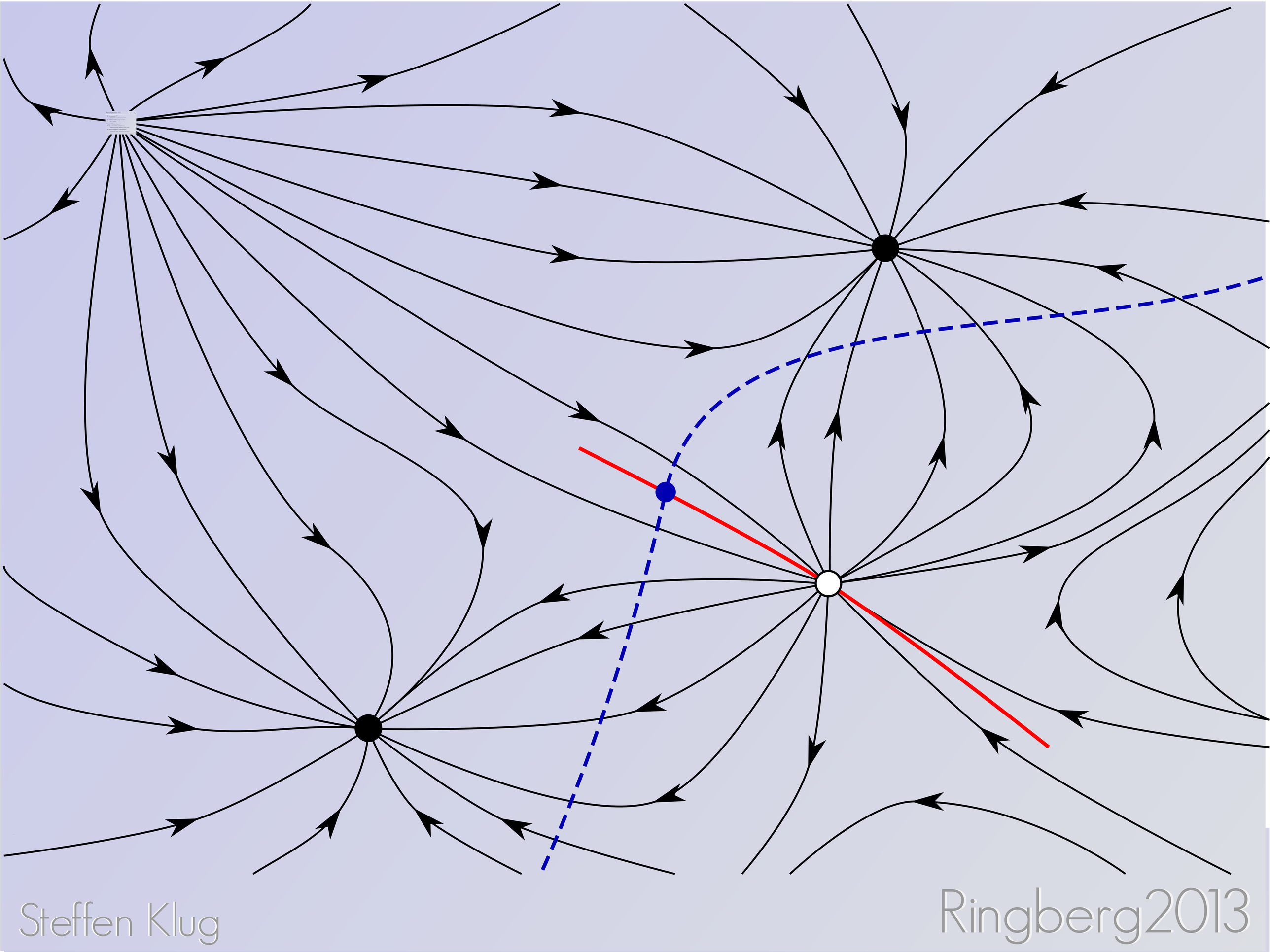
Critical Phenomena, Renormalization & Gauge / Gravity Duality

Ringberg2013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Renormalization ???

- Normalization ???

  physical values should be independent

- Undefined mathematical operations

  $\infty - \infty \neq 0$

Physical / Wilsonian viewpoint:

- Generates effective field theories

  degrees of freedom depend on energy scale

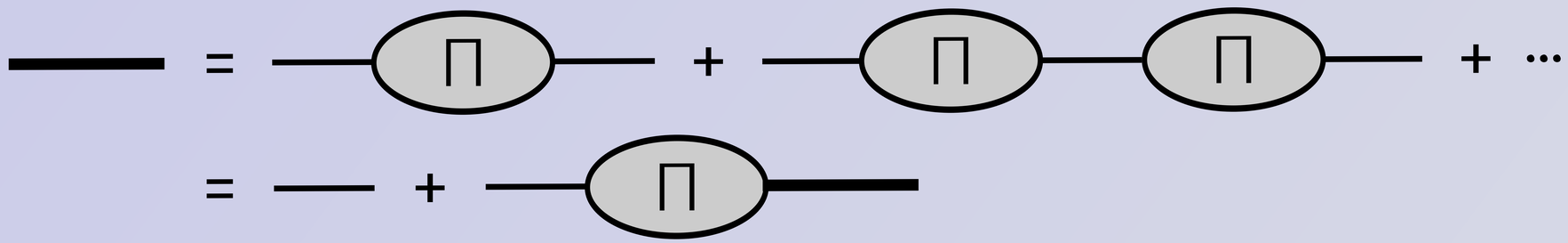
- Utilizes self-similarity / scaling behavior

  connects the 'micro' world to the 'macro world'

Renormalization & Self-Similarity

Examples:

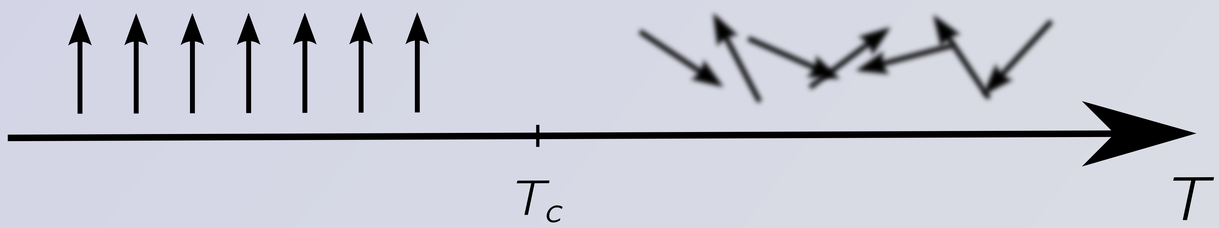
- Polarization of vacuum in QED



$$e^2_{\text{phys}} = \frac{e^2}{1 + e^2 \Pi\left(\mu^2\right)} \approx e^2 \left[1 - e^2 \Pi\left(\mu^2\right)\right]$$

- Fluctuations at critical temperature

$$\mathcal{H} = -\frac{1}{2} \sum_{ij} J_{ij} s_i s_j - H \sum_{i=1}^{N} s_i$$



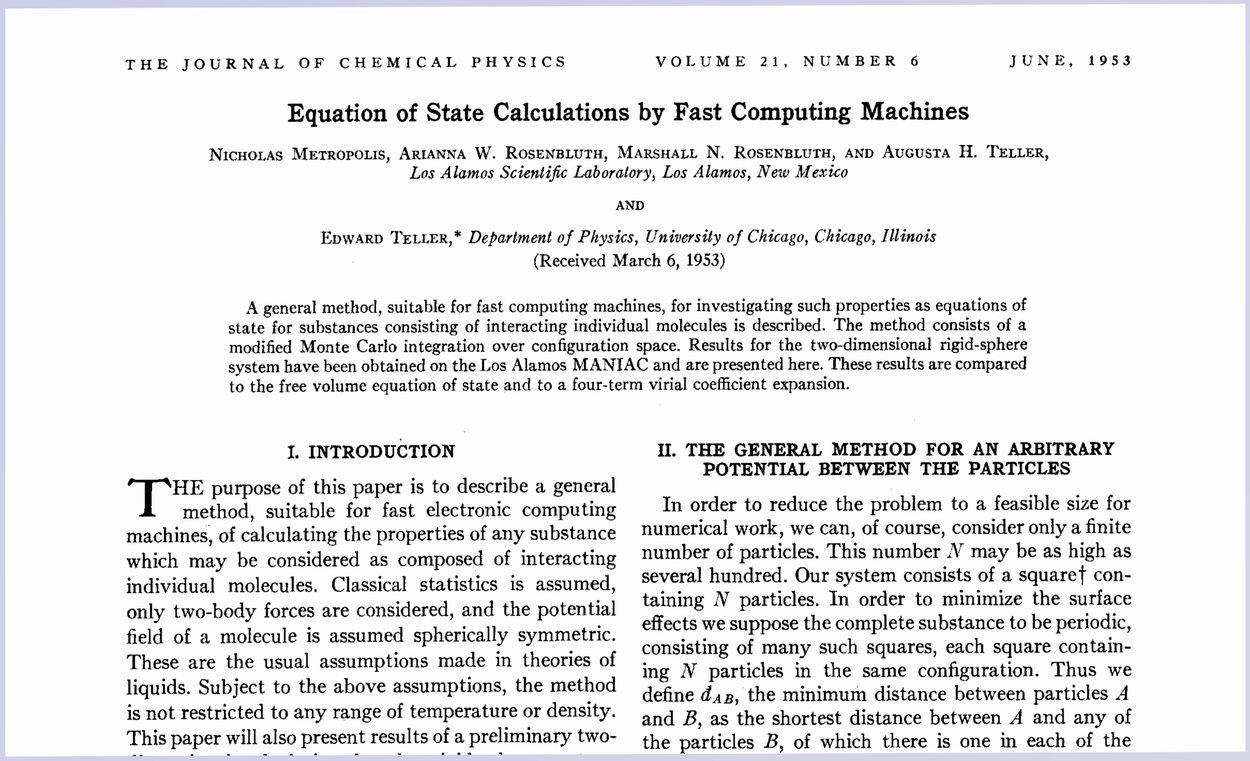
Critical Phenomena

Ising model in 2d

- Metropolis algorithm fun facts:

  18000+ citations till 2012

  2\5 authors are not physicists



THE JOURNAL OF CHEMICAL PHYSICS VOLUME 21, NUMBER 6 JUNE, 1953

Equation of State Calculations by Fast Computing Machines

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(Received March 6, 1953)

A general method, suitable for fast computing machines, for investigating such properties as equations of state for substances consisting of interacting individual molecules is described. The method consists of a modified Monte Carlo integration over configuration space. Results for the two-dimensional rigid-sphere system have been obtained on the Los Alamos MANIAC and are presented here. These results are compared to the free volume equation of state and to a four-term virial coefficient expansion.

I. INTRODUCTION

THE purpose of this paper is to describe a general method, suitable for fast electronic computing machines, of calculating the properties of any substance which may be considered as composed of interacting individual molecules. Classical statistics is assumed, only two-body forces are considered, and the potential field of a molecule is assumed spherically symmetric. These are the usual assumptions made in theories of liquids. Subject to the above assumptions, the method is not restricted to any range of temperature or density. This paper will also present results of a preliminary two-

II. THE GENERAL METHOD FOR AN ARBITRARY POTENTIAL BETWEEN THE PARTICLES

In order to reduce the problem to a feasible size for numerical work, we can, of course, consider only a finite number of particles. This number N may be as high as several hundred. Our system consists of a square† containing N particles. In order to minimize the surface effects we suppose the complete substance to be periodic, consisting of many such squares, each square containing N particles in the same configuration. Thus we define $\bar{d}_{AB}$, the minimum distance between particles A and B, as the shortest distance between A and any of the particles B, of which there is one in each of the

http://www.pha.jhu.edu/~javalab/ising/

Critical Phenomena

- Contineous phase transition

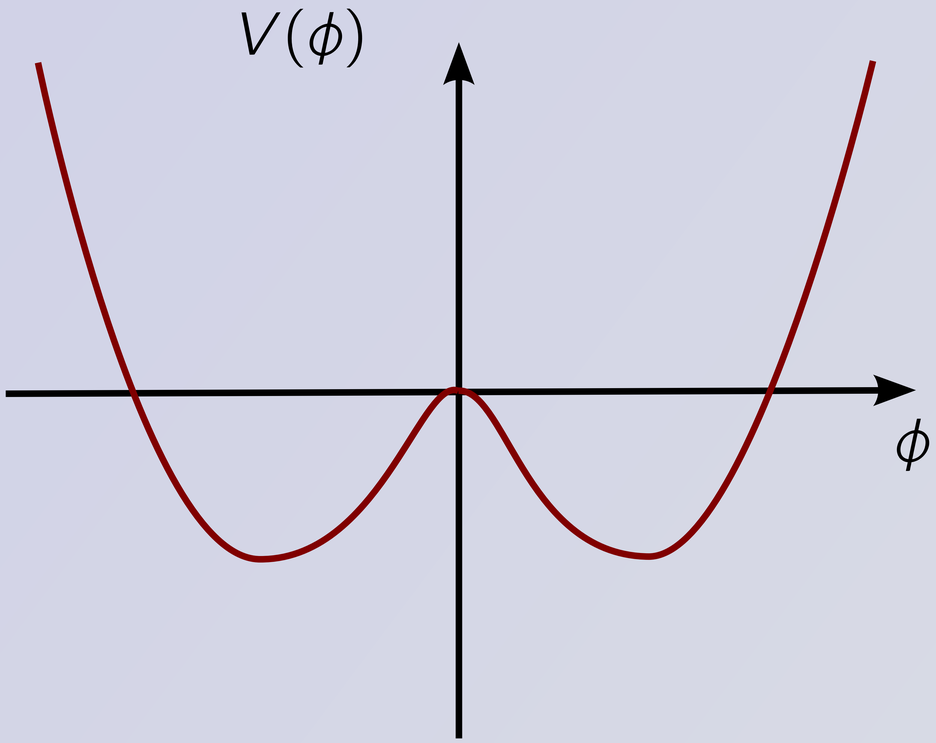
  order/disorder

  $\longrightarrow$ order parameter $|m| \neq 0$

- Critical point

  fluctuations on all length scales

  $\longrightarrow$ coherence length $\xi \to \infty$



- Critical exponents

  $C \sim |T - T_c|^{\alpha}$ $\quad m \sim |T - T_c|^{\beta}$

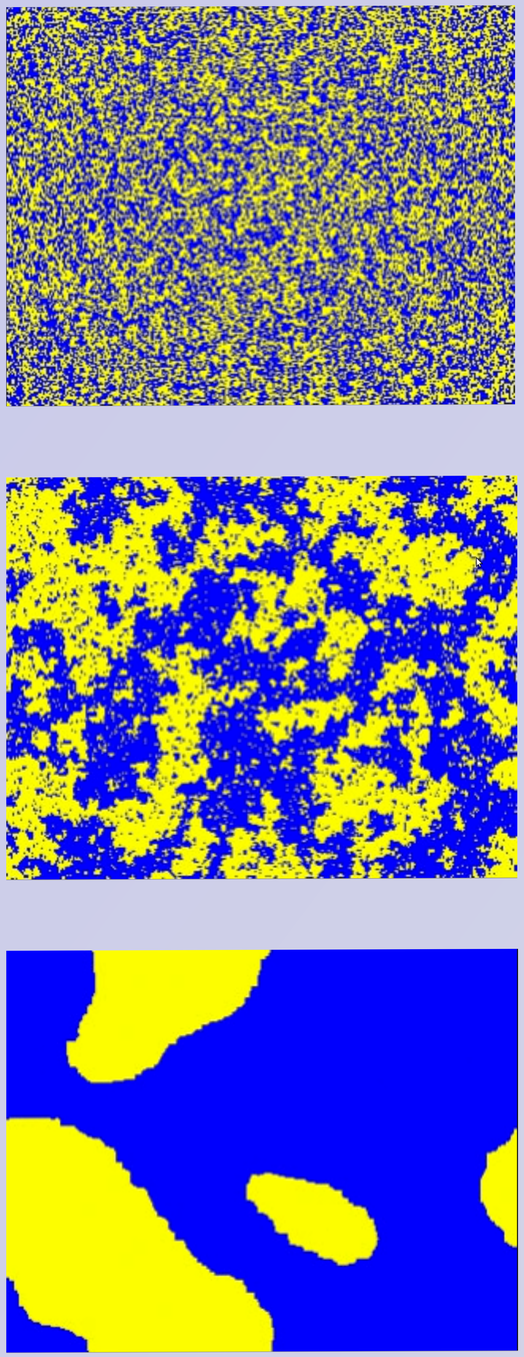
  $\chi \sim |T - T_c|^{\gamma}$ $\quad \xi \sim |T - T_c|^{-\nu}$

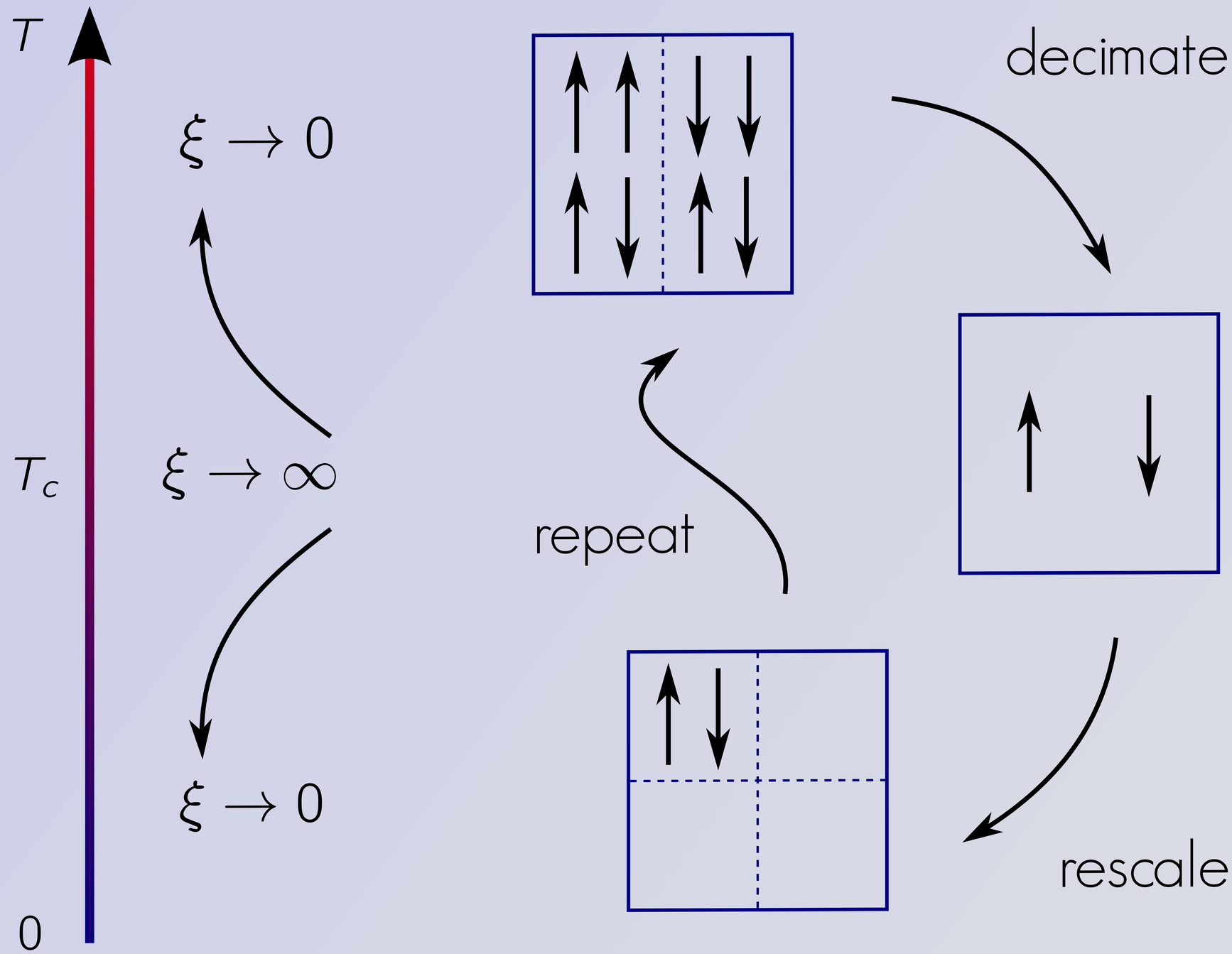
- Universality classes

  Ising $\longrightarrow$ gas /liquids

  XY $\longrightarrow$ superfluid/insulator

Blockspin Renormalization





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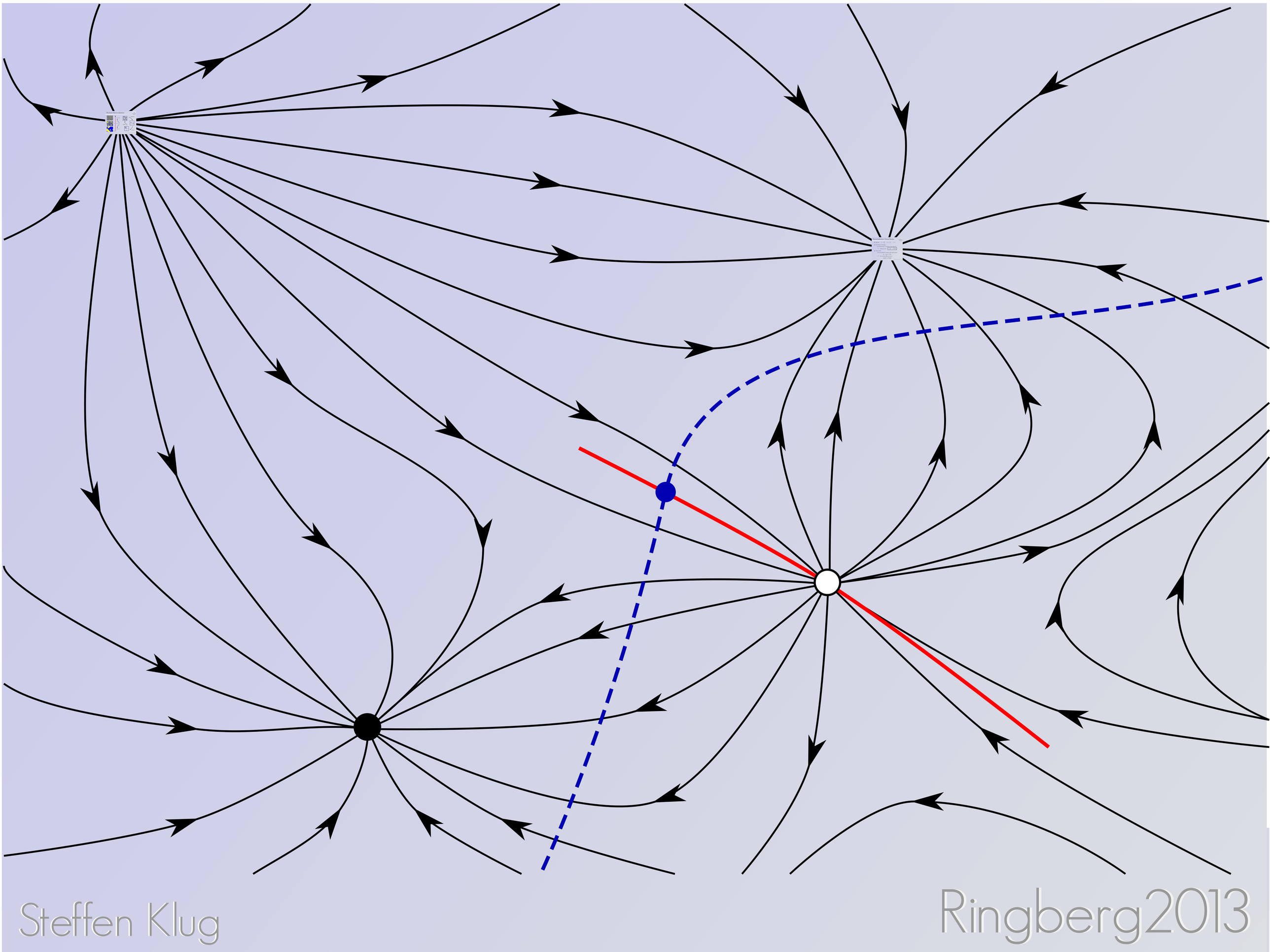
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Renormalization Group Recipe

- Rescale $\quad x \longrightarrow \frac{x}{c} \qquad \lambda \longrightarrow c\lambda \qquad c > 1$

- Decimation / averaging

  e.g. fast fluctuations / momentum-shell inetgration

  $\longrightarrow$ effective action $\quad \boxed{S_{\text{eff}}[\varphi] = g^a O_a[\varphi]}$

- Renormalize...

  ...all occuring fields & compare with original system

$$\varphi \longrightarrow \varphi' = c^{\Delta_\varphi}\varphi$$

$\longrightarrow$ form invariant effective action

$$S_{\text{eff}}[\varphi'] = g'^a O'_a[\varphi']$$

Renormalization Group Flow

- Repeat $\mathbf{g}'(c) = \mathcal{R}_c[\mathbf{g}]$

$$\mathbf{g}'(c) = \lim_{\substack{n\to\infty \\ \ell\to 0}} \Big[\mathcal{R}_c[\mathbf{g}]\Big]^n \approx \lim_{\substack{n\to\infty \\ \ell\to 0}} \left[\mathbb{1} + n\mathcal{R}_\ell\right][\mathbf{g}] = \mathbf{g} + R(\mathbf{g})\mathrm{d}\ell$$

- RG flow equations

$$\boxed{\Rightarrow \frac{\mathrm{d}\mathbf{g}}{\mathrm{d}\ell} = R(\mathbf{g})}$$

- Fixed points

$$\frac{\mathrm{d}\mathbf{g}}{\mathrm{d}\ell}\Big|_{\mathbf{g}=\mathbf{g}^\star} \stackrel{!}{=} 0 \quad \Leftrightarrow \quad R(\mathbf{g}^\star) = 0$$

- Fixed points classification

strong coupling	critical coupling	weak coupling
$g^a \to \infty$ (fluids)	$g^a = g_c$	$g^a \to 0$ (collisions)

Renormalization Group Flow

- Linearization about FP

$$R(\mathbf{g}) = R(g^\star) + \nabla R(\mathbf{g})\big|_{\mathbf{g}=\mathbf{g}^\star}(\mathbf{g} - \mathbf{g}^\star)$$

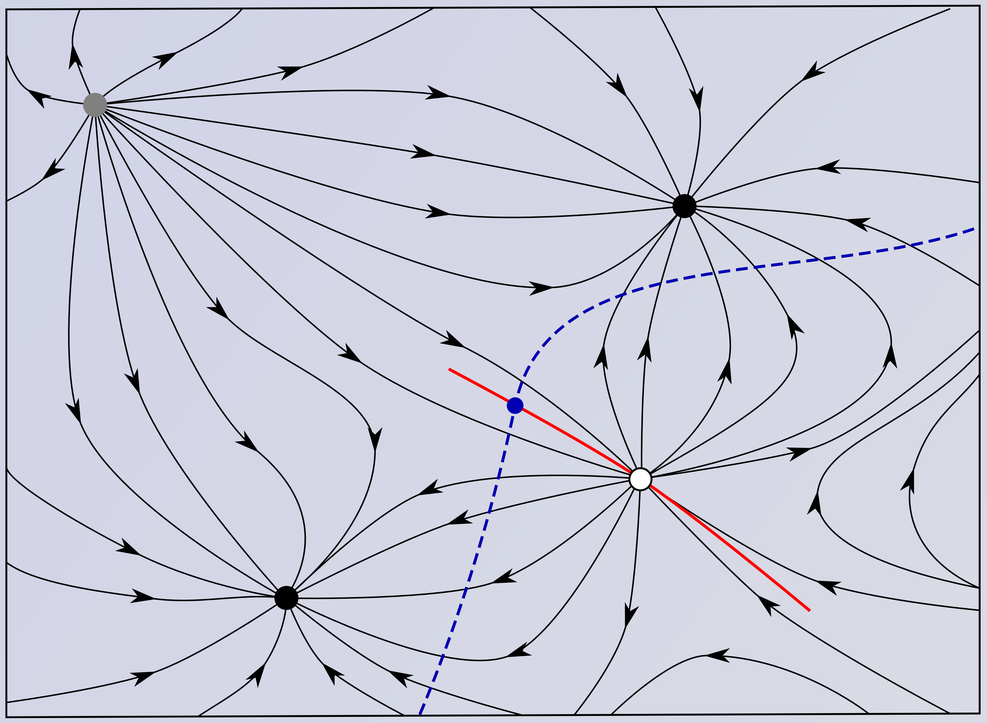
$$\Rightarrow R^a{}_b = \left.\frac{\partial R(g^a)}{\partial g^b}\right|_{g=g^\star}$$

- Scaling fields

$$\frac{\mathrm{d}\, v^a}{\mathrm{d}\ell} = y^a v^a \quad \Rightarrow \quad v^a = C\, \mathrm{e}^{\ell y^a}$$

irrelevant	marginal	relevant
$\mathrm{Re}(y^a) < 0$	$y^a = 0$	$\mathrm{Re}(y^a) > 0$
stable	mixed	unstable

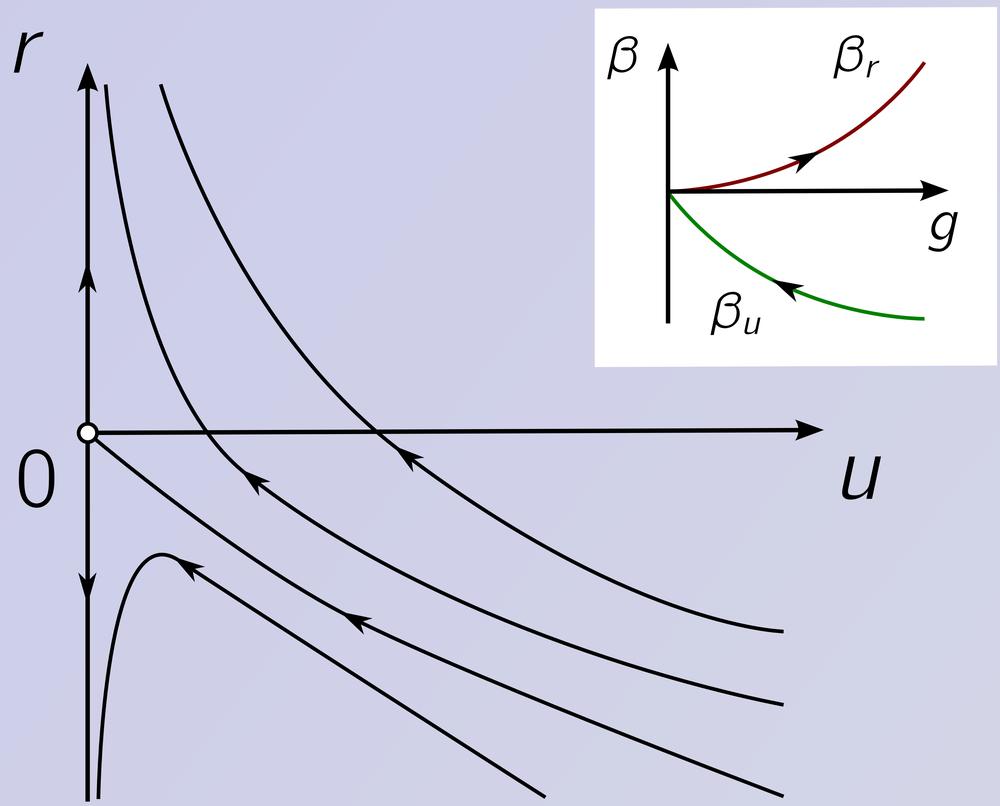
RG flow diagram

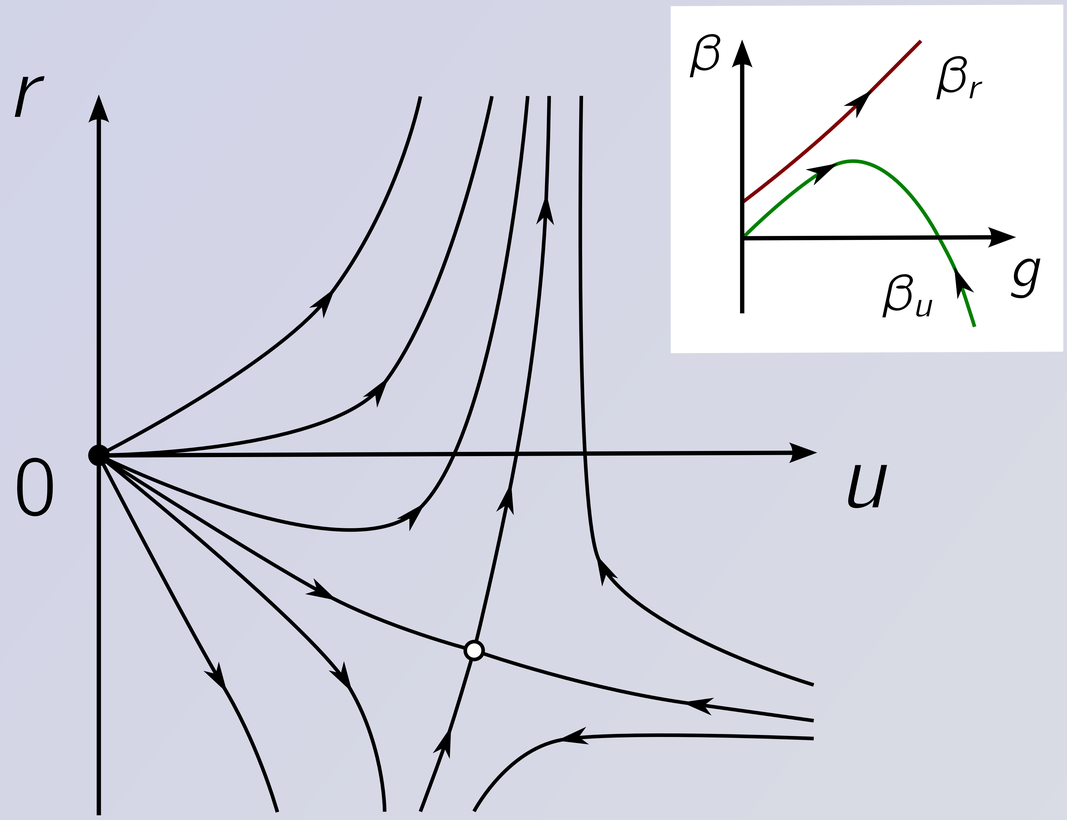


Renormalization Group Flow

Example $$S[\varphi; J] = \int \mathrm{d}^D\mathbf{x} \left[\frac{b}{2} (\nabla\varphi)^2 + \frac{r}{2}\varphi^2 + \frac{u}{4!}\varphi^4 - \varphi J \right]$$

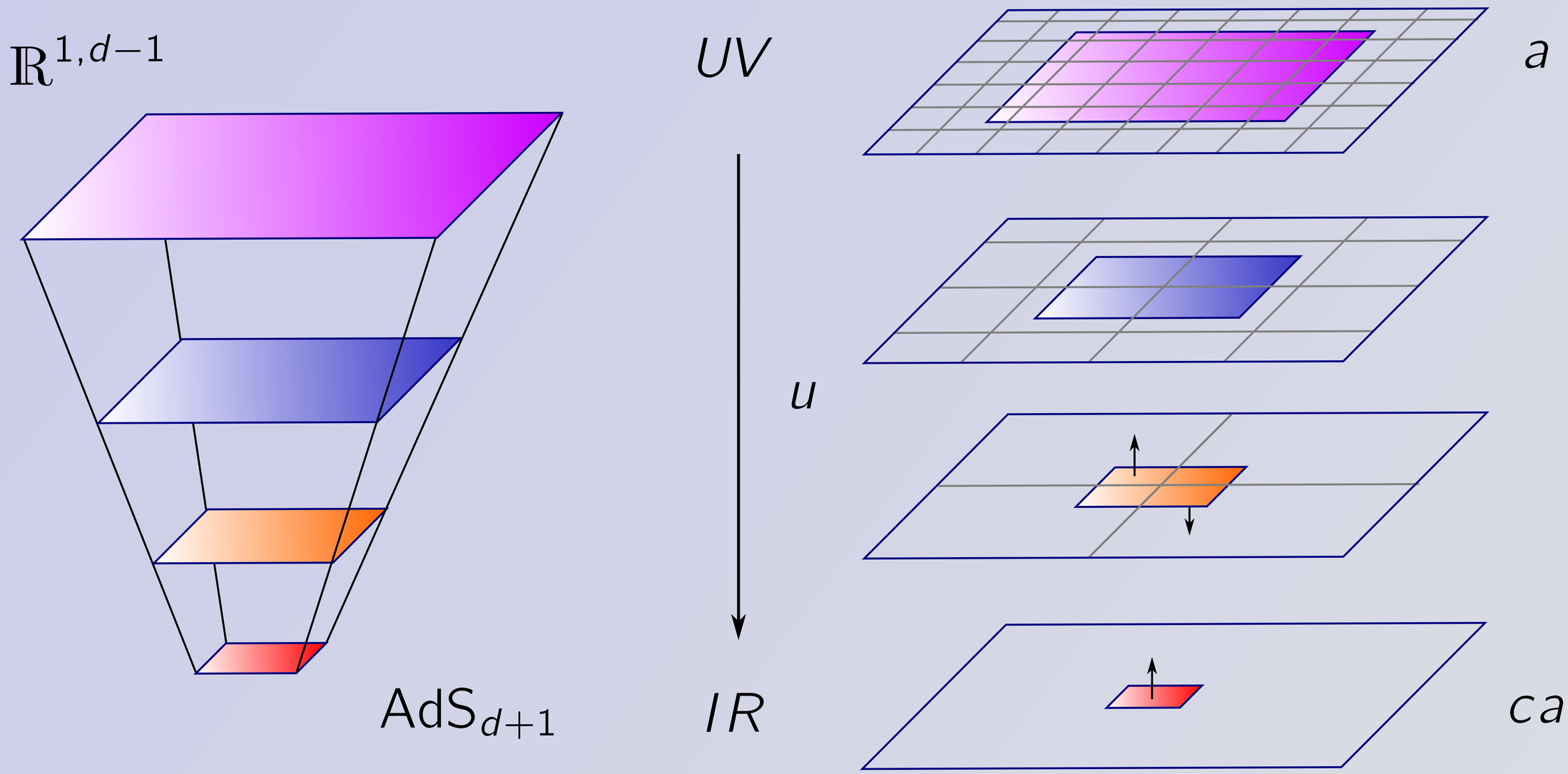
- Gaussian fixed point
  $D > 4 \quad r^\star = 0 \quad u^\star = 0$
- Wilson-Fisher fixed point
  $D > 4 \quad r^\star \neq 0 \quad u^\star \neq 0$





Emergent Holography

- Geometrization of RG equation $\frac{\mathrm{d}g^a(x,u)}{\mathrm{d}u} = R(g^a(x,u),u)$



AdS Space & Fixed Points

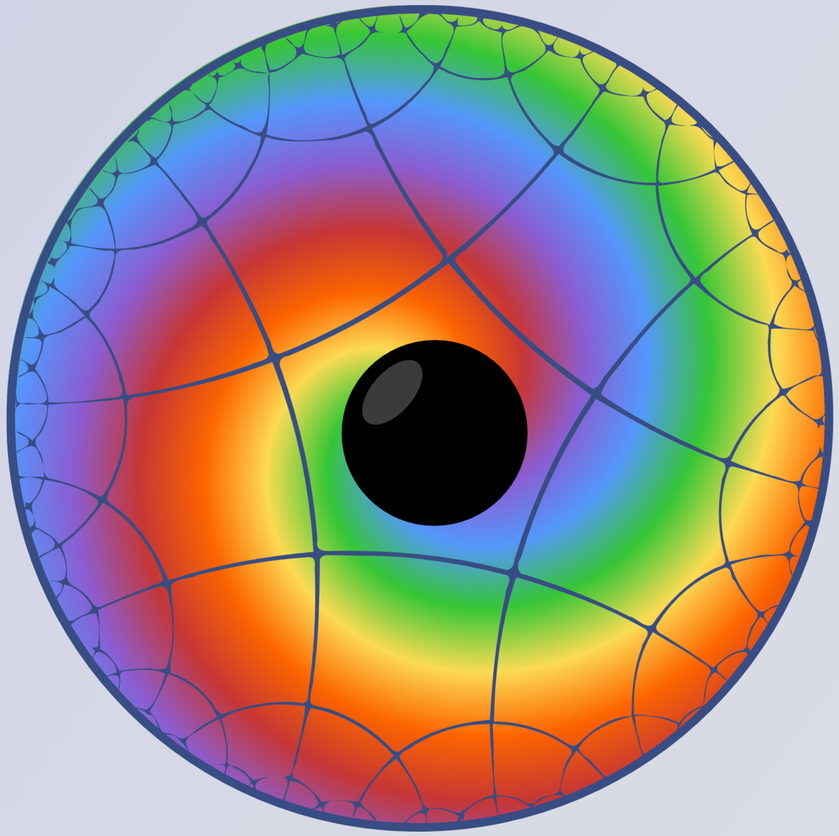
- Geometrization of RG fixed points

$$x \to \lambda x,\ u \to \lambda u \quad \longrightarrow \quad \mathrm{d}s^2 = \frac{L^2}{u^2}\left(\mathrm{d}t^2 + \mathrm{d}\mathbf{x}^2 + \mathrm{d}u^2\right)$$

- AdS_{d+1} spacetime isometries $\equiv$ CFT_d symmetries

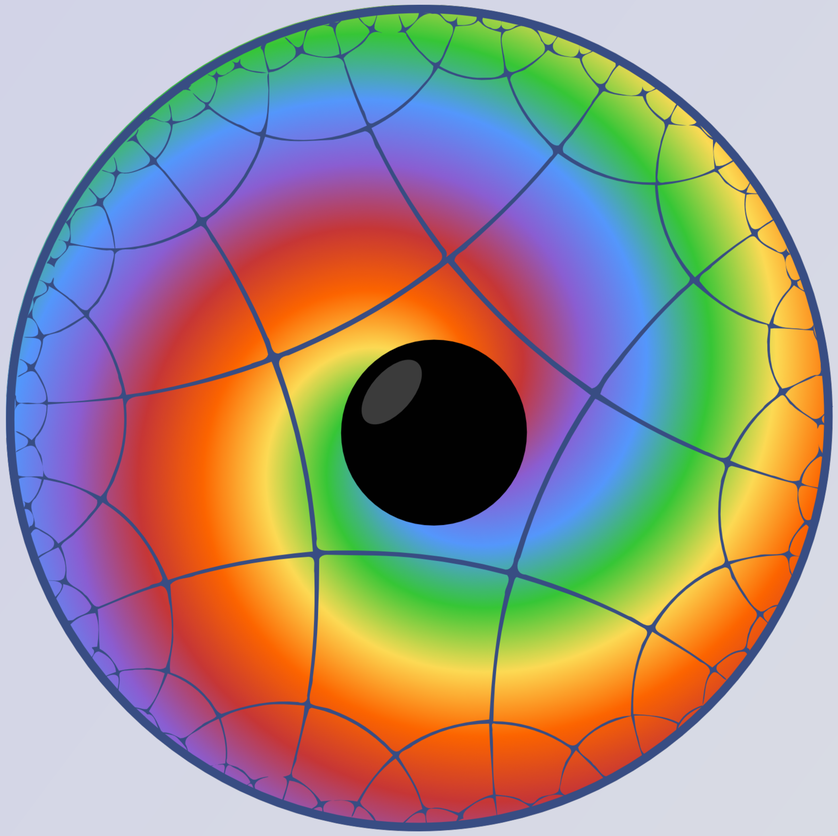
Poincaré symmetry $\equiv$ $SO(d,2)$

scale invariance $\equiv$ conformal group



- Gauge/Gravity Dictionary

$$S_{\text{eff}} = \int \mathrm{d}^d x\, g(x)^a O_a(x)$$

$$g^a(x)\Big|_{\text{UV}} = J^a(x) = \varphi^a(x,u)\Big|_{u=0} = \varphi^a(x)\Big|_{\partial(\text{AdS})}$$

Thank You!!!