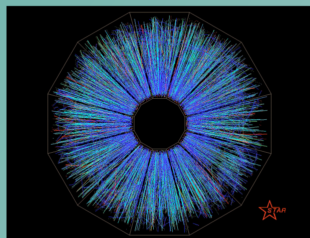
ABC's of Holographic Hydrodynamics

Ringberg2013

Why should we care about Hydro?



heavy ion collisions

turbulences

blood circulation

- describes a vast amount of different systems
- equations (Navier-Stokes) poorly understood (e.g. turbulence, CLAY Millennium Prize)

Outline

- Hydrodynamics
- Gravity and Hydrodynamics
  - Membrane Paradigm
  - Fluid/Gravity Correspondence

What is Hydrodynamics?

Def: effective long-distance description of classical or quantum many-body system at finite temperature

- Conservation laws:

$$\partial_\alpha T^{\alpha\beta} = 0 \qquad \partial_\alpha J^\alpha = 0$$

- Constitutive relations:

$$T_{\alpha\beta} \equiv T_{\alpha\beta}(T, u_\sigma, \mu) \qquad J_\alpha \equiv J_\alpha(T, u_\sigma, \mu)$$

What do we want?

Goal: - systematic construction of $T_{\alpha\beta}$ and J_{α}

- in terms of $T(x),\ u^{\alpha}(x),\ \mu(x)$

- in agreement with the symmetries

Conserved Quantities

$$\partial_\alpha T^{\alpha\beta} = 0 \qquad \partial_\alpha J^\alpha = 0$$

spacetime symmetries:
translations, rotations and boosts
$d + 1$ equations

internal symmetries:
e.g. $U(1)$ baryon number symmetry
1 equation

$d + 2$ equations

Noether theorem:
conservation laws related to
continuous symmetries of the
fundamental microscopic theory

Why can we use Hydro Fields?

- Assumption: many body systems always equilibrate locally over a finite time scale t_m
- for $t > t_m$ local equilibrium at every point x
- parameters characterising local equilibrium: $T(x),\ u^\alpha(x),\ \mu(x)$
- these parameters vary on length scales $L > l_m$
- effective dynamical fields at late times
- evolution governed by hydro equations

Relation btw. Equations and Fields

- $T_{\alpha\beta}$ $(d+1)(\mathrm{d}+2)/2$
  J_{α} $(d+1)$
  components } equations < components

- Hydrodynamical limit (long-wavelength limit):
  $T_{\alpha\beta}$ and J_{α} expressed in terms of $d+2$ fields:

  - local temperature $T(x)$
  - local fluid velocity $u_{\alpha}(x)$ (Note: $u_{\alpha}u^{\alpha}=-1$)
  - local chemical potential $\mu(x)$

Constitutive Relations

- Now we can systematically construct $T_{\alpha\beta}$ and J_α out of the hydrodynamic variables:

$$T^{\alpha\beta} = \epsilon u^\alpha u^\beta + P(\eta^{\alpha\beta} + u^\alpha u^\beta) + \Pi^{\alpha\beta}$$
$$J^\alpha = q u^\alpha + \Upsilon^\alpha$$

- Equations hold only locally in spacetime
- ϵ, P, q depend on T, μ, u^α
- $\Pi^{\alpha\beta}$, Υ^α depend on gradients of T, μ, u^α

Higher-Order Hydrodynamics

- $\Pi^{\alpha\beta} = \sum_{n=1}^{\infty} l_m^n \Pi_{(n)}^{\alpha\beta}$
- $\Pi_{(n)}^{\alpha\beta}$ n-th order in derivative of hydro fields
- every derivative suppresses fields by $1/L$
- magnitude of n-th term relative to 0th order:

$$\left(\frac{l_m}{L}\right)^n \quad \text{with} \quad L \gg l_m$$

First-Order Hydrodynamics

$$\Pi^{ab}_{(1)} = -\eta \left(\partial^a u^b + \partial^b u^a - \frac{2}{d} \delta^{ab} \partial \cdot u \right) - \zeta \delta^{ab} \partial \cdot u$$

$$\Upsilon^a = -\sigma T \partial^a \left(\frac{\mu}{T} \right)$$

Constraint by:
- Lorentz invariance
- Local form of 2nd law of thermodynamics: $\partial_\alpha J^\alpha_s \geq 0$ (entropy current)

transport coefficients → $\eta \geq 0, \; \zeta \geq 0, \; \sigma \geq 0$

Transport Coefficients

- Determined by the underlying microscopic theory
- weakly coupled theory: Computed using linear response and fluctuation-dissipation theorem
- strongly coupled theory: AdS/CFT

linear response: describes how an observable changes due to small ext. pert.

fluctuation-dissipation theorem: quantifies the relation between thermal fluc. of a system in equilibrium and response of the system to ext. pert.

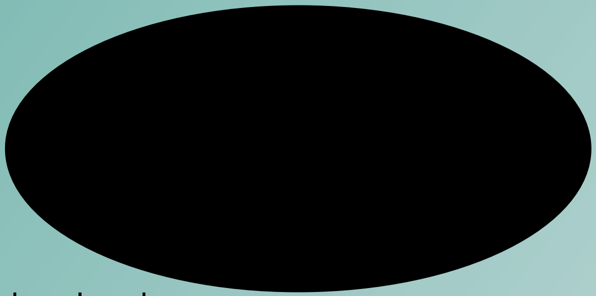
Hydrodynamics and Gravity

- Dynamics of membranes in curved spacetime are governed by fluid dynamic equations in long-wavelength limit

  Einstein's equations in $d+1$ dim
  contain fluid equations in d dim

- In the following:
  1st: Membrane Paradigm (membrane close to b.h. hor.)
  2nd: Fluid/Gravity Duality (AdS Boundary = membrane)

Membrane Paradigm



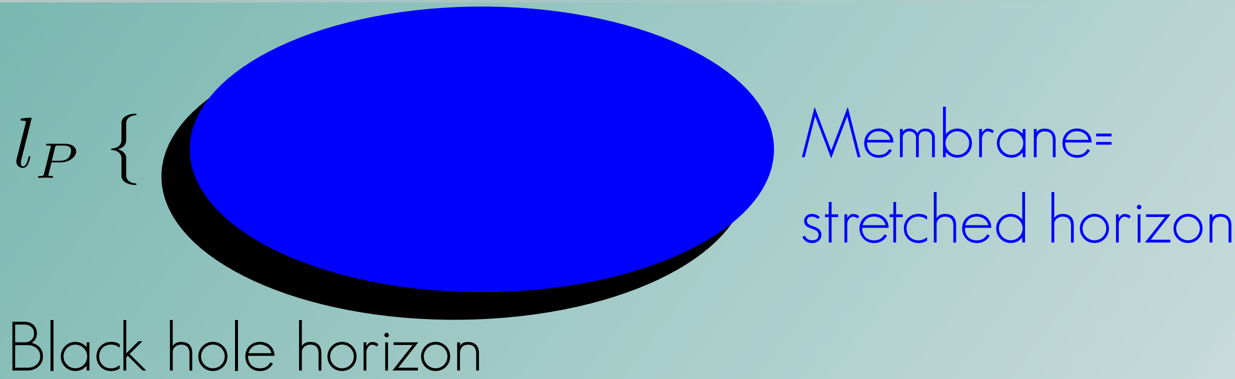
Black hole horizon

- At the b.h. horizon:
  entropy density computed using QFT diverges

$$\neq \frac{A_H}{4G\hbar} = S_{BH}$$

- QFT not valid at this point, need Quant. Grav.
- Stretched horizon = cutoff for QFT

Membrane Paradigm



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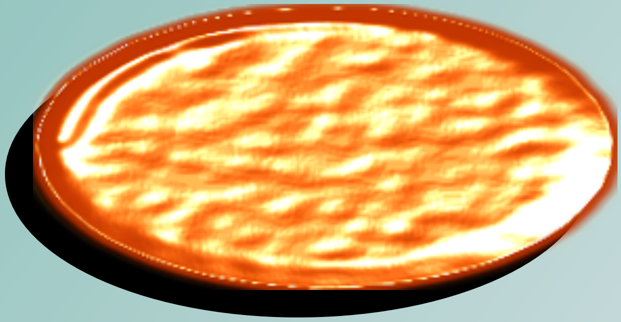
Stretched Horizon

- Stretched horizon has dynamics of its own
- Behaves like a hot conducting viscous fluid
- Governed by Navier-Stokes Equation

  (non-relativistic hydrodynamic equation)

Aside: $\frac{\eta}{s} = \frac{1}{4\pi}$



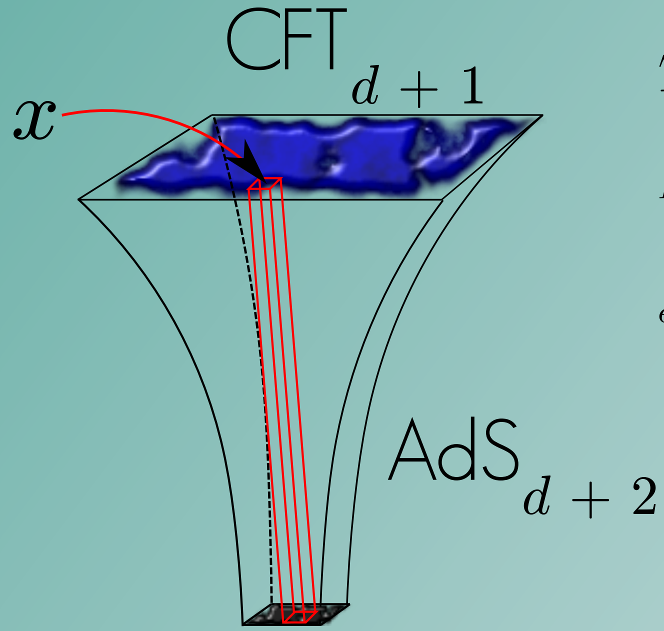
Membrane=
stretched horizon

Black hole horizon

Fluid/Gravity Correspondence

- Low energy dynamics of planar black holes in asymptotically AdS given by relativistic conformal fluids
- Unperturbed black hole solution to Einstein equations describes thermal equilibrium of dual theory
- Long-wavelength perturbation of black hole corresponds to hydrodynamic expansion

Fluid dynamics in N=4 SYM



$$\partial_\mu T^{\mu\nu} = 0$$

$$T^{\alpha\beta} = \epsilon u^\alpha u^\beta + P(\eta^{\alpha\beta} + u^\alpha u^\beta) + \Pi^{\alpha\beta}$$

$$P = \frac{\pi^{d-1}}{16\pi G_N^{(d+2)}} T^{d-1} \qquad \eta = \frac{1}{16\pi G_N^{(d+2)}} \left(\frac{4\pi}{d} T\right)^d$$

$$\epsilon = \frac{\pi^{d-1}}{16\pi G_N^{(d+2)}} (d-2) T^{d-1}$$

$$\Uparrow$$

$$R_{MN} - \frac{1}{2} R g_{MN} + \Lambda g_{MN} = 0$$

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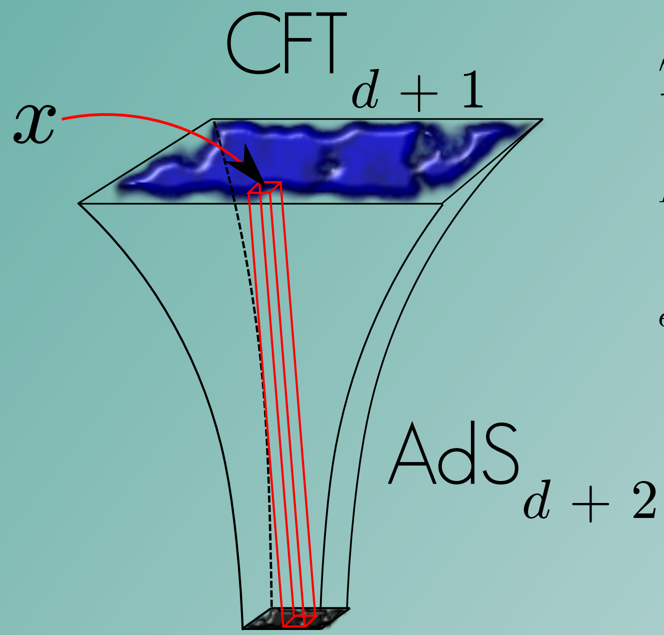

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$$\boxed{\frac{\eta}{s} = \frac{1}{4\pi}}$$

$\Uparrow$ claimed to be lower bound for fluids found in nature (Kovtun et.al.)

$$R_{MN} - \frac{1}{2} R g_{MN} + \Lambda g_{MN} = 0$$

Further Results from Fluid/Gravity

- Systematic treatment of superfluids (i.e. Goldstone modes)

- Anomaly effects found in hydrodynamics:

- stir really fast

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- Anomaly effects found in hydrodynamics:

- stir really fast

- sugar jumps out of the mug

Conclusion

- Hydrodynamics is found in a vast number of different systems.
- There is a systematic way to construct hydrodynamic constitutive relations.
- There is a close connection to gravity.
- New results were found in the context of AdS/CFT correspondence
- Chance to win 1mio Dollars!

Puzzle

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Hot Chocolate Effect

- pour hot milk into a mug

Puzzle

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- stir in chocolate powder

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- pour hot milk into a mug
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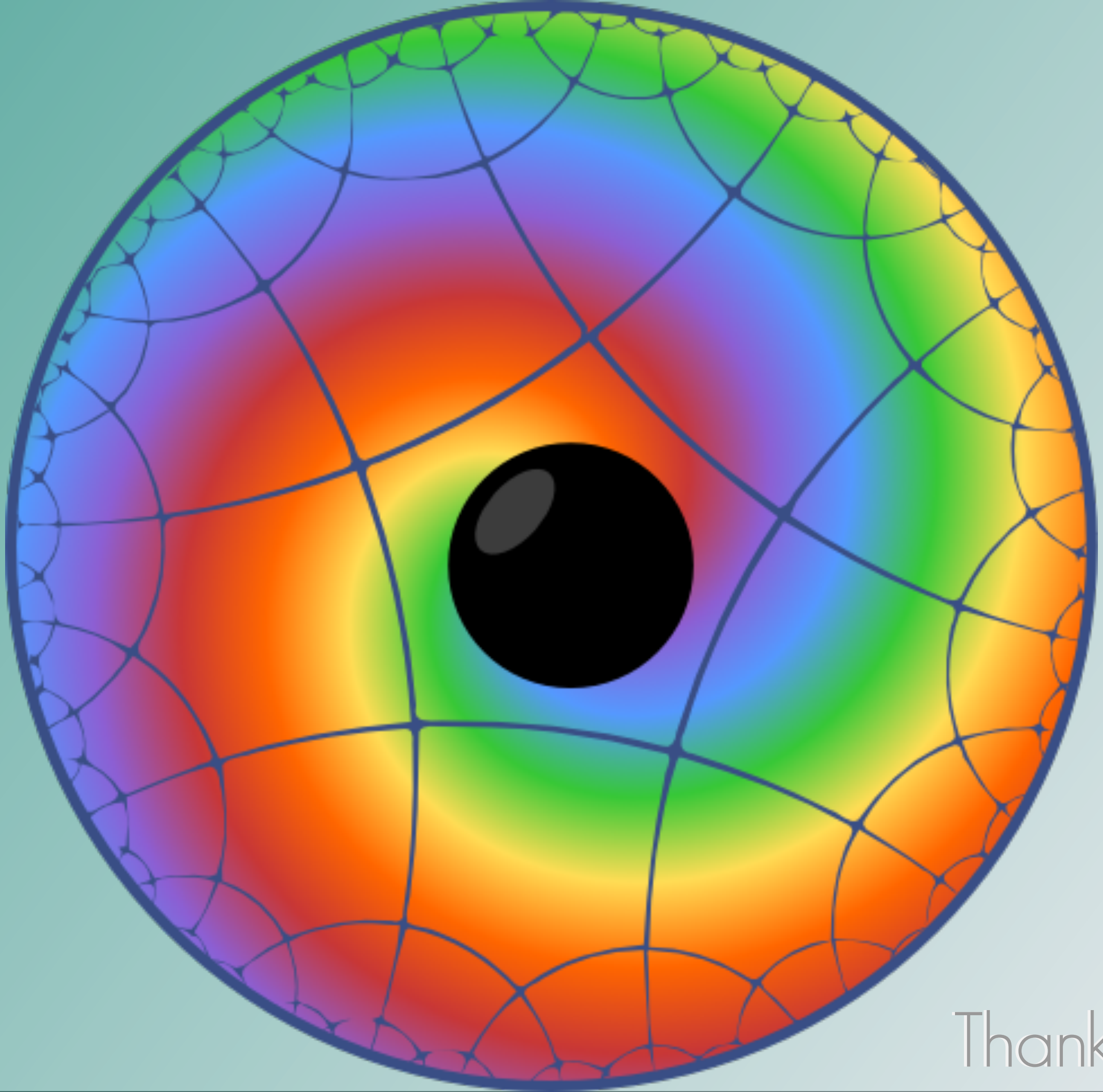
- tap the bottom of the mug with a spoon while the milk is still in motion

Puzzle

Hot Chocolate Effect

- pour hot milk into a mug
- stir in chocolate powder
- tap the bottom of the mug with a spoon while the milk is still in motion
- pitch of taps will increase progressively with no relation to speed or force of tapping

WHY??



Thank You!