# MY FAVOURITE INTRODUCTIONS TO ADS / CFT

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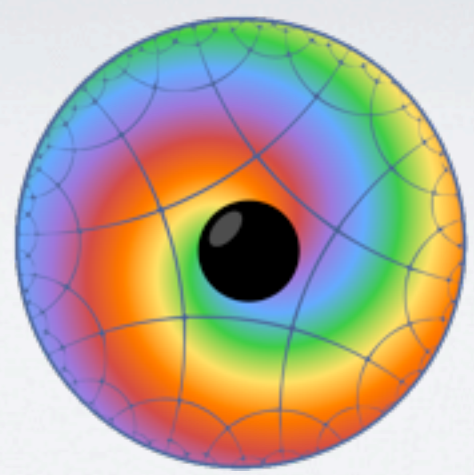
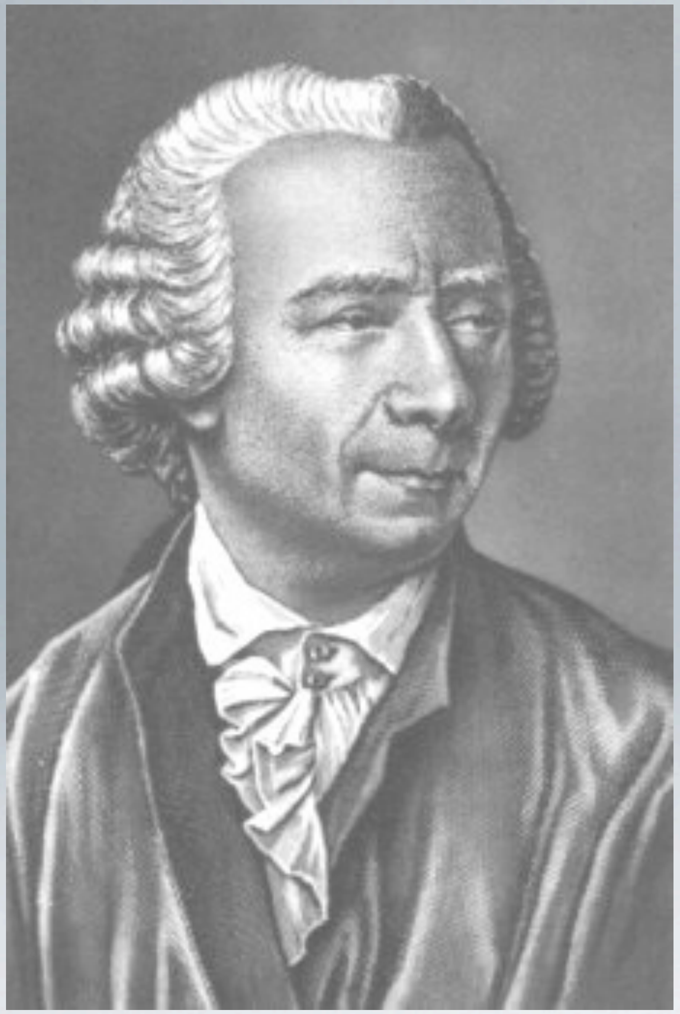
IMPRS Young Scientist Workshop at Ringberg Castle
2013 / 7 / 22



# GREATEST EQUATION EVER ?
# EULER'S EQUATION



Leonhard Euler

$$e^{i\pi} + 1 = 0$$

- fundamental constants
- basic operations

# GREATEST EQUATION EVER ?
# MALDACENA'S EQUATION

Joseph Polchinski

www.kitp.ucsb.edu/joep

AdS = CFT

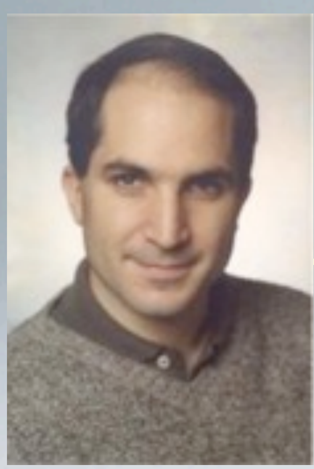
- Maxwell's eq., non-abelian
- Dirac, Klein-Gordon equations
- QM, QFT, GR
- SUSY, Strings, extra dimensions

# MY GOALS

- Give an introduction to many of the ideas connected to AdS / CFT without going into too much detail.

- Avoid very concrete examples like (really learn it!)

www.sns.ias.edu/~malda/



$$SU(N)\ \mathcal{N}=4 \text{ Super-Yang-Mills theory}$$
$$= \text{Type IIB Superstring theory on } \mathrm{AdS}_5 \times \mathrm{S}^5$$

- Vary the degree of difficulty.

# LARGE N THEORIES (1) *('t Hooft 1974)*

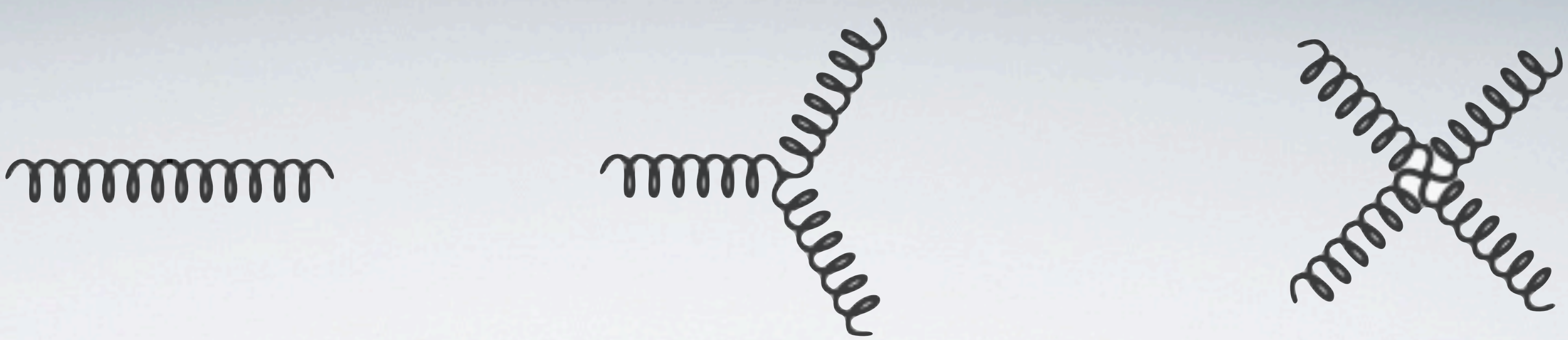
Let us talk about a *SU(N) gauge theory*. This is a theory similar to QCD (which has N=3). It has gluons (instead of photons in QED), which interact with each other:


www.particlezoo.net

$$\mathcal{L} = -\frac{1}{4}(F^a_{\mu\nu})^2$$

$$\sim \frac{1}{g^2}(\partial A)^2 + \frac{1}{g^2}(\partial A)[A, A] + \frac{1}{g^2}[A, A][A, A]$$

Feynman rules:



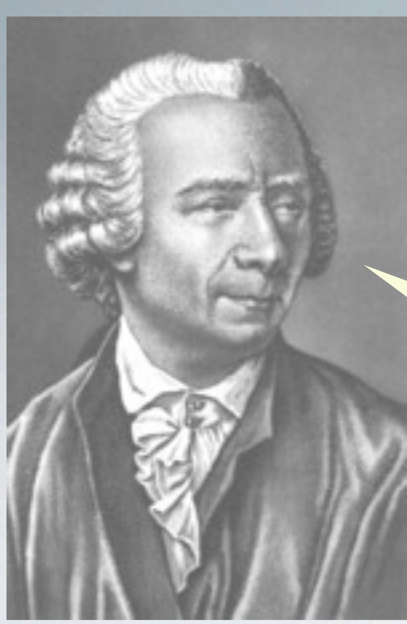
propagator: $\sim g^2$ both vertices: $\sim \frac{1}{g^2}$

# LARGE N THEORIES (2)

We can now define a new coupling constant (yes, we can!):

$$\lambda = g^2 N$$

In terms of this each propagator (E) gets $\frac{\lambda}{N}$ and each vertex (V) gets $\frac{N}{\lambda}$. Furthermore, loops (F) get $N$. So each Feynman diagram comes with a factor of



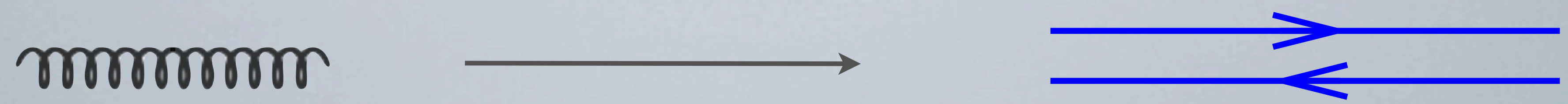
$$V - E + F = 2 - 2g$$

$$N^{V-E+F}\lambda^{E-V} = N^{2-2g}\lambda^{E-V}$$

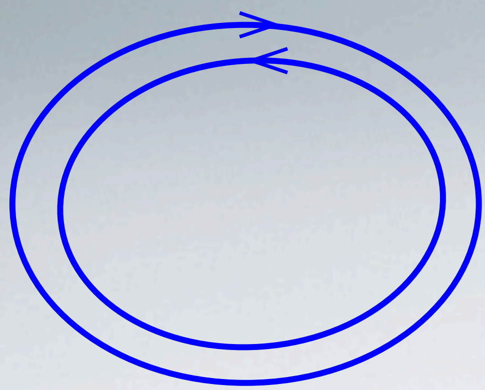
In the limit $N \to \infty$ the diagrams are ordered wrt $N$.

# LARGE N THEORIES (3)

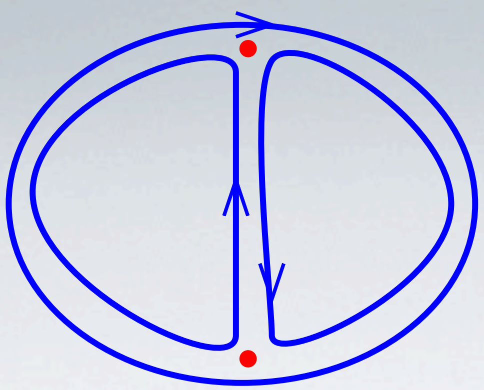
Let's do a bit of this counting. A propagator can be written in double-line notation:



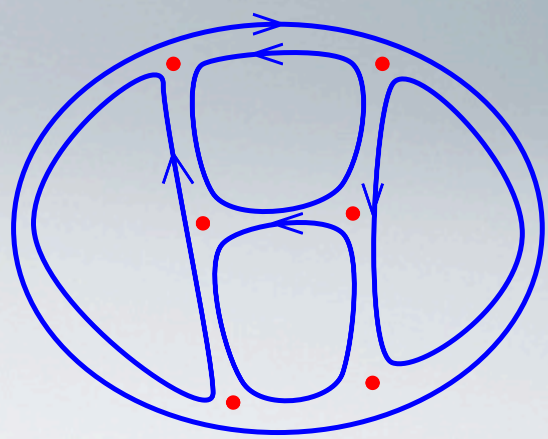
Then the dominant (planar) diagrams look like this:



$\propto N^2$

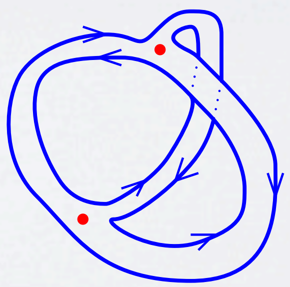


$\propto \lambda N^2$



$\propto \lambda^3 N^2$

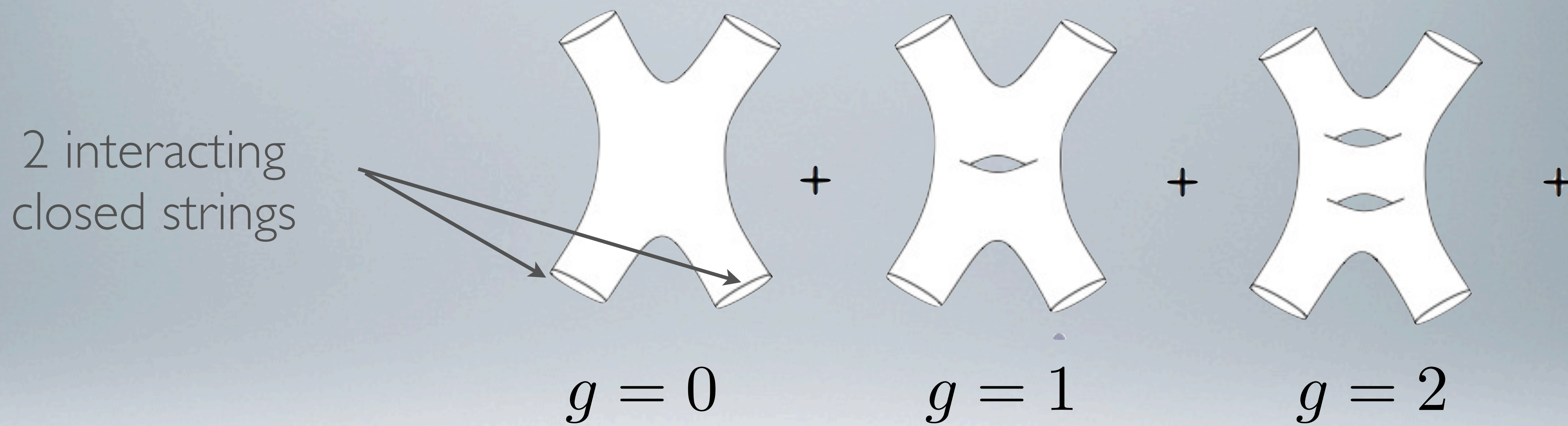
A subdominant one (non-planar) is



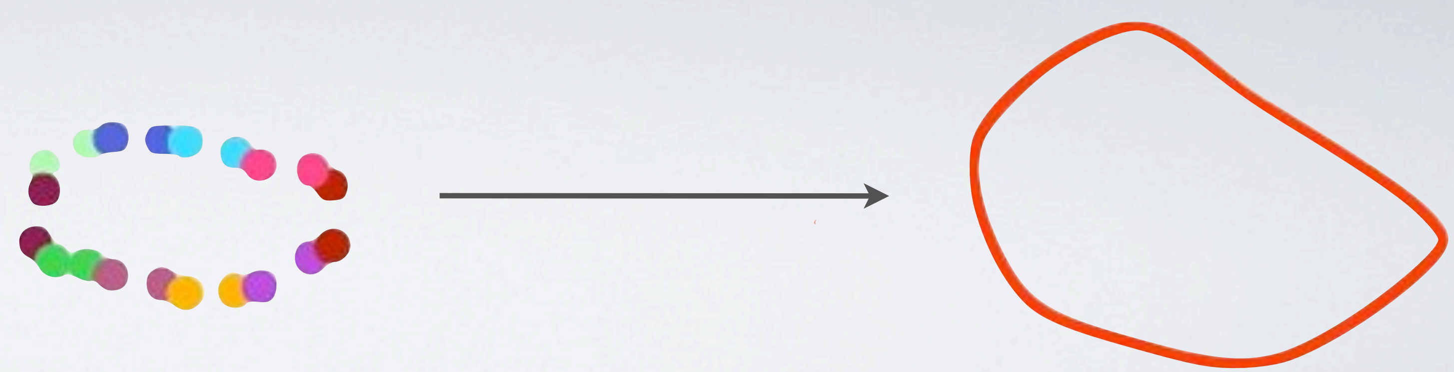
$\propto g^2 N = \lambda N^0$

# LARGE N THEORIES (4)

This is exactly the way, diagrams in perturbative string theory are ordered. It is according to *topology*:




Gluons have charge and anticharge; glueballs can be seen as closed strings:



# LARGE N THEORIES (5)

Could a large N expansion be good for QCD (N=3)?

A priori this should not be discarded. Actually, the QED fine structure constant is (Witten ~ 70s):

$$\alpha = \frac{e^2}{4\pi} = \frac{1}{137} \quad \Rightarrow \quad e \approx \frac{1}{3}$$

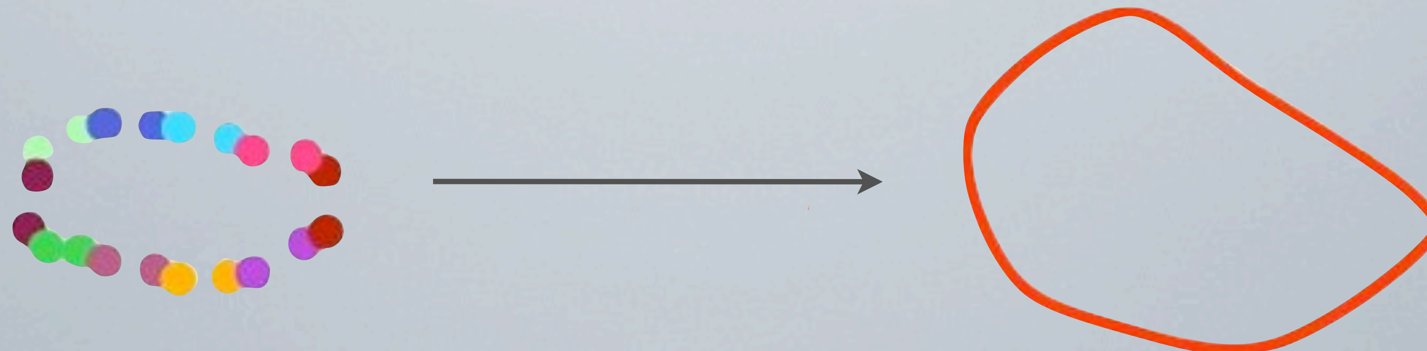
www.crafoordprize.se

**Every large N theory is basically a string theory on a different background.**

However, the question which background is very difficult!

www.nndb.com/people/945/000099648/

# WEINBERG & WITTEN THEOREM



Since some oscillation mode of the string describes the graviton, this basically means a graviton is made of gauge bosons.

$$\mathrm{Tr}\left(A_\mu A_\nu\right) \quad \text{"} \Leftrightarrow \text{"} \quad g_{\mu\nu}$$

This seems to contradict the Weinberg & Witten theorem from 1980. But it is actually evaded since gauge bosons and graviton live in spacetimes with <u>*different dimension*</u>!

# HOLOGRAPHIC PRINCIPLE *('t Hooft '93, Susskind '94)*

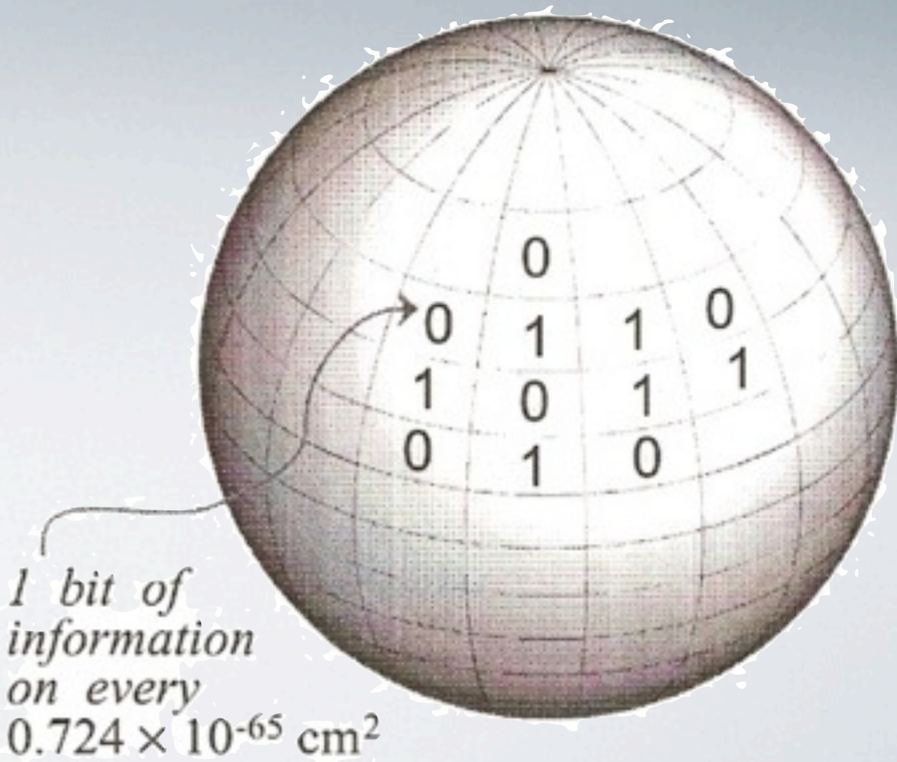
Usually, in thermodynamics, the entropy scales with the *volume* of the observed system:

$$S \propto V$$

Black holes behave differently. Their entropy scales with the *area* of the horizon (in Planck units):

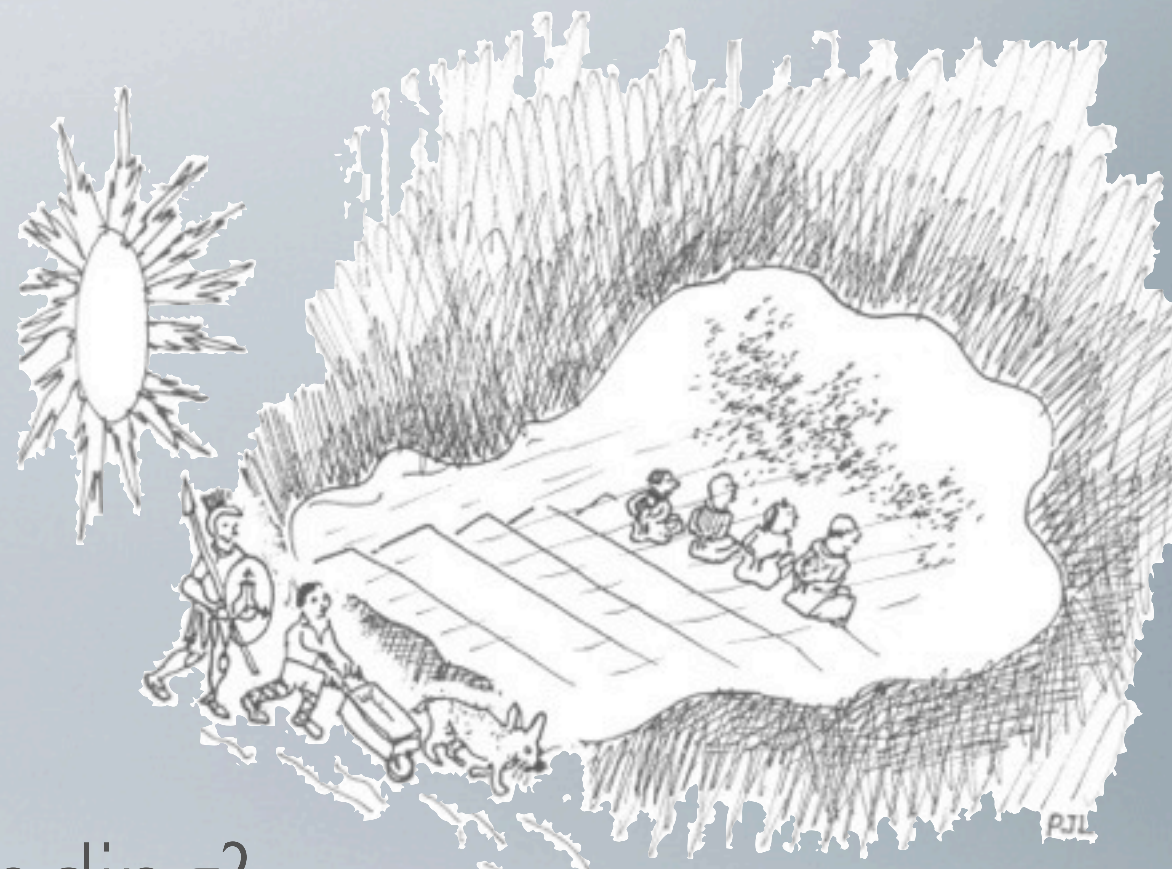
$$S = \frac{A}{4G}$$

This must be a general feature in a quantum theory of gravity.




scienceandnonduality.wordpress.com/

# Plato's allegory of the cave



www.thetruthaboutforensicscience.com/

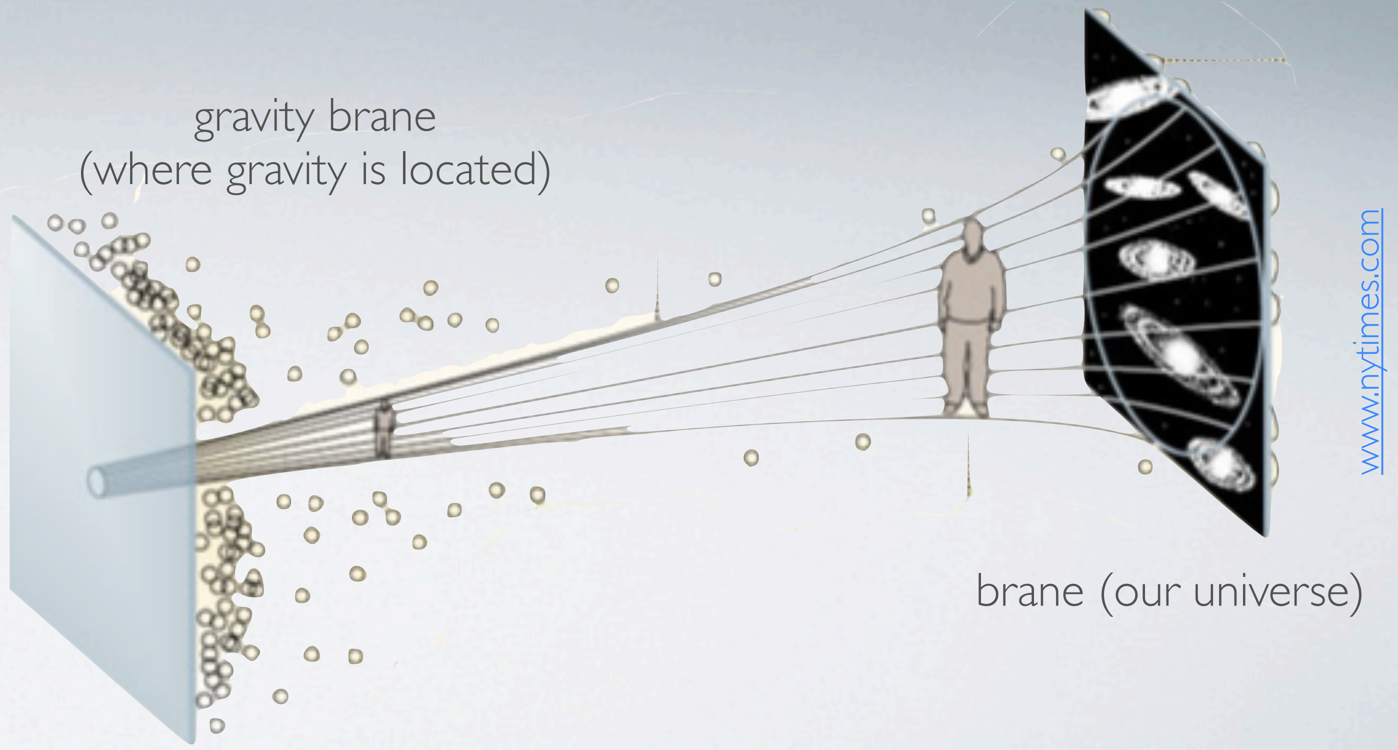
- What is reality?
- How limited is our understanding?
- Chained prisoners can only see the shadows on and the echoes off the wall. They perceive this as real, not just as a reflection of true reality.
- In holography, *both* descriptions (the people *and* their shadows) are real and carry the same information!

# NEWTON'S LAW *(Duff, Liu 2000)*

One may compute 1-loop corrections to the graviton propagator.
Let us have photons, fermions and scalars run in the loop.
For a particular theory (N=4 SYM) the correction then is:

$$V(r) = \frac{GmM}{r}\left(1 + \frac{2N^2G}{3\pi r^2}\right)$$

Identical to the one in the Randall-Sundrum model for extra dimensions:



# RENORMALIZATION GROUP (I)

news.cornell.edu

What could be the extra dimension?
Hint: RG equations are local in scale:

$$\mu \frac{\partial}{\partial \mu} g = \beta(g(\mu))$$

Let's use a simplified case (conformal).
That's the CFT in AdS / CFT:

$$\beta = 0$$

Such theories should be *scale invariant*, i.e. the following must be a symmetry.

$$x^\mu \to \lambda \, x^\mu$$

Let the extra dimension coordinate r scales like an energy.

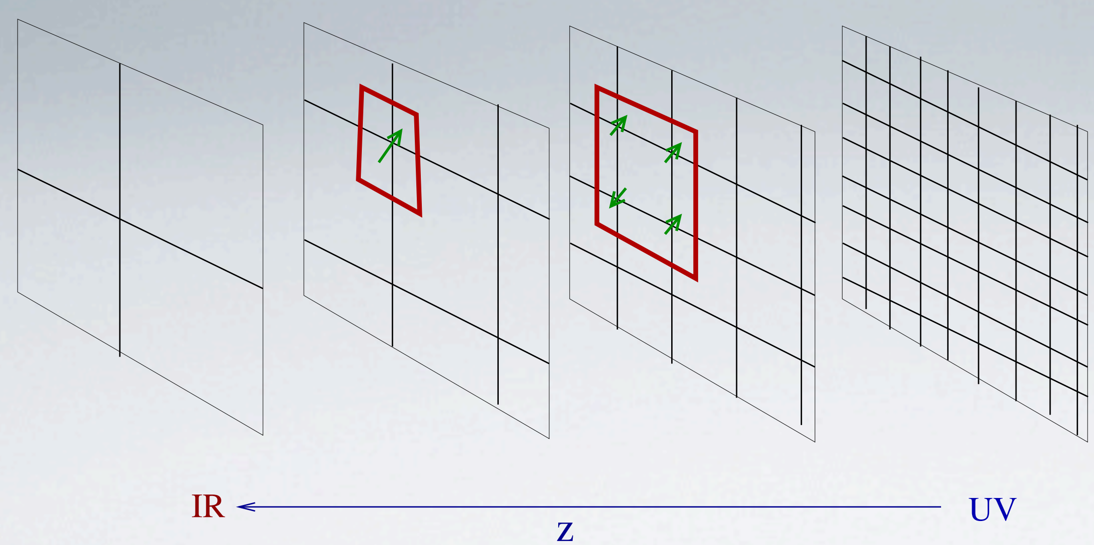
$$r \to \lambda^{-1} \, r$$

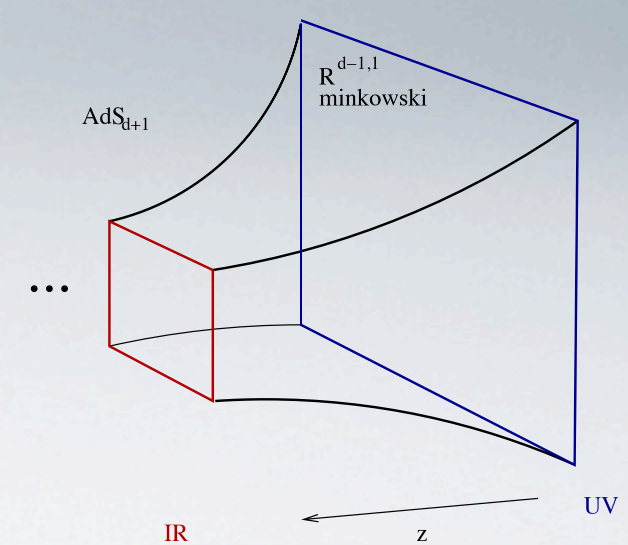
# RENORMALIZATION GROUP (2)

A Poincaré-invariant metric which also has this symmetry is:

$$ds^2 = \frac{r^2}{L^2}\,\eta_{\mu\nu}dx^\mu dx^\nu + \frac{L^2}{r^2}\,dr^2$$

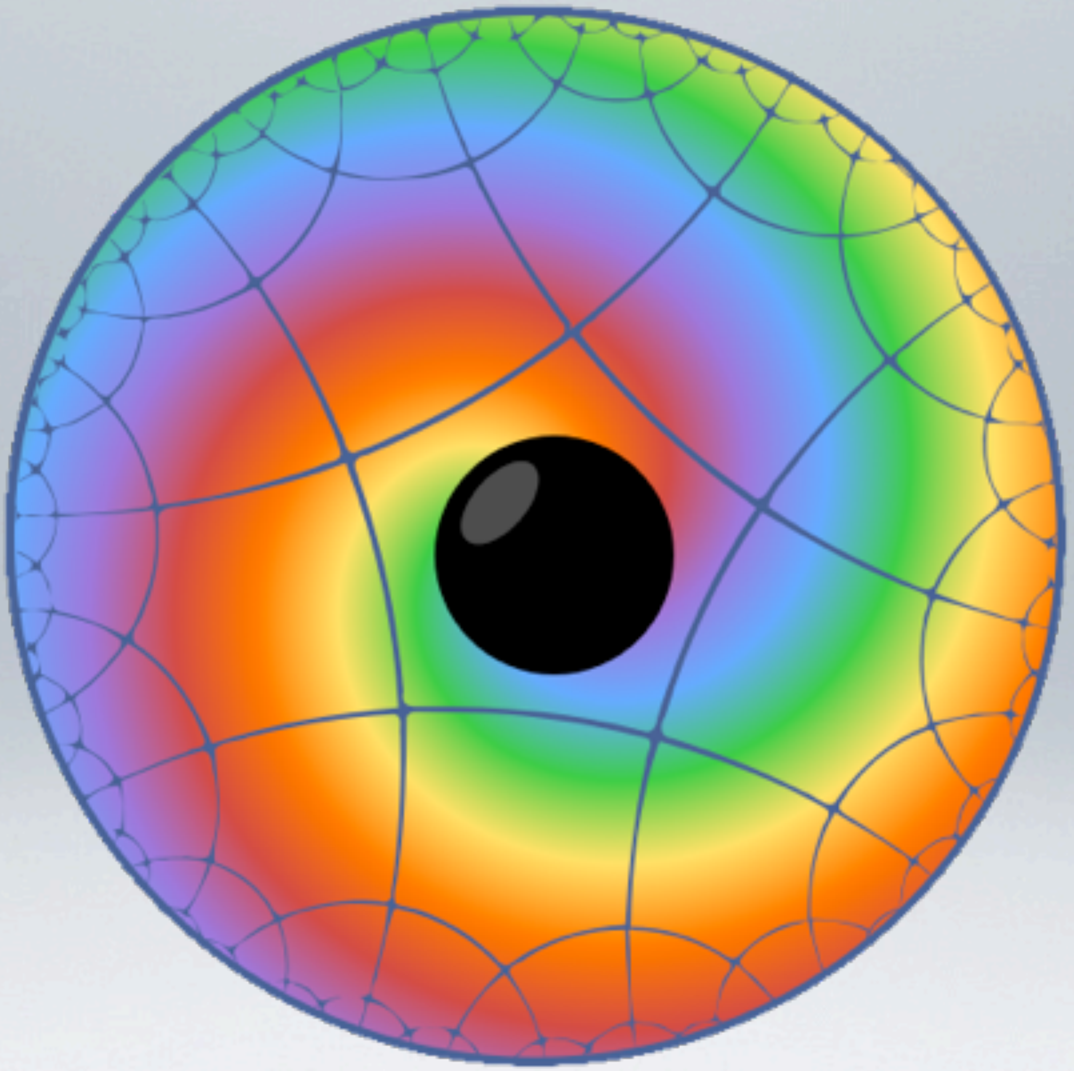
That is the metric of AdS space (that's the ... in ...).




Kadanoff block spin transformation <=> AdS space

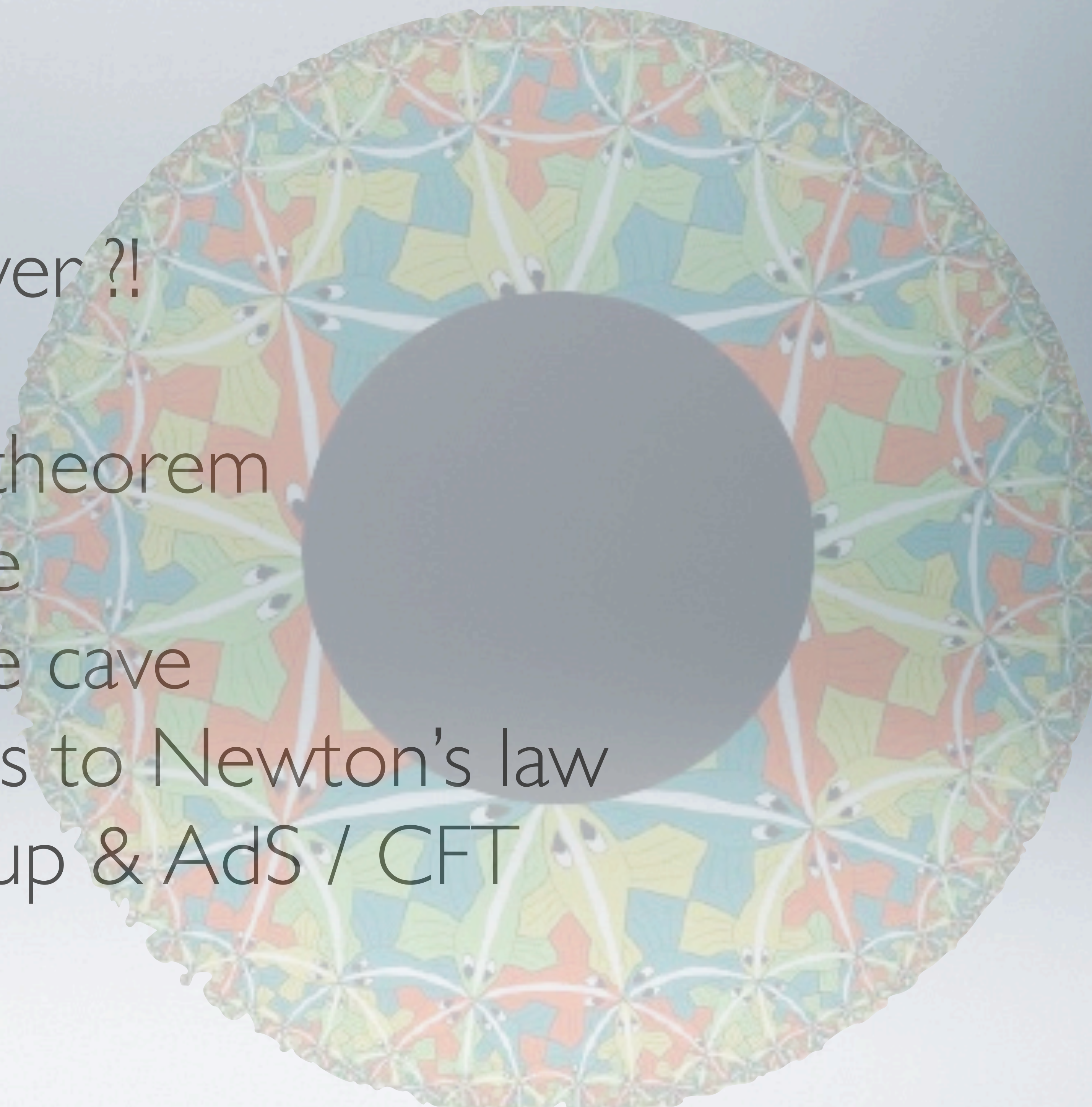
# OUR CONFERENCE LOGO *(Strydom 2013)*



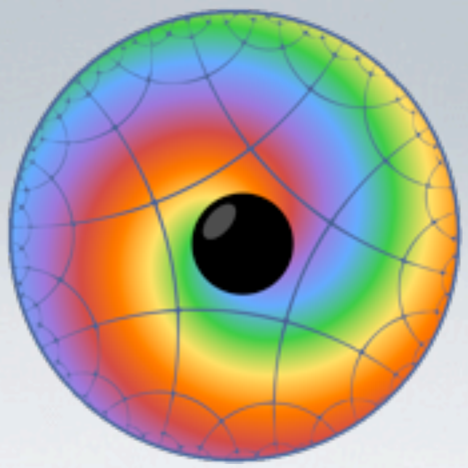
- large number of colours (large N)
- black hole in AdS space (holographic principle)

# SUMMARY

- Greatest equation ever ?!
- Large N theories
- Weinberg & Witten theorem
- Holographic Principle
- Plato's allegory of the cave
- Quantum corrections to Newton's law
- Renormalization group & AdS / CFT



# THANK YOU FOR LISTENING!



# REFERENCES

I took pictures / explanations from the following sources:


- J. Polchinski: Introduction to Gauge / Gravity Duality
- J. McGreevy: Holographic duality with a view toward many-body physics
- J. Maldacena: The gauge string duality (Talk at Xth Quark Confinement and the Hadron Spectrum)
- J. Casalderrey-Solana et al.: Gauge / String Duality, Hot QCD and Heavy Ion Collisions
- I. Klebanov, J. Maldacena: Solving quantum field theories via curved spacetimes
- D. Tong: String Theory