Phenomenological two-loop applications with SecDec 2.1

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http://secdec.hepforge.org/

"We have it!"

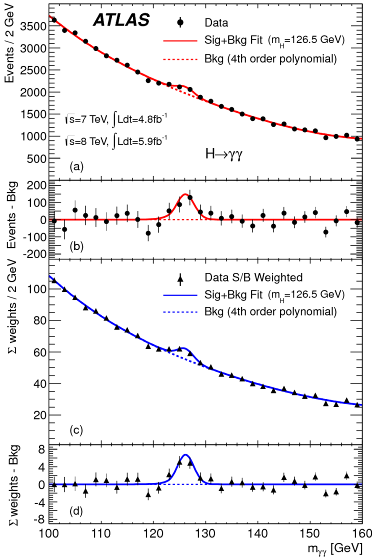
- But which one?

  Standard Model Higgs,
  supersymmetric Higgs,
  composite Higgs, ...

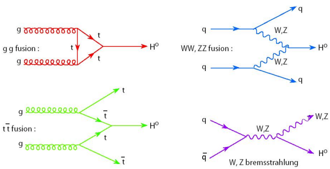
- And how do we find out?

  Experimental side:
  more measurements, more statistics

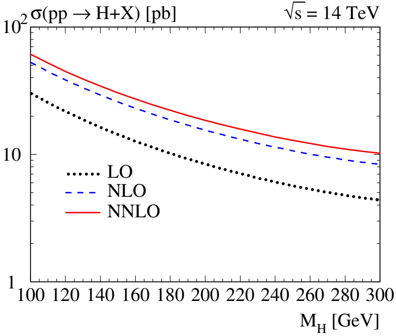
  Theoretical side:
  increase predictivity
  One way: increase theoretical precision

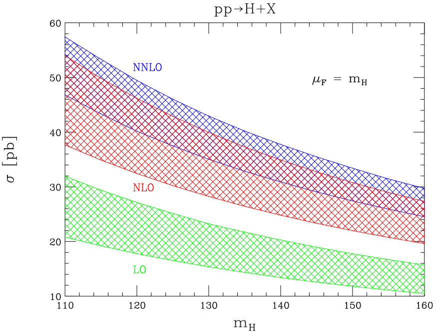


Success of higher order corrections



Example: Higgs production cross section





Harlander & Kilgore '02

Anastasiou, Melnikov, Petriello '05

Assumption: The found Higgs boson is just one out of two

- Minimal extension of the SM: 2 Higgs doublets

$$H_1 = \begin{pmatrix} v_1 + \frac{1}{\sqrt{2}}(\phi_1^0 + \mathrm{i}\chi_1^0) \\ -\phi_1^- \end{pmatrix} \qquad H_2 = \begin{pmatrix} \phi_2^+ \\ v_2 + \frac{1}{\sqrt{2}}(\phi_2^0 + \mathrm{i}\chi_2^0) \end{pmatrix}$$

Assumption: The found Higgs boson is just one out of two

Further constraints on a 2 Higgs doublet model are given in the MSSM

- MSSM Higgs potential (incl. soft SUSY breaking terms)

$$V = m_1|H_1|^2 + m_2|H_2|^2 - m_{12}(\epsilon_{ab}H_1^a H_2^b + h.c.)$$
$$+\frac{1}{8}(g_1^2 + g_2^2)(|H_1|^2 - |H_2|^2)^2 + \frac{1}{2}g_2^2|H_1^\dagger H_2|^2$$

- MSSM Higgs potential fixed by g_1, g_2, the vevs in $\tan\beta \equiv \frac{v2}{v1}$ and soft SUSY breaking term in $M_A^2 = m_{12}^2(\tan\beta + \cot\beta)$

In the MSSM, the prediction of the Higgs masses is possible

The neutral $\mathcal{CP}$-even Higgs boson masses

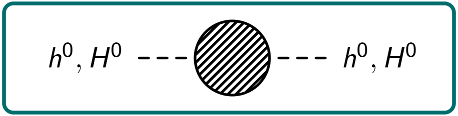
The Higgs masses can be read off from the tree-level mass matrix

$$M_{\text{Higgs}}^{2,\text{tree}} = \begin{pmatrix} M_A^2 \sin^2\beta + M_Z^2 \cos^2\beta & -(M_A^2 + M_Z^2)\sin\beta\cos\beta \\ -(M_A^2 + M_Z^2)\sin\beta\cos\beta & M_A^2\cos^2\beta + M_Z^2 \sin^2\beta \end{pmatrix}$$

For $\tan\beta \gg 1$ ($\Rightarrow \cos\beta \to 0$, $\sin\beta \to 1$)

$$\lim_{\tan\beta\to\infty} M_{\text{Higgs}}^{2,\text{tree}} = \begin{pmatrix} M_A^2 & 0 \\ 0 & M_Z^2 \end{pmatrix} = \begin{pmatrix} m_{H,\text{tree}}^2 & 0 \\ 0 & m_{h,\text{tree}}^2 \end{pmatrix}$$

$\Rightarrow$ We are interested in higher order self-energy corrections to the Higgs boson masses



Public codes implementing the rMSSM corrections

FeynHiggs Frank, Hahn, Heinemeyer, Hollik, Rzehak, Weiglein '00 '03 '07

SoftSusy Allanach '02 SPheno Porod '03

CPsuperH Carena, Choi, Drees, Ellis, Lee, Pilaftsis, Wagner '04 '09

Suspect Djouadi, Kneur, Moultaka '07

H3m Kant, Harlander, Mihaila, Steinhauser '10

Summary of the implemented rMSSM corrections:

1-loop complete

2-loop $\mathcal{O}(\alpha_s\alpha_t)$, $\mathcal{O}(\alpha_t^2)$, $\mathcal{O}(\alpha_s\alpha_b)$, $\mathcal{O}(\alpha_t\alpha_b)$, $\mathcal{O}(\alpha_b^2)$, gaugeless limit and $p^2 = 0$

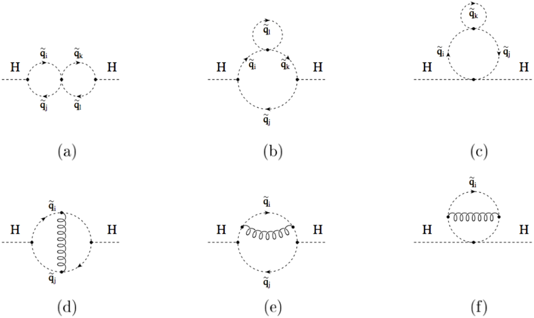
3-loop $\mathcal{O}(\alpha_s^2\alpha_t)$, gaugeless limit and $p^2 = 0$

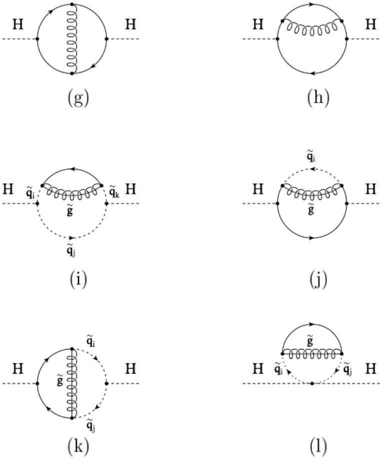
dominant correction @ 2-loop: $\mathcal{O}(\alpha_s\alpha_t)$ $(p^2 = 0)$

⟶ **next improvement:** $\mathcal{O}(\alpha_s\alpha_t)$ for $p^2 \neq 0$

Higgs boson self-energy diagrams for $\mathcal{O}(\alpha_s\alpha_t)$

$H = H^0, h^0$





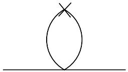
24 diagrams

Renormalization at two-loop

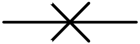
The two-loop integrals may contain two ultraviolet (UV) divergences which need to be cancelled with a proper renormalization.

The renormalization comprises

- 1-loop subrenormalization



- 2-loop counter terms

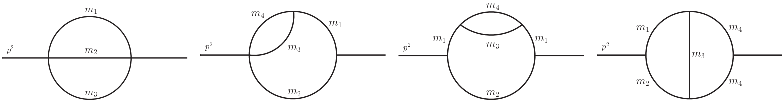


Now we have $\mathcal{O}(100)$ diagrams!

Treatment of loop integrals

Tensor reduction to only scalar integrals with the TwoCalc package possible G. Weiglein et al. '93

- Many of the resulting integrals (mainly 1-loop) are known analytically
- Full analytic results unknown for some two-loop topologies



T_{234} T_{1234} T_{11234} T_{12345}

- These integrals are treated numerically with SecDec

The program SecDec 2.1

SecDec is a tool to numerically compute various sorts of integrals contributing to higher-order computations.

It can tackle:

- General Feynman integrals and more general parametric functions for arbitrary kinematics

Feynman graph — or — parametric function

General Feynman integral

- **Generic Feynman integrals** in D dimensions at L loops with N propagators to power ν_j of rank R with $N_\nu = \sum_{j=1}^{N} \nu_j$, e.g. scalar multi-loop integral in Feynman parametrization

$$G = \frac{(-1)^{N_\nu}}{\prod_{j=1}^{N} \Gamma(\nu_j)} \Gamma(N_\nu - LD/2) \int_0^\infty \prod_{j=1}^{N} dx_j \; x_j^{\nu_j - 1} \, \delta(1 - \sum_{l=1}^{N} x_l) \frac{\mathcal{U}^{N_\nu - (L+1)D/2}(\vec{x})}{\mathcal{F}^{N_\nu - LD/2}(\vec{x})}$$

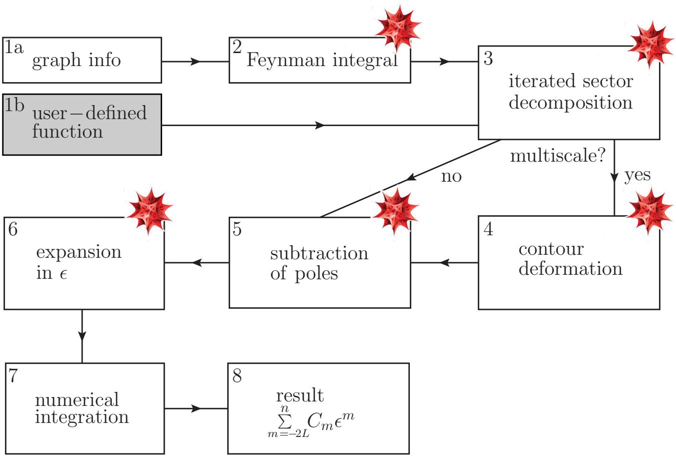
- Extension to physical kinematics including mass thresholds since SecDec 2.0: Limitation of multi-scale integrals to the Euclidean region lifted! SB, Carter, Heinrich '12

NEW in SecDec 2.1

- Computation of contracted **tensor** integrals at in principle arbitrary rank possible SB & Heinrich '13

$$T_{12345}^{\mathrm{Rank3}} = \iint \mathrm{d^D}k_1 \, \mathrm{d^D}k_2 \, \frac{p_{1\mu} k_1^\mu k_{1\nu} k_2^\nu}{D_1 D_2 D_3 D_4 D_5}$$

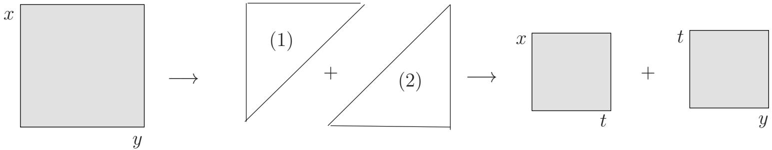
Operational sequence of the SecDec 2.1 program



Numerical integration:

CUBA library Hahn et al. '04 '11 or BASES Kawabata '95

The method of sector decomposition



$$\int_0^1 dx_1 \int_0^1 dx_2 \, \frac{1}{(x_1+x_2)^{2+\epsilon}}$$

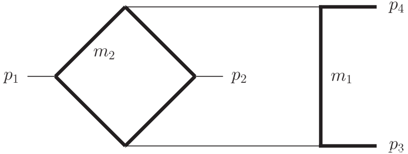
$$= \int_0^1 dx_1 \int_0^1 dx_2 \, \frac{1}{(x_1+x_2)^{2+\epsilon}} \left(\theta(x_1-x_2)+\theta(x_2-x_1)\right)$$

$$= \int_0^1 dx_1 \int_0^{x_1} dx_2 \, \frac{1}{(x_1+x_2)^{2+\epsilon}} + \int_0^1 dx_2 \int_0^{x_2} dx_1 \, \frac{1}{(x_1+x_2)^{2+\epsilon}}$$

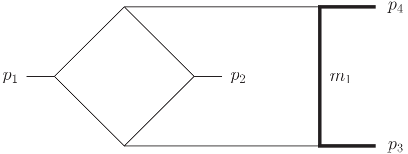
$$= \int_0^1 dx_1 \int_0^1 dt \, \frac{x_1}{(x_1+x_1 t)^{2+\epsilon}} + \int_0^1 dx_2 \int_0^1 d\tilde{t} \, \frac{1}{x_2^{1+\epsilon}(\tilde{t}+1)^{2+\epsilon}}$$

Recent successes of SecDec

- Fast evaluation of e.g. two-loop bubbles (needed for the MSSM Higgs masses!)
- With some analytical preparations beforehand we managed to compute two of the most complicated non-planar two-loop boxes entering the fermionic corrections in $gg \to t\bar{t}$ @ NNLO



(a) ggtt1



(b) ggtt2

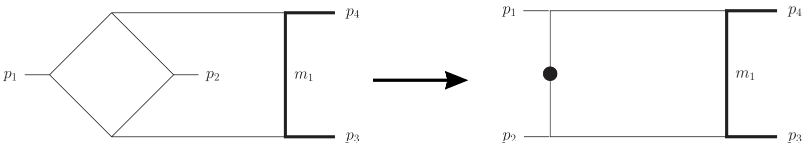
- Various groups from diverse fields are using SecDec for numerical calculations/checks

Analytical manipulations beforehand

Goals for better numerical convergence:

1) decrease number of numerical integration parameters
2) turn linear divergences $x^{-2-\epsilon}$ into logarithmic ones
3) decrease number of functions

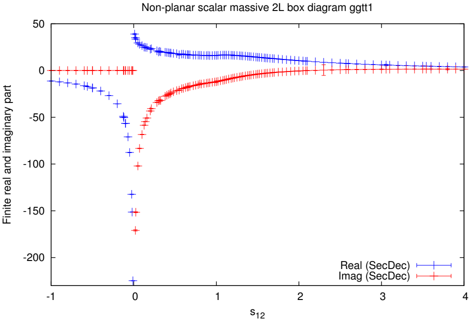
Achieving goal 1: Integrate out one loop first



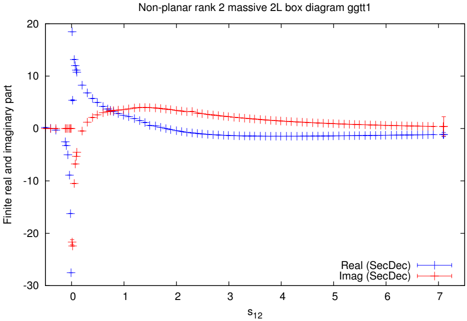
$\rightarrow$ analytical integration of one Feynman parameter is possible

Achieving goal 2 & 3: Using a new transformation, a more even distribution of divergences among Feynman parameters is possible

Results for the non-planar massive two loop diagram ggtt1



Scalar integral



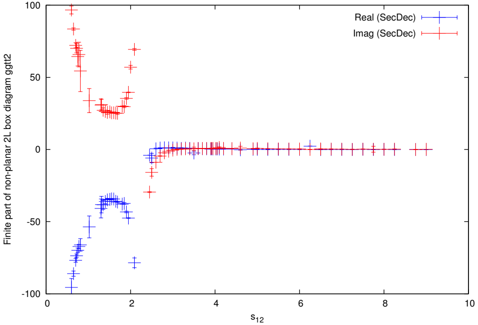
Rank 2

$m_1^2 = m_2^2 = 1,\ p_3^2 = p_4^2 = m_1^2,\ p_1^2 = p_2^2 = 0,\ s_{23} = -1.25$

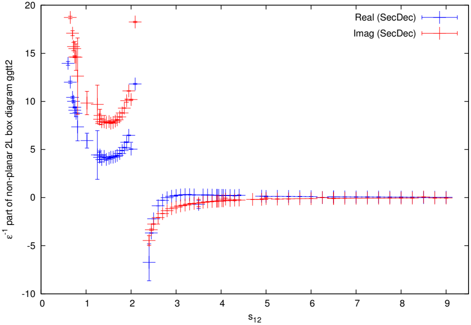
timings: 11-60 secs (scalar) & 5-10 secs (rank 2) far from threshold (th.)
1600 secs (scalar) & 700 secs (rank 2) very close to th.

rel. accuracy: 10^{-3}, abs. accuracy: 10^{-5}

Result for the non-planar massive two loop diagram ggtt2



Finite part



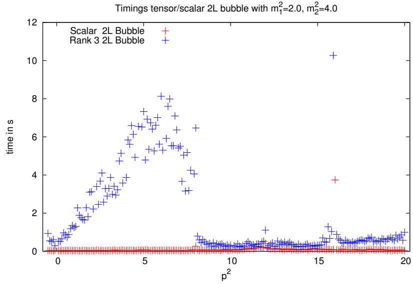
$1/\epsilon$ pole

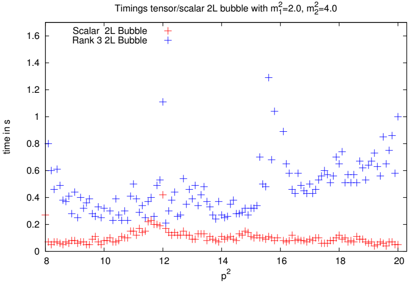
$m_1^2 = 1,\ p_1^2 = p_2^2 = 0,\ p_3^2 = p_4^2 = m_1^2,\ s_{23} = -1.25$

timings: 250-4000 secs, rel. accuracy $5{\cdot}10^{-3}$, abs. accuracy: 10^{-5}

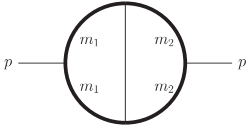
analytic results: Manteuffel & Studerus '13

2-loop bubble with 2 mass scales - timings

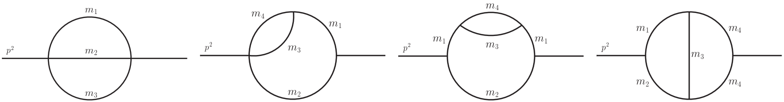




- relative & absolute accuracy 0.1%
- Scalar integral is finite, rank 3 integral has $\mathcal{O}(\epsilon^{-2})$ poles
- Intel Core i7 Processor

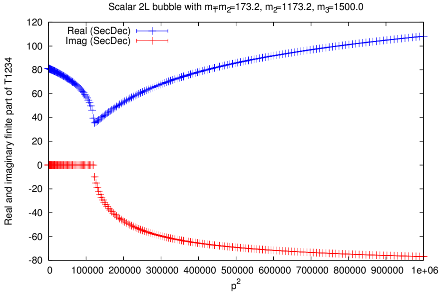


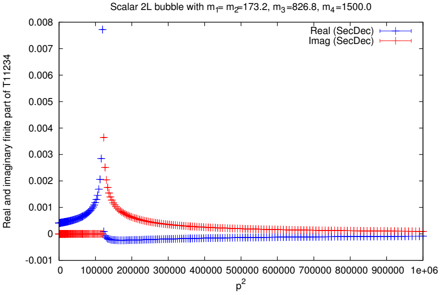
Back to MSSM Higgs boson masses...



T_{234} T_{1234} T_{11234} T_{12345}

- The integrals which are not available in analytical form are provided by SecDec, e.g.





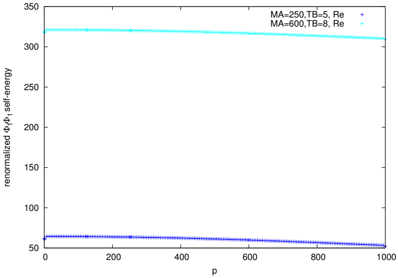
T_{1234}, finite part T_{11234}, finite part

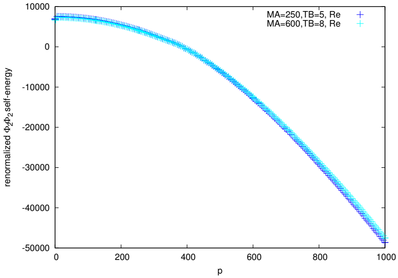
Renormalized self-energies

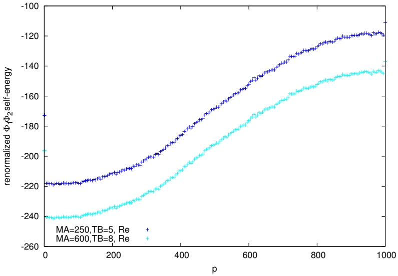
- Join the analytical and numerical components to compute the renormalized neutral $\mathcal{CP}$-even Higgs-boson self-energies

$$\hat{\Sigma}^{(2)}_{\phi_1^0\phi_1^0}(p^2) = \Sigma^{(2)}_{\phi_1^0\phi_1^0}(p^2) + p^2\delta Z^{(2)}_{H_1} - \delta V^{(2)}_{\phi_1^0\phi_1^0}$$
$$\hat{\Sigma}^{(2)}_{\phi_2^0\phi_2^0}(p^2) = \Sigma^{(2)}_{\phi_2^0\phi_2^0}(p^2) + p^2\delta Z^{(2)}_{H_2} - \delta V^{(2)}_{\phi_2^0\phi_2^0}$$
$$\hat{\Sigma}^{(2)}_{\phi_1^0\phi_2^0}(p^2) = \Sigma^{(2)}_{\phi_1^0\phi_2^0}(p^2) - \delta V^{(2)}_{\phi_1^0\phi_2^0}$$

Results for the renormalized self-energies







(preliminary)

The neutral MSSM Higgs-boson masses @ 2-loop

From the Higgs doublet states to the physical Higgs boson states in the MSSM

$$\begin{pmatrix} H^0 \\ h^0 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \phi_1^0 \\ \phi_2^0 \end{pmatrix}$$

- The new self-energy corrections are included in the inverse Higgs-boson propagator matrix

$$\Gamma \equiv \Delta_{\text{Higgs}}^{-1} = \begin{pmatrix} p^2 - m_{H,\text{tree}}^2 + \hat{\Sigma}_H(p^2) & \hat{\Sigma}_{hH}(p^2) \\ \hat{\Sigma}_{hH}(p^2) & p^2 - m_{h,\text{tree}}^2 + \hat{\Sigma}_h(p^2) \end{pmatrix}$$

  with renormalized self-energies $\hat{\Sigma}$ up to the two-loop level

- The propagator poles m_H^2 and m_h^2 are solutions to $\text{Det}(\Gamma) = 0$

Summary & Outlook

Summary

- Momentum dependent 2-loop corrections to the MSSM Higgs masses using SecDec 2.1 are around the corner
- SecDec 2.1 is a useful tool to compute various sorts of integrals: multi-loop integrals, contracted tensor integrals and user-defined polynomial integrals
- SecDec 2.1 provides valuable checks for non-planar 2-loop 4-point master integrals entering $t\bar{t}$@NNLO computations

Outlook

- Further applications to other massive two-loop amplitudes
- Combination with new unitarity inspired reduction of 2-loop amplitudes

Backup

Install SecDec 2.1

- **Download**:
  http://secdec.hepforge.org
- **Install**:
  tar xzvf SecDec.tar.gz
  cd SecDec-2.1
  ./install
- **Prerequisites**:
  Mathematica (version 6 or above), Perl, Fortran and/or C++ compiler

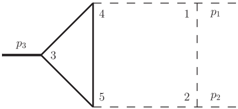
User Input I

- param.input: parameters for integrand specification and numerical integration

```
################ input parameters for sector decomposition ################
#------------------------------------------------------------------------------------------------
# subdirectory for the mathematica output files (will be created if non-existent) :
# if not specified, a directory with the name of the graph given below will be created by default
subdir=2loop
#------------------------------------------------------------------------------------------------
# if outputdir is not specified: default directory for
# the output will have integral name (given below) appended to directory above,
# otherwise specify full path for Mathematica output files here
outputdir=
#------------------------------------------------------------------------------------------------
# graphname (can contain underscores, numbers, but should not contain commas)
graph=P126
#------------------------------------------------------------------------------------------------
# number of propagators:
propagators=6
#------------------------------------------------------------------------------------------------
# number of external legs:
legs=3
#------------------------------------------------------------------------------------------------
# number of loops:
loops=2
#------------------------------------------------------------------------------------------------
# construct integrand (F and U) via topological cuts (only possible for scalar integrals)
# default is 0 (no cut construction used)
cutconstruct=1
#########################################################
# parameters for subtractions and epsilon expansion
#########################################################
```

User Input II

- template.m: definition of the integrand (Mathematica syntax)




```
(******* USER INPUT for construction of integrand ****************************)

(******* Use with cutconstruct=1 *********************************************)

proplist={{ms[1],{3,4}},{ms[1],{4,5}},{ms[1],{5,3}},
          {0,{1,2}},{0,{1,4}},{0,{2,5}}};

(******* Use with cutconstruct=0 *********************************************)
(*
momlist={k1,k2};
proplist={k1^2-ms[1],(k1+p3)^2-ms[1],(k1-k2)^2-ms[1],
         (k2+p3)^2,(k2+p1+p3)^2,k2^2};
numerator={1};
*)

(******* Propagator powers (optional) ****************************************)

powerlist=Table[1,{i,Length[proplist]}];

(******* On-shell conditions (optional) *************************************)
onshell={ssp[1]->0,ssp[2]->0,ssp[3]->sp[1,2],sp[1,3]->0,sp[2,3]->0};

(******* Set Dimension *******************************************************)

Dim=4-2*eps;
(***************************************************************************)
```

Program Test Run

- ./launch -p param.input -t template.m

```
********** This is SecDec version 2.0 **********
Authors: Sophia Borowka, Jonathon Carter, Gudrun Heinrich
**************************************************
graph = P126
primary sectors 1,2,3,4,5,6, will be calculated
calculating F and U . . .
done
written to /home/pcl335a/sborowka/Work/SecDecBeta/loop/2loop/P126/FUN.m

results of the decomposition will be written to
/home/pcl335a/sborowka/Work/SecDecBeta/loop/2loop/P126
doing sector decomposition . . .
done

working on pole structure: 2 logarithmic poles, 0 linear poles, 0 higher poles
C++ functions created for pole structure 2l0h0
compiling 2l0h0/epstothe0 ...
doing numerical integrations in P126/2l0h0/epstothe0
compiling 2l0h0/epstothe-1 ...
doing numerical integrations in P126/2l0h0/epstothe-1
compiling 2l0h0/epstothe-2 ...
doing numerical integrations in P126/2l0h0/epstothe-2
working on pole structure: 1 logarithmic pole, 0 linear poles, 0 higher poles
C++ functions created for pole structure 1l0h0
compiling 1l0h0/epstothe0 ...
doing numerical integrations in P126/1l0h0/epstothe0
compiling 1l0h0/epstothe-1 ...
doing numerical integrations in P126/1l0h0/epstothe-1
working on pole structure: 0 logarithmic poles, 0 linear poles, 0 higher poles
C++ functions created for pole structure 0l0h0
compiling 1l0h0/epstothe0 ...
doing numerical integrations in P126/0l0h0/epstothe0
Output written to /home/pcl335a/sborowka/Work/SecDecBeta/loop/2loop/P126/P126_pfull.res

To remove intermediate files, execute the command /home/pcl335a/sborowka/Work/SecDecBeta/loop/launchcleanP126
```

Get the Result

- resultfile P126_full.res

```
************************************************
***OUTPUT: P126 p5 ***************
 point: 7.0
 ext. legs: 0.0 0.0 7.0
 prop. mass: 1.0 0. 0. 0. 0. 0.
Prefactor=-Exp[-2EulerGamma*eps]
************************************************
****** eps^-2 coeff ******

result        =0.07563683
              +0.1003924148 I
error         =0.000493522517701388
              + 0.00139691015080074 I
CPUtime (all eps^-2 subfunctions)  =0.04
CPUtime (longest eps^-2 subfunction)  =0.01
.
.
.

****** eps^0 coeff ******

result        =0.906978296750816
              -0.908781551612644 I
error         =0.00754504726896407
              + 0.0442867373250588 I
CPUtime (all eps^0 subfunctions)  =2.44
CPUtime (longest eps^0 subfunction)  =0.51

************************************************

Time taken for decomposition = 2.005725

Total time for subtraction and eps expansion = 41.5057 secs
Time taken for longest subtraction and eps expansion = 17.8613 secs
```

Analytical side: Two-loop renormalization for neutral $\mathcal{CP}$-even Higgs-boson self-energies

Feynman diagrammatic calculation performed in the gaugeless limit

- Renormalization corresponds to FeynHiggs renormalization
- Mass renormalization in the OS scheme:

$$\delta M_A^{2(2)},\ \delta t_1^{(2)},\ \delta t_2^{(2)},\ \delta m_{\tilde{t}_1}^{(1)},\ \delta m_{\tilde{t}_2}^{(1)},\ \delta m_t^{(1)}$$

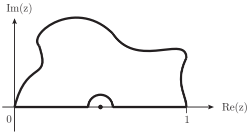
- Field renormalization in the $\overline{DR}$ scheme:

$$\delta Z_{H_1}^{(2)},\ \delta Z_{H_2}^{(2)},\ \delta \tan\beta^{(2)}$$

- Resulting input parameters: $m_t, \mu, X_t, M_{SUSY}, m_{\tilde{g}}, \tan\beta, m_A$

  $X_t = A_t - \mu \cot\beta$ and A_t the soft SUSY breaking parameters

Deformation of the integration contour to integrate mass thresholds



- Integrand is analytically continued into the complex plane

$$\mathcal{F}(\vec{t}) \to \mathcal{F}(\vec{t} + \mathrm{i}\vec{y}(\vec{t})) = \mathcal{F}(\vec{t}) + \mathrm{i}\sum_j y_j(\vec{t})\frac{\partial \mathcal{F}(\vec{t})}{\partial t_j} + \mathcal{O}(y(\vec{t})^2)$$

- The integration contour is deformed by

$$\vec{t} \to \vec{z} = \vec{t} + \mathrm{i}\vec{y} \ ,$$

$$y_j(\vec{t}) = -\lambda t_j(1 - t_j)\frac{\partial \mathcal{F}(\vec{t})}{\partial t_j}$$

Soper '99

Soper, Nagy, Binoth; Kurihara et al., Anastasiou et al., Freitas et al., Becker et al.

Convert linear divergences into logarithmic ones

$\vec{x}_{jk}$ denotes all Feynman parameters excluding x_j and x_k

Assume $\alpha > 1$ and functions P, Q, R such that a linear divergence appears in x_j in Eq. (1) after sector decomposition

$$\prod_{i=1}^{N} \left\{ \int_0^1 \mathrm{d}x_i \right\} [x_j(P(\vec{x}_{jk}) + x_k Q(\vec{x}_{jk})) + R(\vec{x}_{jk})]^{-\alpha} \tag{1}$$

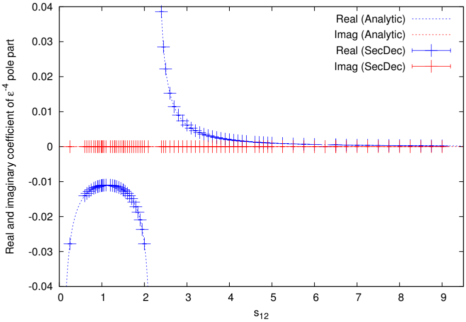
$$= \prod_{i=1}^{N} \left\{ \int_0^1 \mathrm{d}x_i \right\} \frac{1}{x_j} [x_j P(\vec{x}_{jk}) + x_k Q(\vec{x}_{jk}) + R(\vec{x}_{jk})]^{-\alpha}$$

$$- \prod_{i=1}^{N} \left\{ \int_0^1 \mathrm{d}x_i \right\} \frac{1}{x_j} [x_k(x_j P(\vec{x}_{jk}) + Q(\vec{x}_{jk})) + R(\vec{x}_{jk})]^{-\alpha}$$

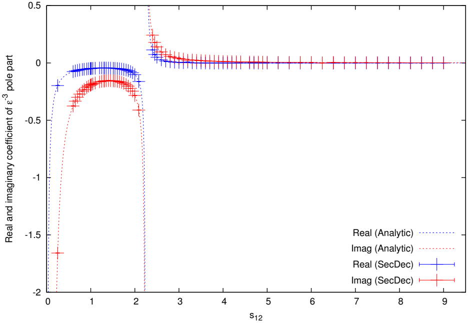
We rest with a logarithmic divergence in x_j.

- Linear divergences can be turned into logarithmic ones
- In the case of *ggtt*2 this leads to a total reduction of number of functions by 2/3

Result for the non-planar massive two loop diagram ggtt2



Leading pole



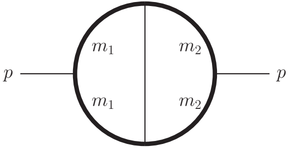
Sub-leading pole

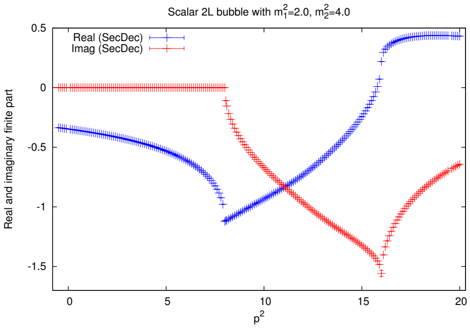
$m_1^2 = 1,\ p_1^2 = p_2^2 = 0,\ p_3^2 = p_4^2 = m_1^2,\ s_{23} = -1.25$

analytic results: Manteuffel & Studerus '12

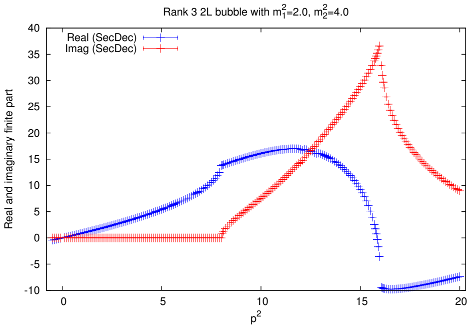
2-loop bubble with 2 mass scales - Results

thresholds at $4 \cdot m_1^2 = 8$ and $4 \cdot m_2^2 = 16$





Scalar integral



Rank 3