Two-loop Form Factor in massless $N=2$ SYM

Felix Bischof

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Form factor in $N=2$ SYM: equality with QCD even for terms of subleading transcendentality?

- $N=2$: more similar to QCD (matter).

$N=2$ Super Yang-Mills

Generators Q^A_α and $\overline{Q}_{\dot\beta A}$; $A=1,2$ related by R-trafo

$$Q^A_\alpha = U^{AB} Q^B_\alpha, \qquad U \in SU(2).$$

Renormalisable Multiplets

	helicities	degeneracy	fields
$\mathcal{V}^{N=2}$	± 1	1	$v_\mu \leftrightarrow \vert p,\lambda\rangle_{cv}$
	$\pm\frac{1}{2}$	2	$\psi,\lambda \leftrightarrow \overline{Q}_{1,2}\vert p,\lambda\rangle_{cv}$
	0	1	$z \leftrightarrow \overline{Q}_1\overline{Q}_2\vert p,\lambda\rangle_{cv}$
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$\Rightarrow$ invariance under 2nd susy generator.

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$$\mathcal{L}_{N=2} = \mathrm{Tr}\left(\frac{1}{8g^2}\int d\theta^2\ W^\alpha W_\alpha + \int d\theta^2 d\bar{\theta}^2\ \overline{\Phi} e^{2gV}\Phi\right)$$
$$+ \int d\theta^2 d\bar{\theta}^2\ \left(\overline{Q}_- e^{2gV} Q_- + Q_+ e^{-2gV}\overline{Q}_+\right)$$
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Substitute eom for auxiliary fields.

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Substitute eom for auxiliary fields.

- No form factor, since no gauge-singlet field.

Effective Higgs-gauge coupling

Add effective Higgs-vector coupling to Lagrangian

$$-\frac{g_h}{4} h \mathrm{Tr}\left(F_{\mu\nu} F^{\mu\nu}\right)$$

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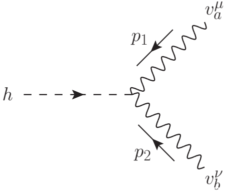
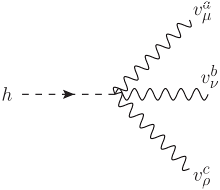


$$= i g_h \delta_{ab}(p_1 \cdot p_2\, g^{\mu\nu} - p_1^\nu\, p_2^\mu)$$



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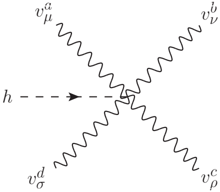

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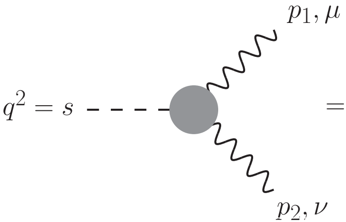
Form Factor

Ingredient of virtual higher-order corrections for inclusive Higgs production cross section.



$$= \Gamma(s)(p_1 \cdot p_2 g^{\mu\nu} - p_1^{\nu} p_2^{\mu})$$

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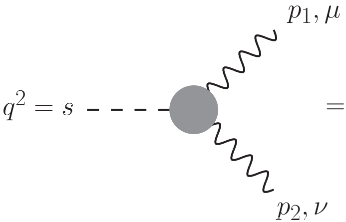

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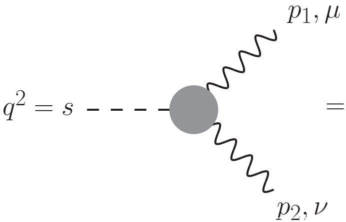

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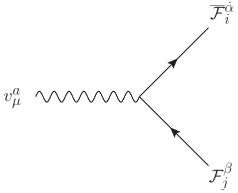
Perturbative Expansion

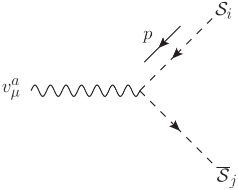
$$\Gamma(\alpha^b, s) = i g_h^b \delta_{ab} \left(1 + \sum_{m=1}^{\infty} \left(\frac{\alpha^b}{4\pi} \right)^m \left(\frac{-s}{\mu_0^2} \right)^{-m\epsilon} S_\epsilon^m \, \Gamma^{(m)} \right).$$

Form Factor: relevant Feynman rules

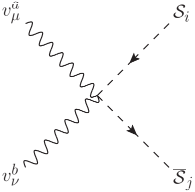


$$= -ig\, T^a_{ij}\, \overline{\sigma}^{\dot\alpha\beta}_\mu$$

Figure : vector-fermion coupling



$$= igT^a_{ji}\, p_\mu$$



$$= ig^2 g_{\mu\nu}\, \{T^a, T^b\}_{ji}$$

Figure : vector-scalar coupling

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- Masters are known as Laurent series in $\epsilon = (4-d)/2$.

  T. Gehrmann and E. Remiddi [hep-ph/0008287] [hep-ph/0101124]