Stringy Geometry

Geometrization of O(d,d)-transformations

Felix Rennecke

Based on
arXiv: 1202.4934, 1205.1522, 1210.1591, 1211.0030, 1304.2784

Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

MOTIVATION

$$S(G,B) = \frac{1}{2\pi\alpha'} \int_{\Sigma} d^2z \, (G_{ab} + B_{ab}) \, \partial X^a \bar{\partial} X^b$$

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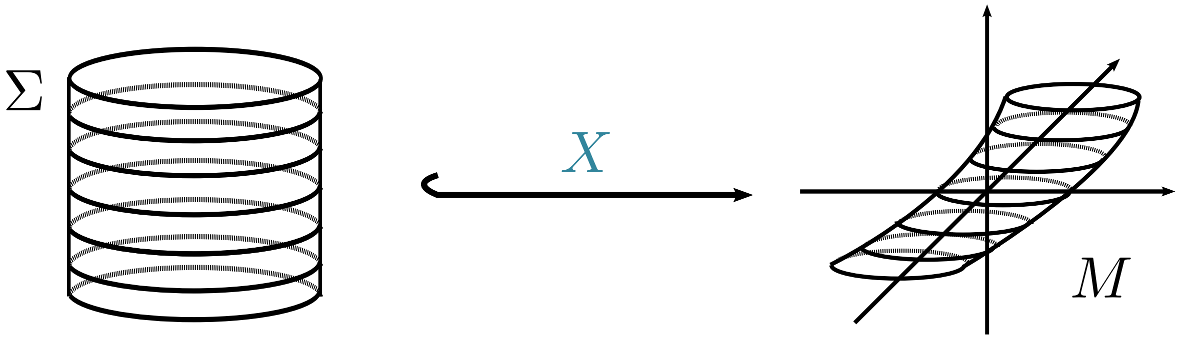

2-dim world-sheet

d-dim space-time

MOTIVATION

Metric determines
shape of spacetime

B-field gives
magnetic field

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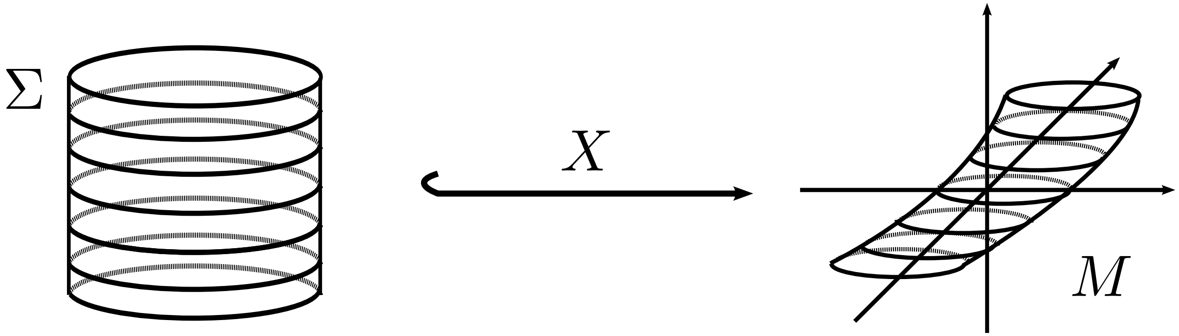

2-dim world-sheet

d-dim spacetime

MOTIVATION

The 2-dim. sigma model *S* describes motion of closed string in *d*-dim. space-time: geodesic motion of a string on

Riemann-Cartan space M

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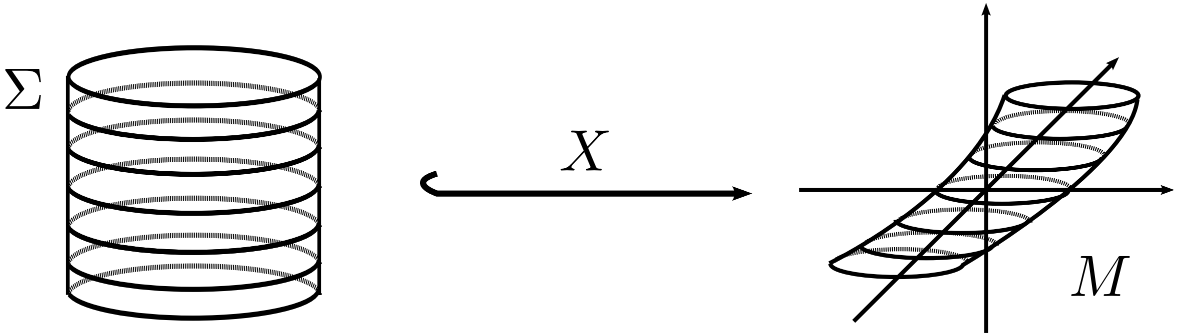

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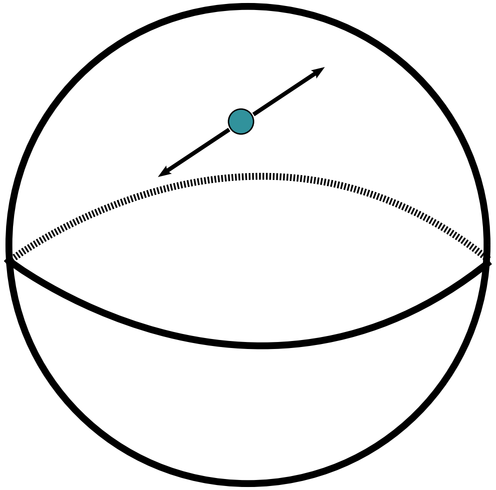
point particle

closed string



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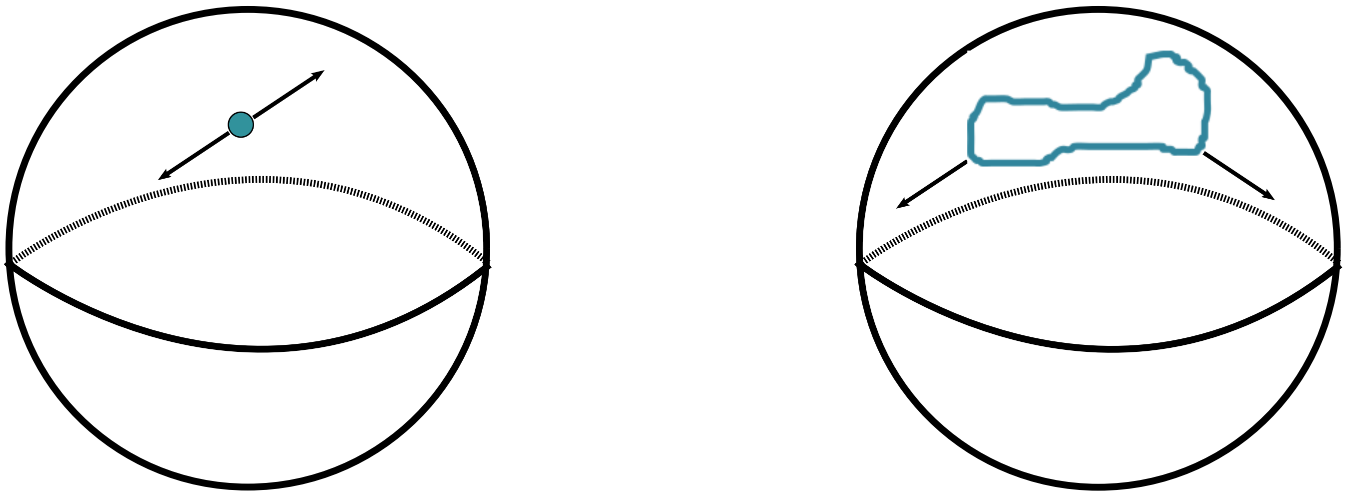

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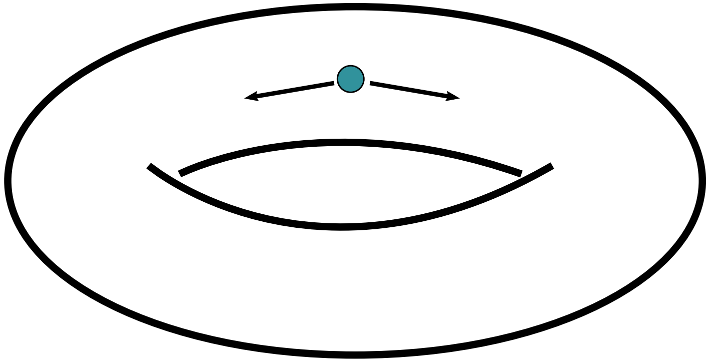

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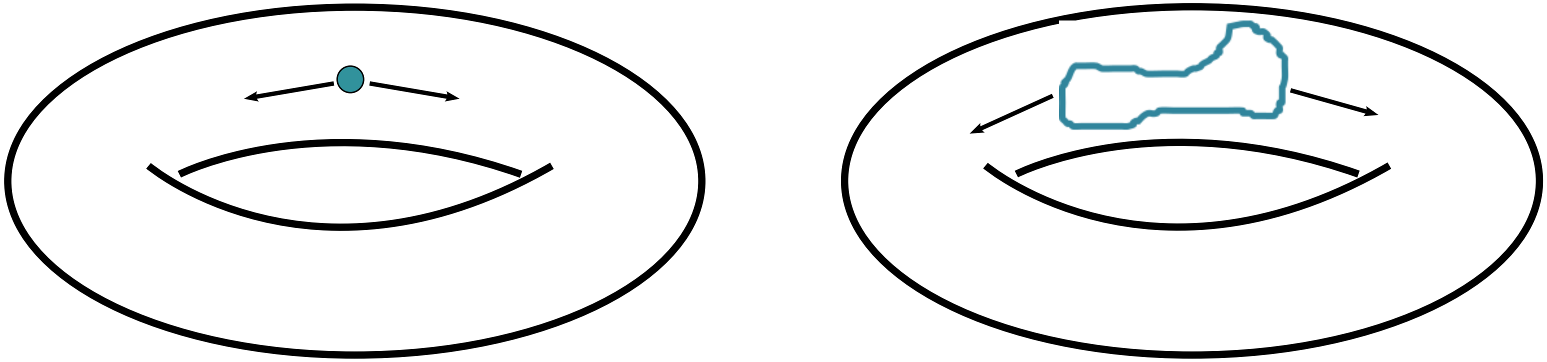

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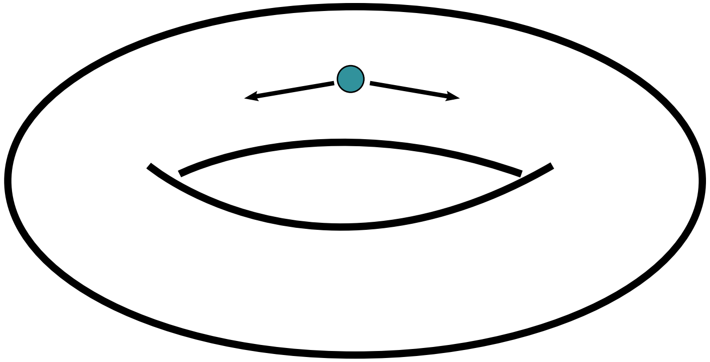

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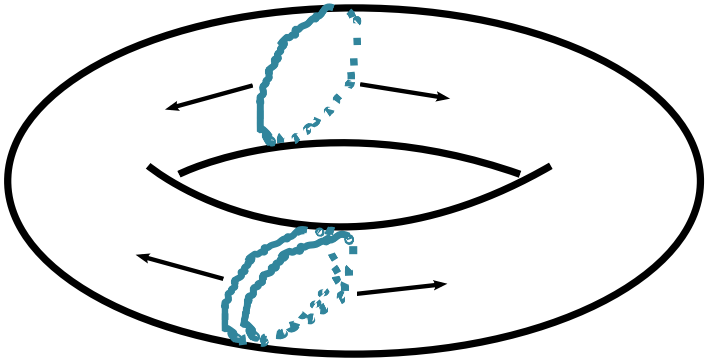

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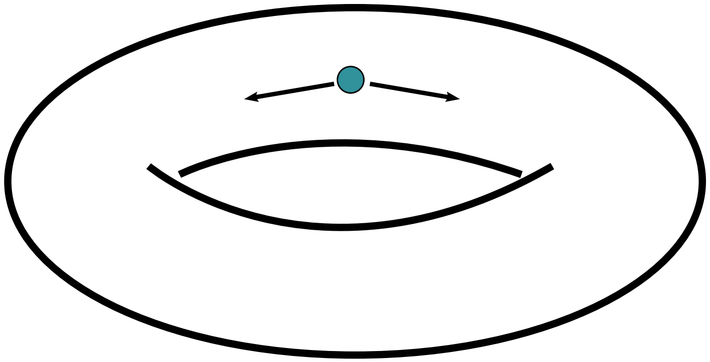

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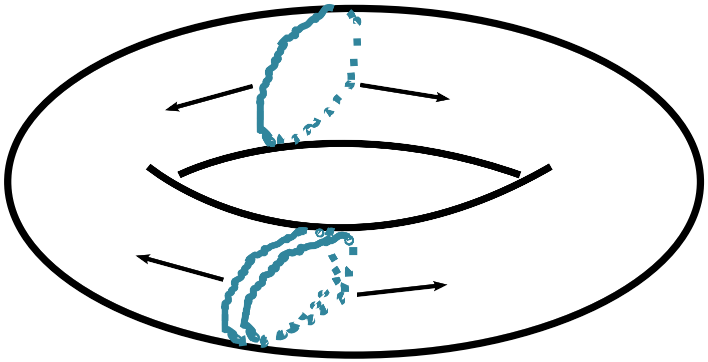

closed string

String probes spacetime very differently – it can wind around certain directions

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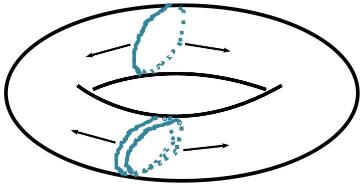

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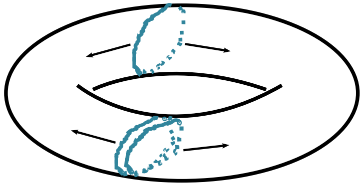

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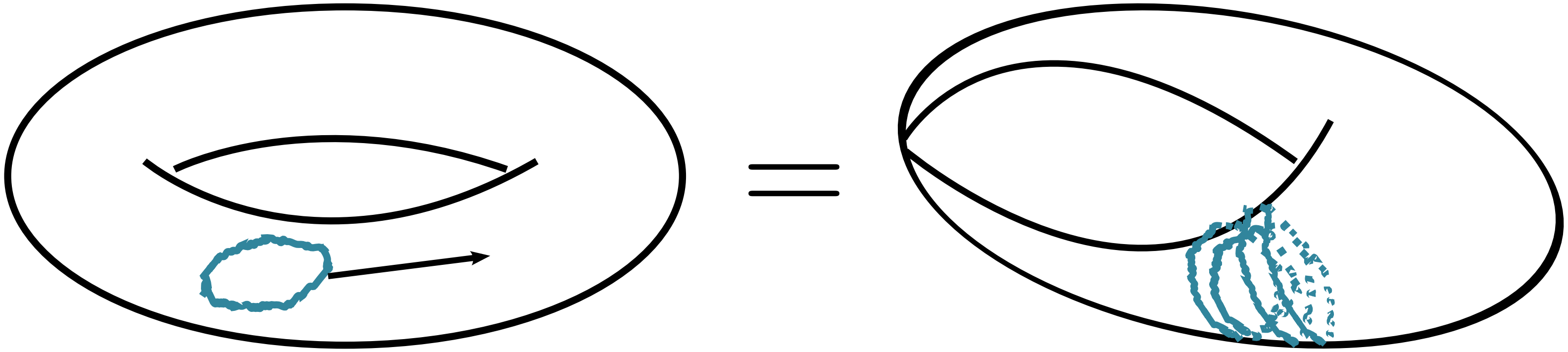

momentum *m* and winding *n* = momentum *n* and winding *m*

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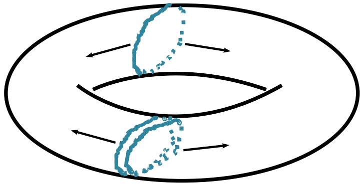

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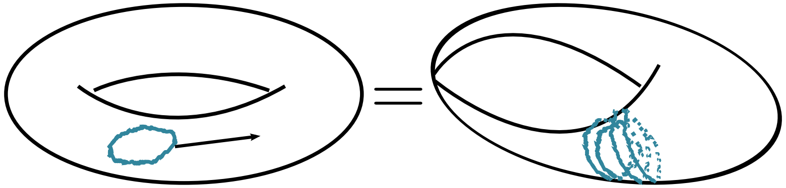

Very different spacetimes can give same
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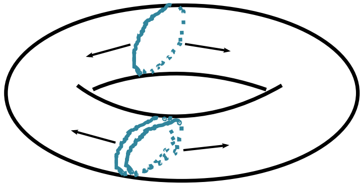

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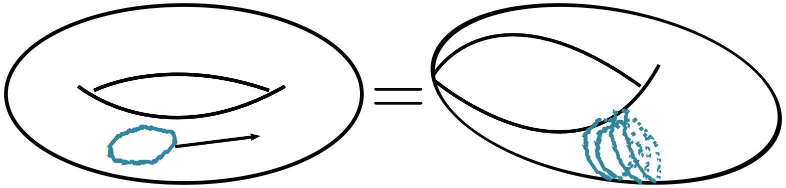

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$$(G, B) \quad \Longleftrightarrow \quad (\widetilde{G}, \widetilde{B})$$

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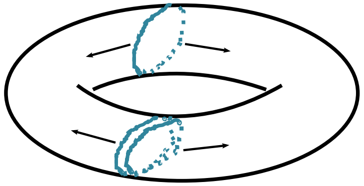

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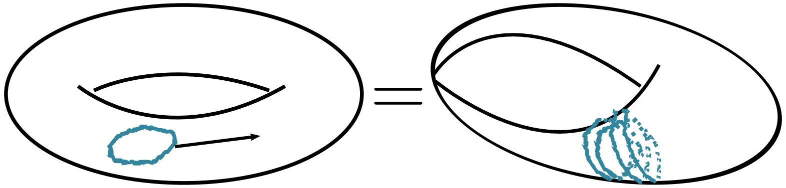

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$$(G, B) \iff (\widetilde{G}, \widetilde{B})$$

related by the Buscher rules

$$\widetilde{G}_{00} = \frac{1}{G_{00}}\,, \quad \widetilde{G}_{0m} = \frac{B_{0m}}{G_{00}}\,, \quad \widetilde{G}_{mn} = G_{mn} - \frac{G_{m0}G_{n0} + B_{m0}B_{0n}}{G_{00}}$$

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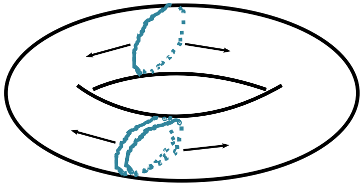

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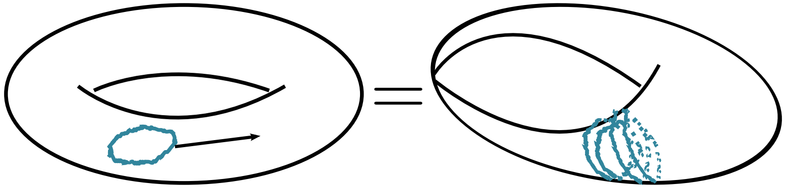

Very different spacetimes can give same theory: T-duality

$$(G, B) \quad \Longleftrightarrow \quad (\widetilde{G}, \widetilde{B})$$

related by the Buscher rules
in terms of generalized metric

$$\mathcal{H}(G, B) = \begin{pmatrix} G - BG^{-1}G & BG^{-1} \\ -G^{-1}B & G^{-1} \end{pmatrix}$$

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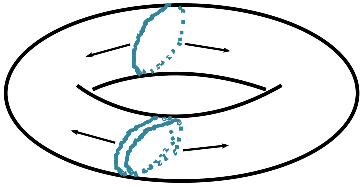

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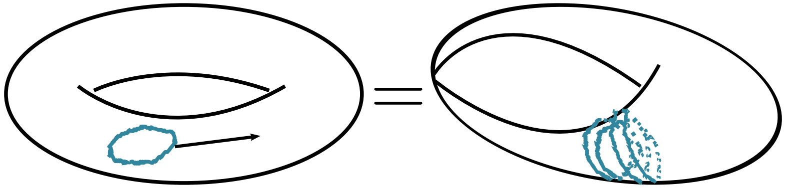

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$$\mathcal{H}(\widetilde{G},\widetilde{B}) = \mathcal{H}'(G,B) = \mathcal{T}^t\mathcal{H}(G,B)\mathcal{T} \quad \text{with} \quad \mathcal{T} = \left(\begin{array}{ccc|ccc} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right)$$

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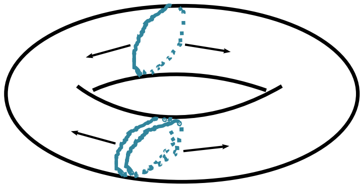

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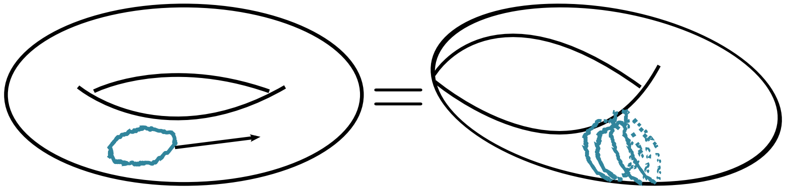

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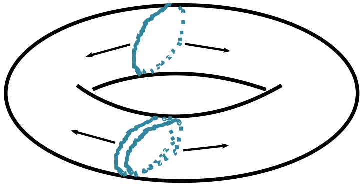

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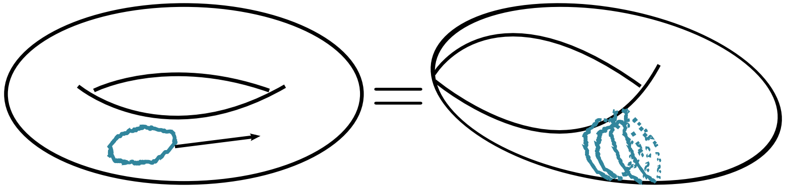

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Simple Example: $\mathbb{R}^2 \times \mathbb{R}^{1,d-3}$

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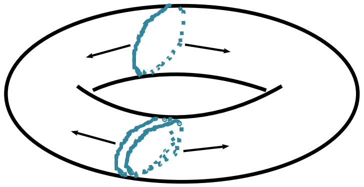

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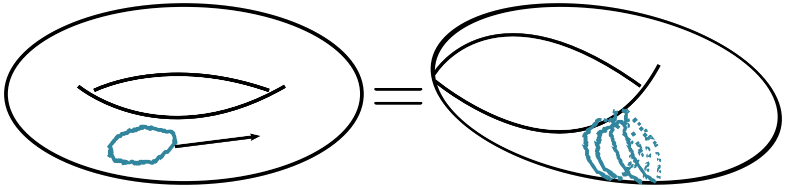

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$$R = 0$$

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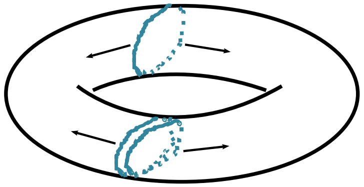

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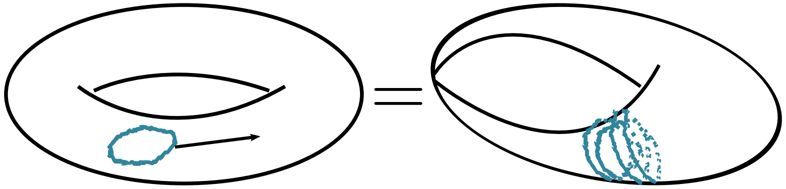

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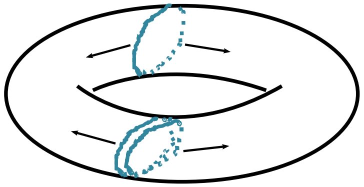

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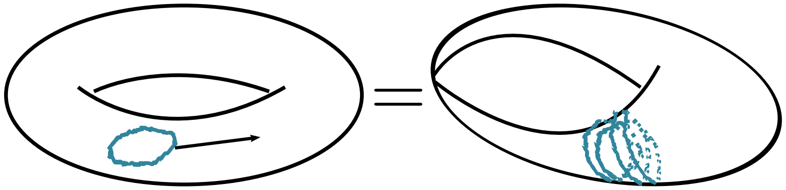

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singular at origin

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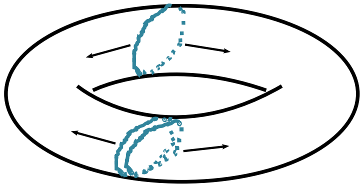

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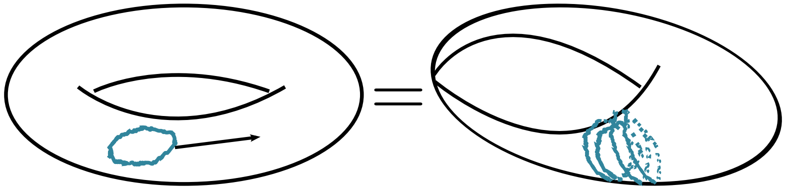

Very different spacetimes can give same theory: T-duality

T-duality can change topology and cause troubles

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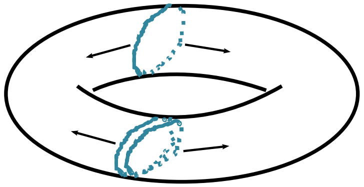

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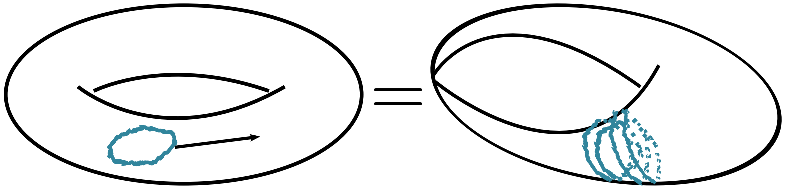

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Better Object: flux $H = dB$

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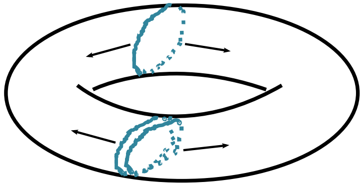

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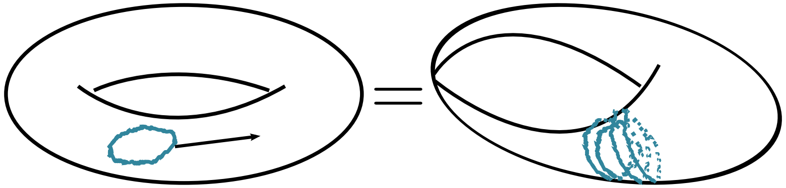

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T-duality relates backgrounds with different metric and flux; e.g. for 3-torus

$$H_{abc} \xleftrightarrow{T_1} f^a{}_{bc} \xleftrightarrow{T_2} Q^{ab}{}_c \xleftrightarrow{?} R^{abc}$$

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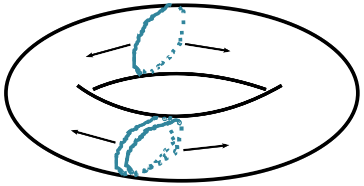

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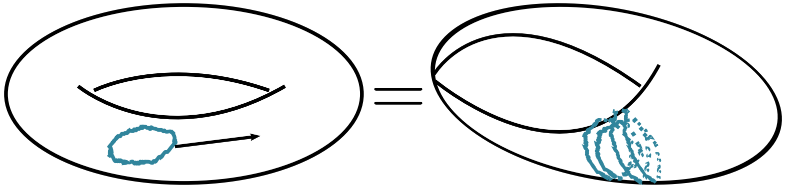

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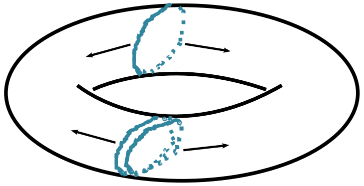

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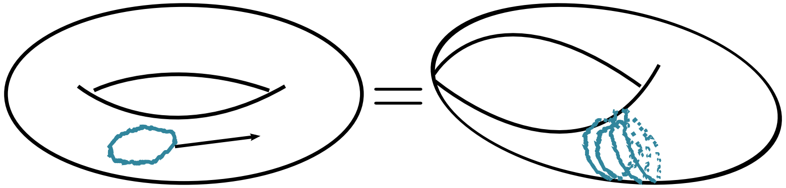

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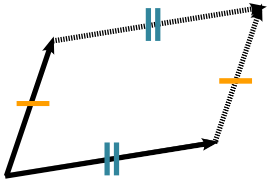
torus



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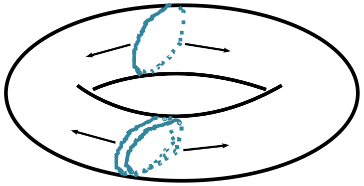

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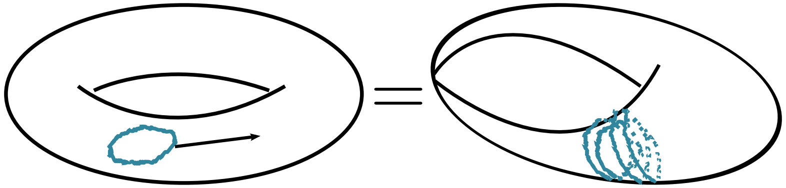

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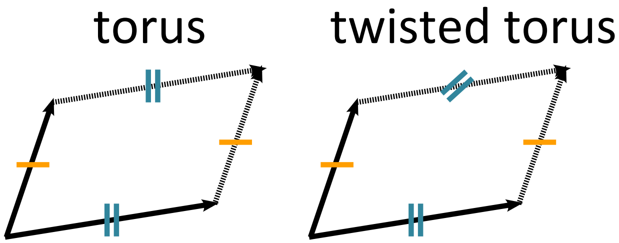

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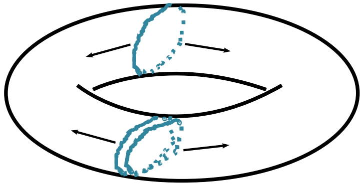

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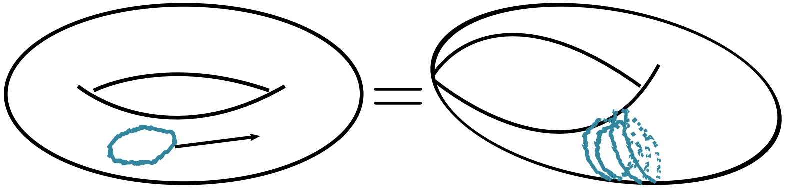

Very different spacetimes can give same theory: T-duality

T-duality can change topology and cause troubles

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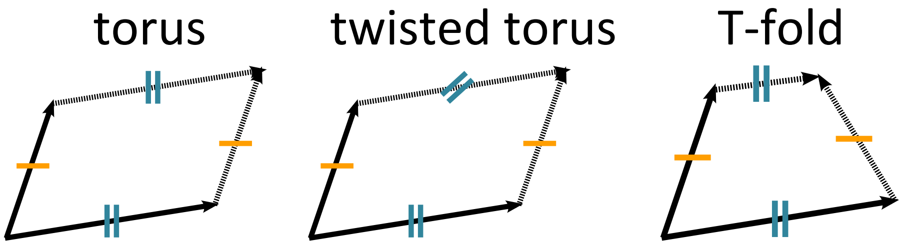

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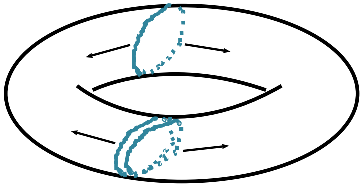

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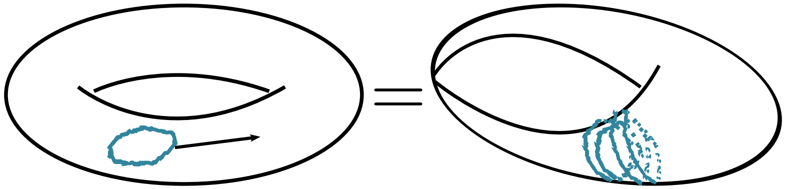

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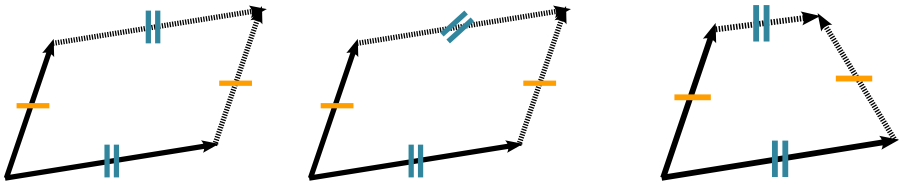
torus twisted torus T-fold unknown



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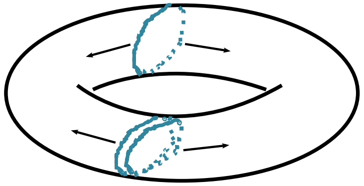

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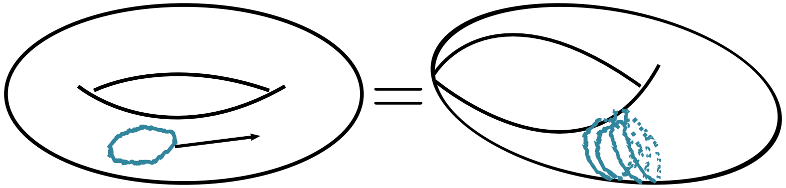

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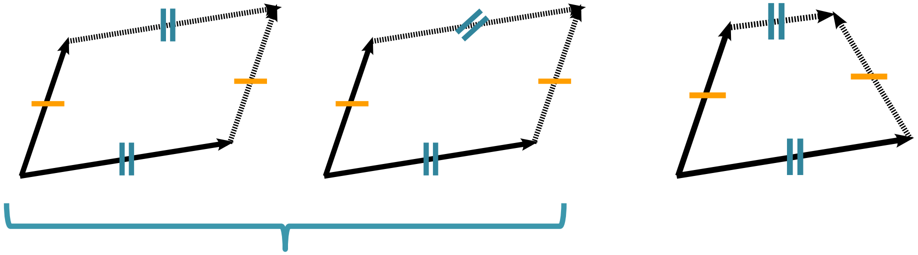

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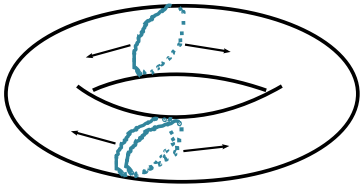

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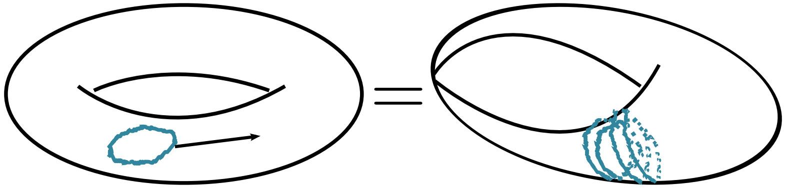

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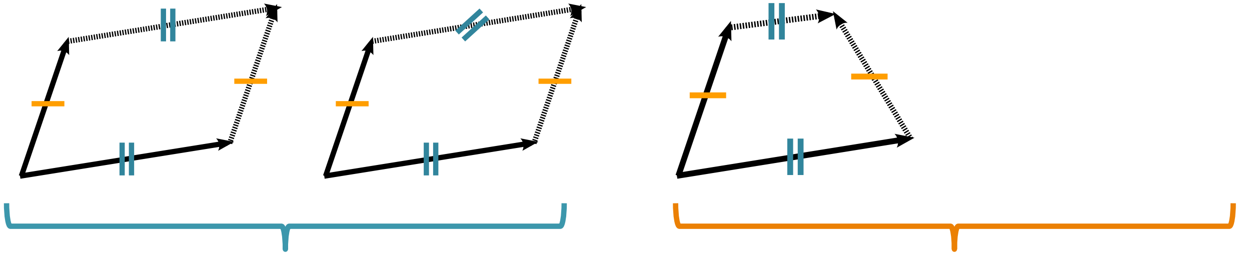

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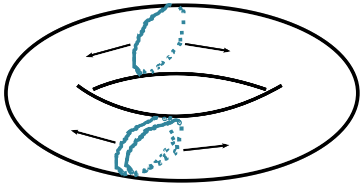

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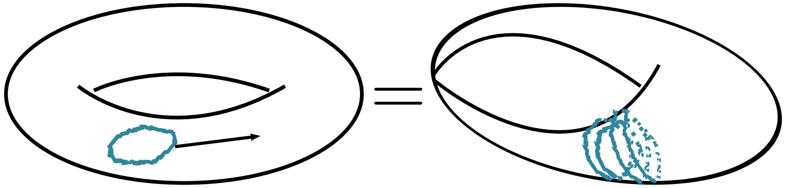

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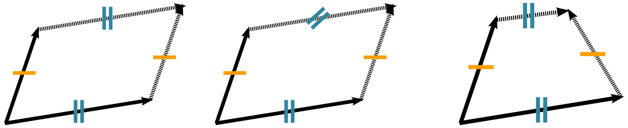

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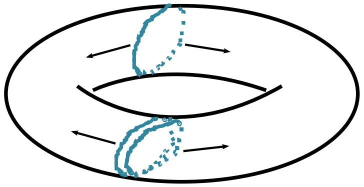

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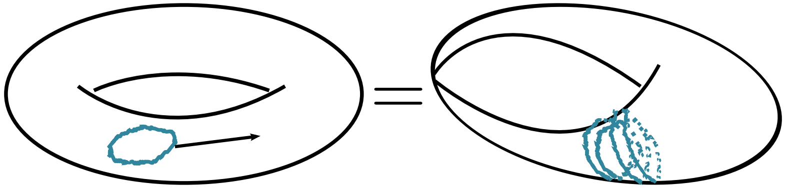

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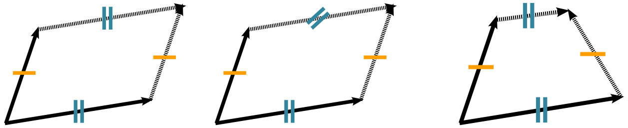

Non-geometric backgrounds

Closed string theory for small energies is

$$S^d_{\text{eff}}(G,B) = \frac{1}{2\kappa^2}\int d^dx\sqrt{|G|}e^{-2\phi}\left(R - \tfrac{1}{12}H_{abc}\,H^{abc}\right) + \ldots$$

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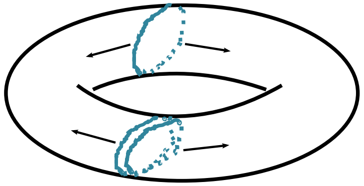

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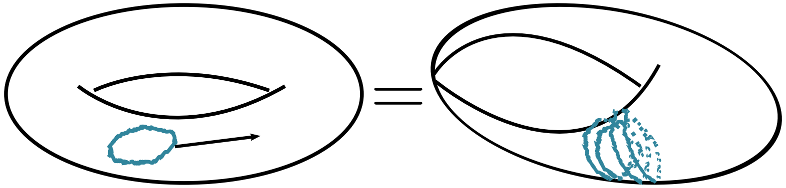

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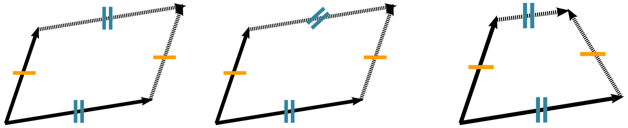

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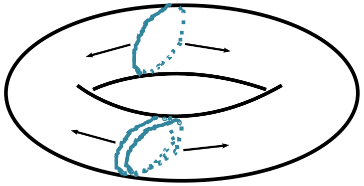

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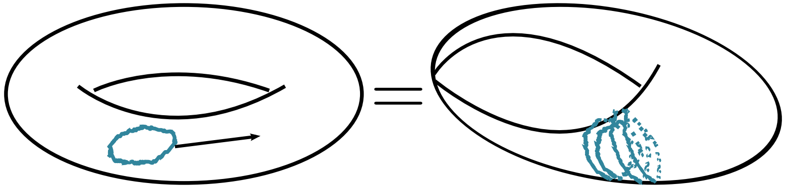

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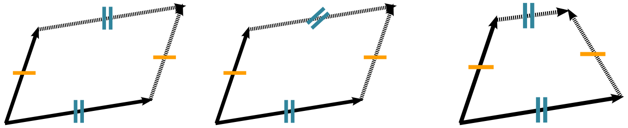

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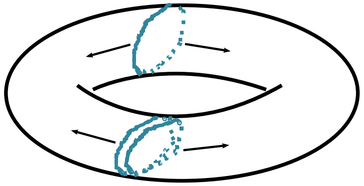
Ricci scalar

H-flux

$$H = dB$$

MOTIVATION



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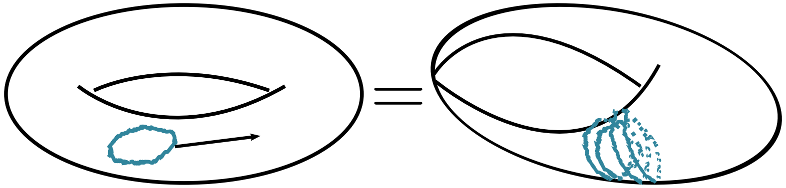

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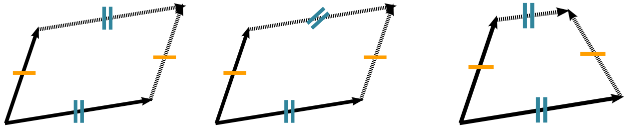

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The effective theory of a closed string is General relativity
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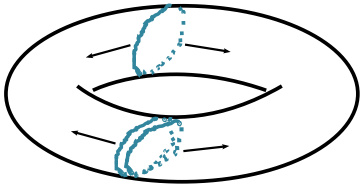

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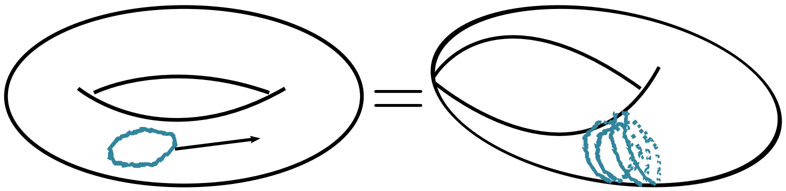

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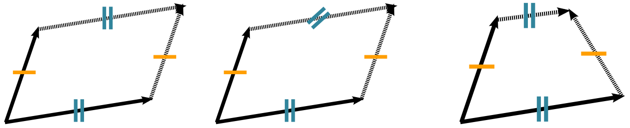

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T-duality and non-geometry can be understood via *O(d,d)-transformations*

all at once: GENERALIZED GEOMETRY

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All possible positions of indices

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$$[X,Y]_L^H = [X,Y]_L - \beta^\sharp(\iota_Y \iota_X H)$$

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$$[a,[b,c]] + [c,[a,b]] + [b,[c,a]] = 0$$

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$$\begin{aligned} [\![X+\xi, Y+\nu]\!] = \quad & [X,Y]^H_L + L^H_X \nu - \iota_\nu\, d^H_\beta X + \iota_Y \iota_X H \\ & + [\xi,\nu]^H_K + \mathcal{L}^H_\xi Y - \iota_Y\, d^H \xi + \iota_\eta \iota_\xi R \end{aligned}$$

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This defines a Courant algebroid

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Yields algebra

$$\begin{aligned} [\![e_a, e_b]\!] &= f^c{}_{ab}\, e_c + H_{abc}\, e^c \\ [\![e_a, e^b]\!] &= Q^{bc}{}_a\, e_c - f^b{}_{ac}\, e^c \\ [\![e^a, e^b]\!] &= R^{abc}\, e_c + Q^{ab}{}_c\, e^c \end{aligned}$$

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Standard geometry	generalized geometry
$([\cdot,\cdot]_L, TM, d)$	$(\llbracket\cdot,\cdot\rrbracket, TM \oplus T^*M, \mathcal{D})$

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There is a sub-structure allowing for gravity theory: Lie algebroids

O(d,d) and LIE ALGEBROIDS

A Lie algebroid is a generalization of a Lie algebra

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O(d,d) transformations act on (*G*,*B*) through generalized metric

$$\mathcal{H}(G,B) = \begin{pmatrix} G - BG^{-1}G & BG^{-1} \\ -G^{-1}B & G^{-1} \end{pmatrix}$$

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$$\mathcal{H}(\widetilde{G},\widetilde{B}) = \mathcal{H}'(G,B) = \mathcal{T}^t\mathcal{H}(G,B)\mathcal{T} \qquad \mathcal{T}^t\begin{pmatrix}0 & \mathbb{1}\\ \mathbb{1} & 0\end{pmatrix}\mathcal{T} = \begin{pmatrix}0 & \mathbb{1}\\ \mathbb{1} & 0\end{pmatrix}$$

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We found: for every *O(d,d)*-transformation there is a Lie algebroid
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T-duality as an *O(d,d)*-transformation: if we understand *O(d,d)* in terms Lie algebroids, connections between fluxes apparent

O(d,d) transformations act on (*G*,*B*) through generalized metric

We found: for every *O(d,d)*-transformation there is a Lie algebroid describing the resulting gravity theory

Non-geometric backgrounds can be patched-up using this construction

OUTLOOK

There is a nice geometric structure behind stringy fluxes

Various interesting routes:

- better understanding of non-geometry: classical aspects
  transformations
- conformal field theory approaches: T-duality simple
- non-associative geometry

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Thank You