

Two-loop Form Factor in massless $N=2$ SYM

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Form factor in $N=2$ SYM: equality with QCD even for terms of subleading transcendentality?

- ▶ $N=2$: more similar to QCD (matter).

$N=2$ Super Yang-Mills

Generators Q_α^A and $\bar{Q}_{\dot{\beta}A}$; $A = 1, 2$ related by R-trafo

$$Q_\alpha^A = U^{AB} Q_\alpha^B, \quad U \in SU(2).$$

Renormalisable
Multiplets

	helicities	degeneracy	fields
$\mathcal{V}^{N=2}$	± 1	1	$v_\mu \leftrightarrow p, \lambda\rangle_{cv}$
	$\pm \frac{1}{2}$	2	$\psi, \lambda \leftrightarrow \bar{Q}_{1,2} p, \lambda\rangle_{cv}$
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Ansatz: Most general $N=1$ Lagrangian with same field content.

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► Impose $SU(2)$ R-symmetry

⇒ invariance under 2nd susy generator.

$N=2$ Super Yang-Mills

$$\begin{aligned}\mathcal{L}_{N=2} = & \text{Tr} \left(\frac{1}{8g^2} \int d\theta^2 W^\alpha W_\alpha + \int d\theta^2 d\bar{\theta}^2 \bar{\Phi} e^{2gV} \Phi \right) \\ & + \int d\theta^2 d\bar{\theta}^2 (\bar{Q}_- e^{2gV} Q_- + Q_+ e^{-2gV} \bar{Q}_+) \\ & + \sqrt{2}ig \int d\theta^2 (Q_+ \Phi Q_- - \bar{Q}_- \bar{\Phi} Q_+)\end{aligned}$$

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Grassmann integration singles out susy-invariant F - and D -terms.

Substitute eom for auxiliary fields.

- ▶ No form factor, since no **gauge-singlet** field.

Effective Higgs-gauge coupling

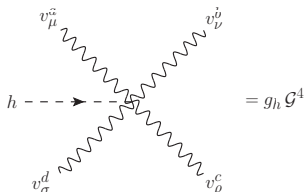
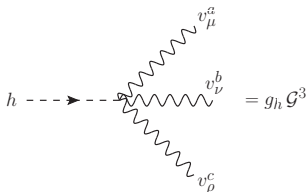
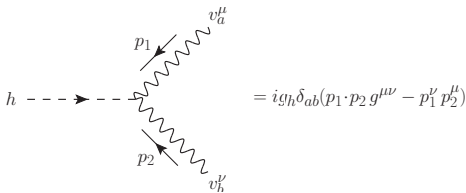
Add **effective Higgs-vector coupling** to Lagrangian

$$-\frac{g_h}{4} h \text{Tr} (F_{\mu\nu} F^{\mu\nu})$$

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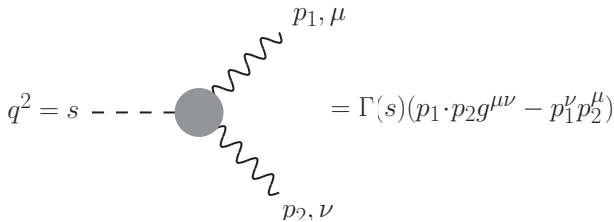
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Form Factor

Ingredient of virtual higher-order corrections for inclusive Higgs production cross section.

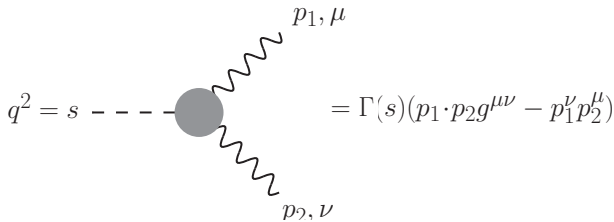


A Feynman diagram representing a form factor vertex. On the left, a dashed line representing an incoming gluon with momentum $q^2 = s$ enters a central grey circular vertex. From this vertex, two wavy lines representing outgoing photons emerge: one pointing upwards and to the right with momentum p_1, μ , and one pointing downwards and to the right with momentum p_2, ν .

$$q^2 = s \text{ --- } \bullet \begin{matrix} \nearrow \text{wavy} & p_1, \mu \\ \searrow \text{wavy} & p_2, \nu \end{matrix} = \Gamma(s)(p_1 \cdot p_2 g^{\mu\nu} - p_1^\nu p_2^\mu)$$

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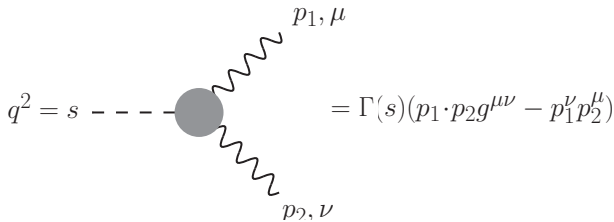
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Tensor structure $(p_1 \cdot p_2 g^{\mu\nu} - p_1^\nu p_2^\mu)$.

Form Factor

Ingredient of virtual higher-order corrections for inclusive Higgs production cross section.



A Feynman diagram showing a central grey circular vertex. A dashed line enters from the left, labeled $q^2 = s$. Two wavy lines exit from the vertex: one goes up and to the right, labeled p_1, μ , and the other goes down and to the right, labeled p_2, ν .

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Perturbative Expansion

$$\Gamma(\alpha^b, s) = i g_h^b \delta_{ab} \left(1 + \sum_{m=1}^{\infty} \left(\frac{\alpha^b}{4\pi} \right)^m \left(\frac{-s}{\mu_0^2} \right)^{-m\epsilon} S_\epsilon^m \Gamma^{(m)} \right).$$

Form Factor: relevant Feynman rules

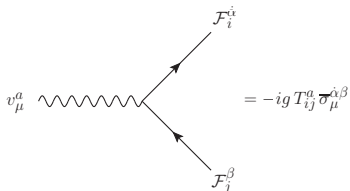


Figure : vector-fermion coupling

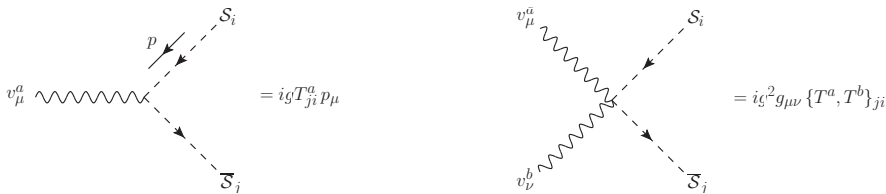


Figure : vector-scalar coupling

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1) Production of all 201 two-loop diagrams. (QGRAF)

P. Nogueira, J. Comput. Phys. **105** (1993) 279

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► Masters are known as Laurent series in $\epsilon = (4 - d)/2$.

T. Gehrmann and E. Remiddi [hep-ph/0008287]

[hep-ph/0101124]