,x';s) = \int \frac{d^n p}{(2\pi)^n} e^{-\omega_p^2 s} e^{ip(x-x')}$

$$d_s = -2 \frac{\partial \log Tr\, K}{\partial \log s}$$

Diffusion and Spectral Dimension

$$d_s = -2\frac{\partial \log Tr\ K}{\partial \log s}$$

Flat (Euclidean) Spacetime $\rightarrow$ ordinary Laplacian operator $\rightarrow d_s = d$

$$s \rightarrow 0 \quad \begin{cases} \textit{Causal Dynamical Triangulation} & d_s \rightarrow 2 \\ \textit{Spacetime noncommutativity} & d_s < 3 \\ \textit{Asymptotic safety} & d_s \rightarrow 2 \end{cases}$$

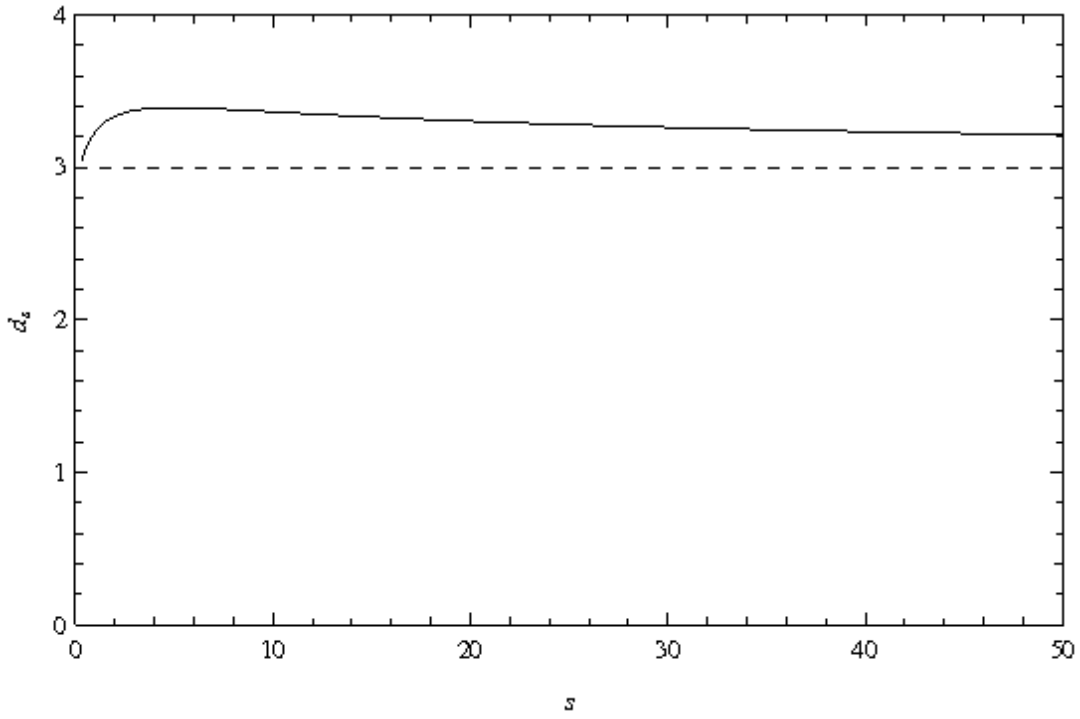
Diffusion and Spectral Dimension

- Spectral dimension in PQFT

$$Tr\ K(x,x';s) = \int \frac{p^2 dp}{(2\pi)^3}\ e^{-\omega_{\boldsymbol{p}}^2 \Delta_\ell^2(\boldsymbol{p})s}$$

$$d_s = \begin{cases} 3 \quad for \quad s \to 0 \\ 3.4 \quad for\ s \sim \ell^2 \\ 3 \quad for\ s \to \infty \end{cases}$$

Superdiffusion for $s \sim \ell^2$



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  (in preparation).