# Higgs Boson Masses in the Complex MSSM

Sebastian Paßehr
in collaboration with Prof. Wolfgang Hollik

Max Planck Institute for Physics, Munich

Munich,
15th of November 2013

# content

# motivation

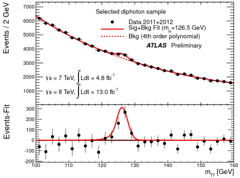
Why do we want a Higgs boson?

- explanation of masses of fundamental particles: explicit mass terms in the Lagrangian are forbidden by gauge and chiral symmetries, instead couplings of the Higgs fields to other fields, e. g.

$$\left(y_e \left(H_1 \cdot L_1\right) E_1^C\right)_{\mathrm{min}} = \frac{1}{2} m_e e_{\mathrm{L}} e_{\mathrm{R}},$$

- restore unitarity of the $S$ matrix,
- experimental hint:

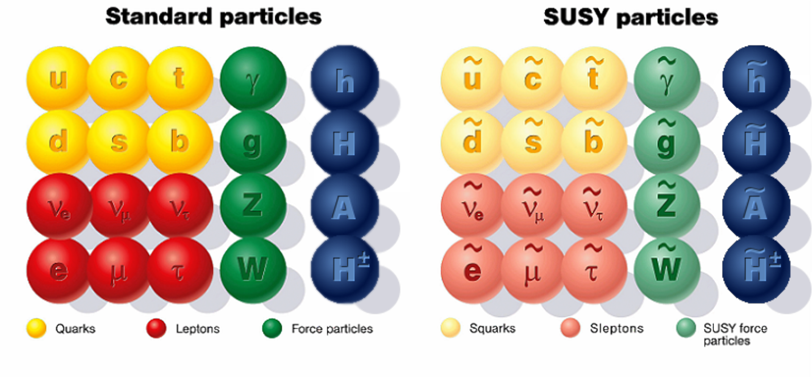
boson with a mass of
$(125.2 \pm 0.3(\mathrm{stat}) \pm 0.6(\mathrm{sys}))$ GeV,
could be a Higgs particle.




[Fleischmann, Epiphany 2013],

# MSSM particles

each fermion has a bosonic superpartner and
each boson has a fermionic superpartner



# Higgs fields in the MSSM

(at least) two complex $SU(2)$-Higgs doublets necessary,

$$H_1 = \begin{pmatrix} \frac{1}{\sqrt{2}}\left(v_1 + \phi_1^0 - \mathrm{i}\chi_1^0\right) \\ -\phi_1^- \end{pmatrix} \quad \text{and} \quad H_2 = \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}}\left(v_2 + \phi_2^0 - \mathrm{i}\chi_2^0\right) \end{pmatrix},$$

$\Rightarrow$ eight bosonic degrees of freedom:

3 Goldstone bosons: used up for gauge boson masses,

5 physical Higgs bosons,

vacuum expectation values $v_1$ and $v_2$, with $v_1^2 + v_2^2 = v^2$,

Higgs potential fixed by Lagrangian density in contrast to the SM:

$$V_{\text{Higgs}} = m_1^2 H_1^\dagger H_1 + m_2^2 H_2^\dagger H_2 - m_{12}^2 \left(H_1 \cdot H_2 + H_1^\dagger \cdot H_2^\dagger\right) + \frac{1}{8}\left(g_1^2 + g_2^2\right)\left(H_2^\dagger H_2 - H_1^\dagger H_1\right)^2 + \frac{1}{2} g_2^2 H_1^\dagger H_1 H_2^\dagger H_2,$$

# Higgs bosons

mass eigenstates (tree level):

$$\begin{pmatrix} H^0 \\ h^0 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \phi_1^0 \\ \phi_2^0 \end{pmatrix},$$

$$\begin{pmatrix} G^0 \\ A^0 \end{pmatrix} = \begin{pmatrix} \cos\beta_n & \sin\beta_n \\ -\sin\beta_n & \cos\beta_n \end{pmatrix} \begin{pmatrix} \chi_1^0 \\ \chi_2^0 \end{pmatrix},$$

$$\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \begin{pmatrix} \cos\beta_c & \sin\beta_c \\ -\sin\beta_c & \cos\beta_c \end{pmatrix} \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix},$$

| | |
|---|---|
| Goldstone bosons | $G^0, G^\pm$, |
| physical *CP* even bosons | $h^0, H^0$, |
| physical *CP* odd boson | $A^0$, |
| physical charged Higgs bosons | $H^\pm$. |

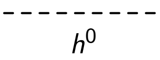
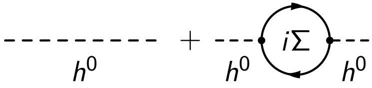
# Higgs boson masses

tree-level masses correlated by the potential:

$$m^2_{H^0,h^0} = \frac{1}{2}\left(m^2_{A^0} + m^2_Z \pm \sqrt{\left(m^2_{A^0} + m^2_Z\right)^2 - (2 m_Z m_{A^0} \cos 2\beta)^2}\right),$$
$$m^2_{H^\pm} = m^2_{A^0} + m^2_W,$$

two free parameters:
conventionally $\tan\beta = \frac{v_2}{v_1}$, real case: $m_{A^0}$, complex case: $m_{H^\pm}$

theoretical upper bound: $m^2_{h^0} \leq (m_Z \cos 2\beta)^2$ .

| | tree level | tree level + one-loop level |
|---|---|---|
| |  |  |
| propagator: | $D(p^2) = \frac{1}{p^2 - m^2}$ | $D(p^2) = \frac{1}{p^2 - m^2 + \hat{\Sigma}(p^2)}$ |
| pole mass: | $\left[p^2 - m^2\right]_{p^2 = m_\mathrm{p}^2} \overset{!}{=} 0$ | $\left[p^2 - m^2 + \hat{\Sigma}(p^2)\right]_{p^2 = m_\mathrm{p}^2} \overset{!}{=} 0$ |

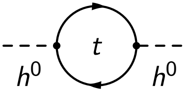




$\Rightarrow$ pole mass is shifted by $\hat{\Sigma}\left(m_\mathrm{p}^2\right)$

# regularization and renormalization

in general loop integrals are divergent




$$\propto \int_{\mathbb{R}^4} \mathrm{d}^4 k \, \frac{1}{\not{k} - m_t} \, \frac{1}{\not{k} - m_t}$$

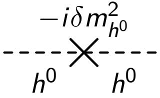
$$\propto \left[ k^2 \right]_0^\infty + \ldots \xrightarrow[|k| \to \infty]{} \infty$$

1st step: regularize integrals—parametrize divergent parts

2nd step: perform renormalization transformation—absorb divergencies

new Feynman diagrams are added,

$\hat{\Sigma} = \Sigma - \delta m_{h^0}^2$




3rd step: apply a consistent renormalization scheme —calculate renormalization constants

# Higgs boson masses at higher orders

real parameters:

$$\mathcal{M} = \begin{pmatrix} m_{h^0}^2 - \hat{\Sigma}_{h^0 h^0} & \\ & m_{H^0}^2 - \hat{\Sigma}_{H^0 H^0} \end{pmatrix}, \quad \hat{\Sigma}_{A^0 A^0} := 0,$$

# Higgs boson masses at higher orders

real parameters:

$$\mathcal{M} = \begin{pmatrix} m_{h^0}^2 - \hat{\Sigma}_{h^0 h^0} & - \hat{\Sigma}_{h^0 H^0} \\ - \hat{\Sigma}_{H^0 h^0} & m_{H^0}^2 - \hat{\Sigma}_{H^0 H^0} \end{pmatrix}, \quad \hat{\Sigma}_{A^0 A^0} := 0,$$

# Higgs boson masses at higher orders

complex parameters $(\phi_{A_t}, \phi_\mu, \dots)$:

$$\mathcal{M} = \begin{pmatrix} m_{h^0}^2 - \hat{\Sigma}_{h^0 h^0} & -\hat{\Sigma}_{h^0 H^0} & -\hat{\Sigma}_{h^0 A^0} \\ -\hat{\Sigma}_{H^0 h^0} & m_{H^0}^2 - \hat{\Sigma}_{H^0 H^0} & -\hat{\Sigma}_{H^0 A^0} \\ -\hat{\Sigma}_{A^0 h^0} & -\hat{\Sigma}_{A^0 H^0} & m_{A^0}^2 - \hat{\Sigma}_{A^0 A^0} \end{pmatrix}, \quad \hat{\Sigma}_{H^\pm H^\pm} := 0,$$

(most general case also includes longitudinal $G^0$ and $Z$),

# Higgs boson masses at higher orders

complex parameters $(\phi_{A_t}, \phi_\mu, \dots)$:

$$\mathcal{M} = \begin{pmatrix} m_{h^0}^2 - \hat{\Sigma}_{h^0 h^0} & -\hat{\Sigma}_{h^0 H^0} & -\hat{\Sigma}_{h^0 A^0} \\ -\hat{\Sigma}_{H^0 h^0} & m_{H^0}^2 - \hat{\Sigma}_{H^0 H^0} & -\hat{\Sigma}_{H^0 A^0} \\ -\hat{\Sigma}_{A^0 h^0} & -\hat{\Sigma}_{A^0 H^0} & m_{A^0}^2 - \hat{\Sigma}_{A^0 A^0} \end{pmatrix}, \quad \hat{\Sigma}_{H^\pm H^\pm} := 0,$$

(most general case also includes longitudinal $G^0$ and $Z$),

meaning of $\hat{\Sigma}$:

$$\hat{\Sigma} = \Sigma\left(p^2\right) - \delta m^2 = \hat{\Sigma}\left(p^2\right),$$

$$\hat{\Sigma}\left(p^2\right) = \hat{\Sigma}^{\text{one loop}}\left(p^2\right) + \hat{\Sigma}^{\text{two loop}}\left(p^2\right) + \dots$$

# new mass eigenstates

$$\mathcal{M} = \begin{pmatrix} m^2_{h^0} - \hat{\Sigma}_{h^0 h^0} & -\hat{\Sigma}_{h^0 H^0} & -\hat{\Sigma}_{h^0 A^0} \\ -\hat{\Sigma}_{H^0 h^0} & m^2_{H^0} - \hat{\Sigma}_{H^0 H^0} & -\hat{\Sigma}_{H^0 A^0} \\ -\hat{\Sigma}_{A^0 h^0} & -\hat{\Sigma}_{A^0 H^0} & m^2_{A^0} - \hat{\Sigma}_{A^0 A^0} \end{pmatrix}$$

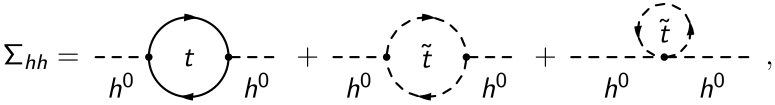
new mass eigenstates $h_1, h_2, h_3$ induced by off-diagonal elements

corresponding masses given by solutions of

$$\det\left[p^2\mathbb{I} - \mathcal{M}\left(p^2\right)\right] = 0.$$

# one-loop corrections

- main contributions come from $t$ and $\tilde{t}$ loops;
  order $\alpha_t$, but proportional to $m_t^4$:




- additional parameters: $\mu, A_t, m_{\tilde{t}_L}, m_{\tilde{t}_R},$
- mass contribution of ca. 40% of tree-level result possible,
- uncertainty of the calculation still too big.

# two-loop corrections

most important parts:
corrections to $m_t$-enhanced one-loop contributions
in a gauge-less limit,

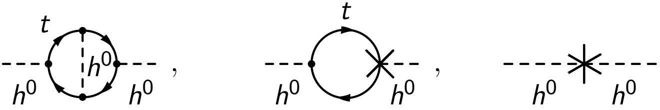
- corrections by gluons and gluinos already known,
  order $\alpha_t \alpha_s$ in an on-shell scheme,

  [Heinemeyer, Hollik, Rzehak, Weiglein, arXiv:hep-ph/0705.0746, 2007],
- corrections by Higgs and Higgsinos already known
  in the real MSSM in the effective potential approach,
  order $\alpha_t^2$ in a $\overline{\text{DR}}$ scheme,

  [Brignole, Degrassi, Slavich, Zwirner, arXiv:hep-ph/0112177, 2002],
- corrections by Higgs and Higgsinos
  in the case of the complex MSSM: in process.

# order $\alpha_t^2$

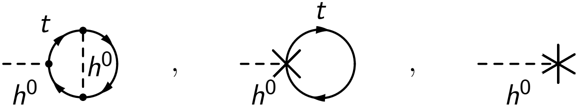
- again: enhancement by additional $m_t^2$,
- Feynman-diagrammatic approach:




(two-loop) $\quad$ (one-loop) $\cdot\,(\delta^{(1)})$, $\quad (\delta^{(2)}) + (\delta^{(1)})^2$,

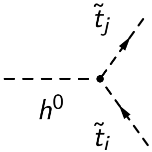
$+$(one-loop)$^2$,

- $(\delta^{(2)}) + (\delta^{(1)})^2$ acquired by renormalizing the Higgs potential, additional Feynman diagrams (tadpoles) necessary:



# complex parameters

complex parameters appear in couplings of Higgs bosons to stops, e. g.




$$= \cdots + A_t(\dots) + \mu(\dots) + A_t^*(\dots) + \mu^*(\dots),$$

stop masses $\mathrm{diag}\left(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2\right) = \mathbf{U}_{\tilde{t}} \mathbf{M}_{\tilde{t}} \mathbf{U}_{\tilde{t}}^\dagger$:

$$\mathbf{M}_{\tilde{t}} = \begin{pmatrix} m_{\tilde{t}_L}^2 + m_t^2 + M_Z^2 \cos 2\beta \left(T_t^3 - Q_t s_w^2\right) & m_t \left(A_t^* - \mu / \tan\beta\right) \\ m_t \left(A_t - \mu^* / \tan\beta\right) & m_{\tilde{t}_R}^2 + m_t^2 + M_Z^2 \cos 2\beta \, Q_t s_w^2 \end{pmatrix}$$

# applied approximations

(similar as for $\alpha_t \alpha_s$ corrections)

1. gauge-less limit: $g_1 = 0$, $g_2 = 0$,
   - only Yukawa-couplings left,
   - $m_W = 0$, $m_Z = 0$,
   - $m_{h^0} = 0$, $m_{G^0} = 0$, $m_{G^\pm} = 0$, $m_{H^0} = m_{H^\pm}$, $m_{A^0} = m_{H^\pm}$,
   - $m_{\tilde{\chi}^0_3} = -\mu$, $m_{\tilde{\chi}^0_4} = \mu$, $m_{\tilde{\chi}^\pm_2} = \mu$,
   - other Charginos and Neutralinos decouple,
   - Higgs mixing angle $\alpha = \beta - \frac{\pi}{2}$,
2. external momentum equal to zero,
   - only two-loop vacuum diagrams; known analytically,
   - renormalisation constants for genuine two-loop counterterms calculated at zero momentum,
   - finding poles of propagator simplifies,
3. bottom mass equal to zero,
   - no mixing in sbottom sector,
   - one sbottom (w. l. o. g. $\tilde{b}_2$) decouples,
   - $m^2_{\tilde{b}_1} = m^2_{\tilde{t}_1} - m^2_t$.

# renormalisation scheme

required renormalisation constants:

1. at one-loop:
   - $\delta m_t$, $\delta m_{\tilde{t}_1}$ and $\delta m_{\tilde{t}_2}$ fixed by on-shell condition,
   - $\delta m_{\tilde{b}_1}$ dependent on top-stop-sector,
   - $\delta A_t$ fixed by on-shell condition for mixing of stops,
   - $\delta\mu$ fixed by on-shell condition for $\tilde{\chi}_2^\pm$ or defined in $\overline{\text{DR}}$ scheme,
   - Higgs field renormalisation constants $\delta Z_{H_1}\big|_{\text{div.}}$ and $\delta Z_{H_2}\big|_{\text{div.}}$ defined in $\overline{\text{DR}}$ scheme,
   - $\delta \tan\beta = \frac{\tan\beta}{2}(\delta Z_{H_2} - \delta Z_{H_1})\big|_{\text{div.}}$ defined in $\overline{\text{DR}}$ scheme,
2. at one-loop and two-loop:
   - tadpoles $\delta t_{h^0}, \delta t_{H^0}, \delta t_{A^0}$ fixed by on-shell conditions,
   - $\delta m_{H^\pm}$ fixed by on-shell condition,
   - $\delta m_{h^0}, \delta m_{H^0}, \delta m_{A^0}$ and $\delta m_{h^0 H^0}, \delta m_{h^0 A^0}, \delta m_{H^0 A^0}$ dependent on tadpoles and $\delta m_{H^\pm}$.

# procedure of calculation

- creation of Feynman diagrams and amplitudes with FeynArts, [Hahn, arXiv:hep-ph/0012260, 2001],
- applying approximations,
- reducing one-loop diagrams to master integrals with FormCalc, [Hahn, arXiv:hep-ph/0901.1528, 2009],
- reducing two-loop diagrams to master integrals with TwoCalc, [Weiglein, Scharf, Böhm, arXiv:hep-ph/9310358, 1993],
- creating counterterms from the Higgs potential,
- applying renormalisation scheme,
- evaluating renormalisation constants with FeynArts and FormCalc,
- expanding master integrals, simplifying result.

# status and outlook

current status:

- all Feynman diagrams are generated and calculated,
- renormalisation constants for complex parameters determined,
- all divergencies cancel,
- already existing result for real parameters confirmed,

outlook:

- inclusion into FeynHiggs,

  [Frank, Hahn, Heinemeyer, Hollik, Rzehak, Weiglein, JHEP 0702, 2007]

  [Degrassi, Heinemeyer, Hollik, Slavich, Weiglein, Eur. Phys. J. C 28, 2003]

  [Heinemeyer, Hollik, Weiglein, Eur. Phys. J. C 9, 1999],
- investigate influence of external momentum.