Gravity = Lots of things[i]

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IMPRS Colloqium

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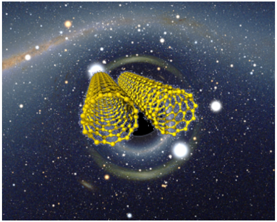
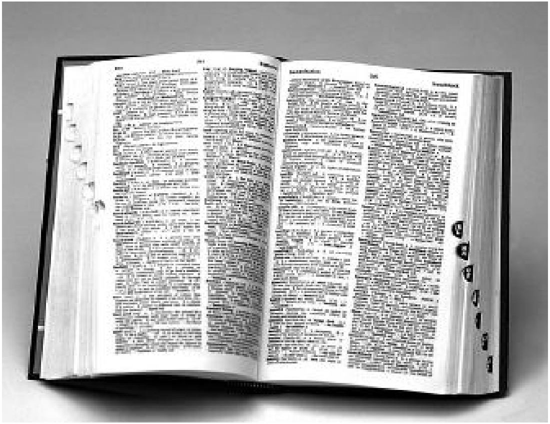


[i]Yet another attempt to sell string theory as a useful piece of maths by means of hope and phantasy.

Outline

1. Present AdS/CFT both in an illustrative and in a rather technical way.
2. Explain why people work on AdS/CFT.
3. Explain how (some) people work on AdS/CFT.
4. Provide concrete example.

What is AdS/CFT about?



AdS/CFT in a nutshell

Gravity + Susy = Supergravity 11d

LOW ENERGY LIMIT

$\Downarrow$

Get M-Theory supergravity multiplet.

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Here have D-branes. Interesting D3-branes

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find geometric structure of SU(N) group

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Stack of D3 branes gravitates Solve supergravity eoms and find resulting metric: We find

$$g = g_{AdS5} + g_{S5}.$$

AdS/CFT in a nutshell

Equivalent descriptions

Two different descriptions of one and the same thing provide two analogies.

$$A = B \quad \& \quad A = C \Rightarrow B = C$$

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AdS/CFT conjecture

$\mathcal{N} = 4$ $SU(N)$ SYM Conformal field theory	$=$	Type IIB superstring theory on $AdS_5 \times S^5$

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AdS/CFT conjecture

$$\begin{matrix} \mathcal{N} = 4\ SU(N) \text{ SYM} \\ \text{Conformal field theory} \end{matrix} \quad = \quad \begin{matrix} \text{Type IIB superstring theory} \\ \text{on } AdS_5 \times S^5 \end{matrix}$$

Matching of parameters

Which point of view is mathematically tractable depends on whether the gauge theory is strongly or weakly coupled:

GRAVITY THEORY	GAUGE THEORY
☺ easy	☹ hard
☹ hard	☺ easy

AdS/CFT in a nutshell

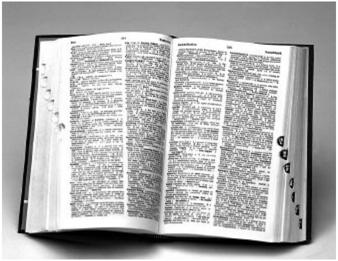
Take limit to make life simpler:

Easy and cool version of AdS/CFT

Einstein gravity		$\mathcal{N} = 4$ $SU(N)$ SYM
(consistent truncation)	$=$	$N \to \infty$, strongly coupled
AdS 5d		∂AdS 4d

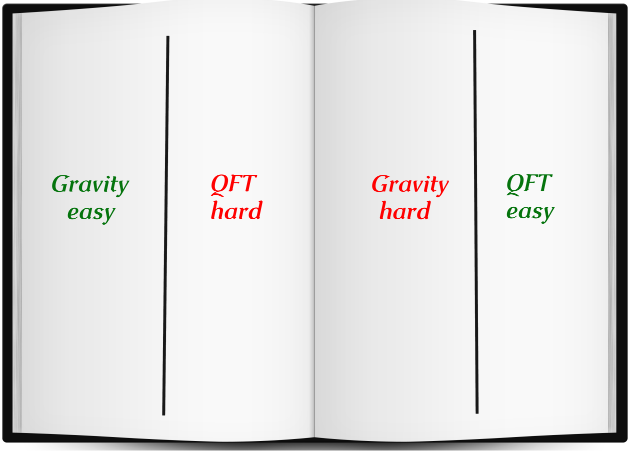
AdS/CFT dictionary

GRAVITY	GAUGE
A_μ	J^μ
$g_{\mu\nu}$	$T^{\mu\nu}$



AdS/CFT: bottomline

If nature provides such a beautiful dictionary, why not using it!?



AdS/CFT:how things are done

There are two possible ways:

Top-down

GRAVITY THEORY $\Longrightarrow$ QFT

Bottom-up

QFT $\Longrightarrow$ GRAVITY THEORY

AdS/CFT Bottom-up: Model building

We may try to model different phenomena, such as superconductivity, superfluidity, hydrodynamics, Kondo effect, quantum Hall effect,...

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Holographic bottom-up study of interfaces

1. Select a set up (background + add-ons):

$$g_{\mu\nu} = g_{\mu\nu}^{(AdS_5 \times S^5\ BH)} \qquad \text{(spacetime)}$$

$$A_\mu \qquad (\mu, \vec{E}, \vec{B})$$

	t	x	y	z	ρ	s_1	s_2	s_3	s_4	χ
D3	×	×	×	×						
D5	×	×	×		×	×	×			

2. Solve equations of motion.
3. Analyse results to extract information.

Equations of motion

In the probe approximation, the dynamics is given by the DBI action:

$$S \propto \int d^6x \sqrt{-\det(g+F])}$$

In our concrete case

$$S = \frac{N_5}{4\pi^2} R^3 \rho^2 f \sqrt{h(1-\chi^2)(S_\chi + S_A + S_{\text{int}})}$$

$$S_\chi = 1 - \chi^2 + \rho^2 \dot{\chi}^2 + \frac{\chi'^2}{hR^2\rho^2}$$

$$S_A = -\frac{(1-\chi^2)}{f^2R^2}\left(\dot{A}^2 h + \frac{A'^2}{R^2\rho^4}\right)$$

$$S_{\text{int}} - \frac{(\dot{\chi}A' - \chi'\dot{A})^2}{f^2R^4\rho^2}$$

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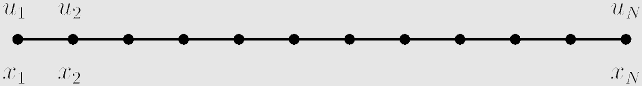
EOMs to solve

```
EqA =
  Collect[
   (2 R^2 ρ^3 f[ρ] h[ρ] f'[ρ] (χ^(0,1)[ρ, x] (-χ^(0,1)[ρ, x] A0^(1,0)[ρ, x] + 2 A0^(0,1)[ρ, x] χ^(1,0)[ρ, x]) + R^2 ρ^2 h[ρ] A0^(1,0)[ρ, x] (-1 + χ[ρ, x]^2 + ρ^2 χ^(1,0)[ρ, x]^2)) +
      h[ρ] (4 R^2 ρ^4 h[ρ]^2 (-1 + χ[ρ, x]^2) A0^(1,0)[ρ, x]^3 -
         ρ h'[ρ] A0^(1,0)[ρ, x] (ρ^2 χ^(0,1)[ρ, x]^2 A0^(1,0)[ρ, x]^2 - 2 ρ^2 A0^(0,1)[ρ, x] χ^(0,1)[ρ, x] A0^(1,0)[ρ, x] χ^(1,0)[ρ, x] + A0^(0,1)[ρ, x]^2 (3 - 3 χ[ρ, x]^2 + ρ^2 χ^(1,0)[ρ, x]^2)) +
         2 h[ρ] (ρ A0^(1,0)[ρ, x]^2 ((-1 + χ[ρ, x]^2) A0^(0,2)[ρ, x] + ρ (R^2 ρ^3 (-1 + χ[ρ, x]^2) h'[ρ] - χ^(0,1)[ρ, x]^2) A0^(1,0)[ρ, x]) +
            2 ρ A0^(0,1)[ρ, x] A0^(1,0)[ρ, x] (ρ χ^(0,1)[ρ, x] A0^(1,0)[ρ, x] χ^(1,0)[ρ, x] - (-1 + χ[ρ, x]^2) A0^(1,1)[ρ, x]) +
            A0^(0,1)[ρ, x]^2 (A0^(1,0)[ρ, x] (-4 + 4 χ[ρ, x]^2 - ρ^2 χ^(1,0)[ρ, x]^2) + ρ (-1 + χ[ρ, x]^2) A0^(2,0)[ρ, x]))) +
      R^2 ρ f[ρ]^2 (2 ρ^2 h'[ρ] χ^(0,1)[ρ, x]^2 A0^(1,0)[ρ, x] + 2 R^2 ρ^3 h[ρ]^2 (A0^(1,0)[ρ, x] (2 - 2 χ[ρ, x]^2 + 3 ρ^2 χ^(1,0)[ρ, x]^2) - ρ (-1 + χ[ρ, x]^2) A0^(2,0)[ρ, x]) +
         h[ρ] (A0^(0,2)[ρ, x] (2 - 2 χ[ρ, x]^2 + 2 ρ^2 χ^(1,0)[ρ, x]^2) +
            ρ (R^2 ρ^3 h'[ρ] A0^(1,0)[ρ, x] (3 - 3 χ[ρ, x]^2 + ρ^2 χ^(1,0)[ρ, x]^2) + 2 χ^(0,1)[ρ, x] (2 χ^(1,0)[ρ, x] (2 A0^(0,1)[ρ, x] - ρ A0^(1,1)[ρ, x]) +
                  χ^(0,1)[ρ, x] (A0^(1,0)[ρ, x] + ρ A0^(2,0)[ρ, x])))))) /. {R → 1}, ρ, Simplify];
EqC =
  Collect[
   (R^3 ρ^2 (-1 + χ[ρ, x]^2) (2 R^2 ρ^4 f[ρ] h[ρ] f'[ρ] χ^(1,0)[ρ, x] (χ^(0,1)[ρ, x]^2 + R^2 ρ^2 h[ρ] (1 - χ[ρ, x]^2 + ρ^2 χ^(1,0)[ρ, x]^2)) +
        h[ρ] (4 R^2 ρ^4 h[ρ]^2 (-1 + χ[ρ, x]^2) A0^(1,0)[ρ, x]^2 (χ[ρ, x] + ρ χ^(1,0)[ρ, x]) -
           ρ^2 h'[ρ] (ρ^2 χ^(0,1)[ρ, x]^2 A0^(1,0)[ρ, x]^2 χ^(1,0)[ρ, x] + A0^(0,1)[ρ, x]^2 χ^(1,0)[ρ, x] (1 - χ[ρ, x]^2 + ρ^2 χ^(1,0)[ρ, x]^2) -
              2 A0^(0,1)[ρ, x] χ^(0,1)[ρ, x] A0^(1,0)[ρ, x] (-1 + χ[ρ, x]^2 + ρ^2 χ^(1,0)[ρ, x]^2)) +
           2 h[ρ] (2 χ[ρ, x]^3 A0^(0,1)[ρ, x]^2 - χ[ρ, x] (3 ρ^2 χ^(0,1)[ρ, x]^2 A0^(1,0)[ρ, x]^2 - 6 ρ^2 A0^(0,1)[ρ, x] χ^(0,1)[ρ, x] A0^(1,0)[ρ, x] χ^(1,0)[ρ, x] +
                 A0^(0,1)[ρ, x]^2 (2 + 3 ρ^2 χ^(1,0)[ρ, x]^2)) + ρ χ[ρ, x]^2 (ρ A0^(1,0)[ρ, x]^2 (χ^(0,2)[ρ, x] + R^2 ρ^4 h'[ρ] χ^(1,0)[ρ, x]) +
                 2 A0^(0,1)[ρ, x] A0^(1,0)[ρ, x] (χ^(0,1)[ρ, x] - ρ χ^(1,1)[ρ, x]) + A0^(0,1)[ρ, x]^2 (2 χ^(1,0)[ρ, x] + ρ χ^(2,0)[ρ, x])) -
              ρ (ρ A0^(1,0)[ρ, x]^2 (χ^(0,2)[ρ, x] + ρ (R^2 ρ^3 h'[ρ] + χ^(0,1)[ρ, x]^2) χ^(1,0)[ρ, x]) - 2 A0^(0,1)[ρ, x] A0^(1,0)[ρ, x] (χ^(0,1)[ρ, x] (-1 + ρ^2 χ^(1,0)[ρ, x]^2) + ρ χ^(1,1)[ρ, x]) +
                 A0^(0,1)[ρ, x]^2 (2 χ^(1,0)[ρ, x] + ρ^2 χ^(1,0)[ρ, x]^3 + ρ χ^(2,0)[ρ, x])))) +
        R^2 ρ^2 f[ρ]^2
         (2 ρ^2 h'[ρ] χ^(0,1)[ρ, x]^2 χ^(1,0)[ρ, x] + 2 R^2 ρ^2 h[ρ]^2 (-2 χ[ρ, x]^3 + χ[ρ, x] (2 + 3 ρ^2 χ^(1,0)[ρ, x]^2) - ρ χ[ρ, x]^2 (4 χ^(1,0)[ρ, x] + ρ χ^(2,0)[ρ, x]) +
              ρ (4 χ^(1,0)[ρ, x] + 3 ρ^2 χ^(1,0)[ρ, x]^3 + ρ χ^(2,0)[ρ, x])) +
           h[ρ] (6 χ[ρ, x] χ^(0,1)[ρ, x]^2 - χ[ρ, x]^2 (2 χ^(0,2)[ρ, x] + R^2 ρ^4 h'[ρ] χ^(1,0)[ρ, x]) + 2 χ^(0,2)[ρ, x] (1 + ρ^2 χ^(1,0)[ρ, x]^2) +
              ρ (R^2 ρ^3 h'[ρ] χ^(1,0)[ρ, x] (1 + ρ^2 χ^(1,0)[ρ, x]^2) + 2 χ^(0,1)[ρ, x] (-2 ρ χ^(1,0)[ρ, x] χ^(1,1)[ρ, x] + χ^(0,1)[ρ, x] (5 χ^(1,0)[ρ, x] + ρ χ^(2,0)[ρ, x]))))))) /. {R → 1},
   ρ, Simplify];
```

Solve PDEs: numerics

The analytic approach is not feasable so we resort to numerics.

Discretization



- Discretize spacetime.
- Functions take values at grid points (seed).
- **Derivatives are substituted by operators (matrices).**
- Once equation has turned into numbers use favourite resolution method (Newton-Raphson).

Differentiation matrices

Easy example

3 grid points: x_1, x_2, x_3
2nd order finite difference for derivative: $x_i' = \frac{x_{i+1} - x_{i-1}}{2h}$
Boundary conditions: periodic.

$$\begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} = \frac{1}{h} \begin{pmatrix} 0 & 1/2 & -1/2 \\ -1/2 & 0 & 1/2 \\ 1/2 & -1/2 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \tag{1}$$

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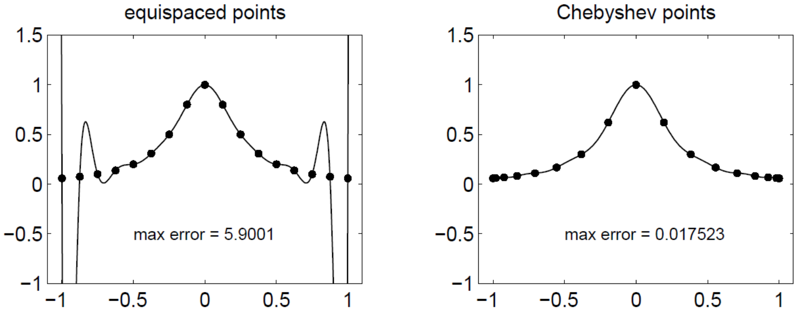
Spectral methods

Use unevenly distributed points: Chebyshev points.

$$x_j = \cos(j\pi/N) \qquad j = 0, 1, ...N$$

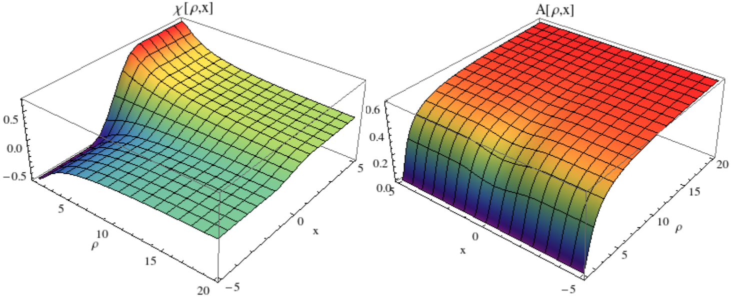
Use N-th order polynomial interpolation.

Even vs uneven

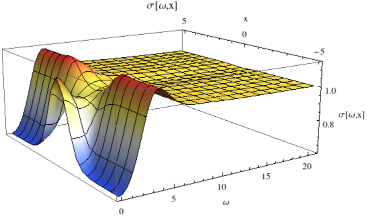


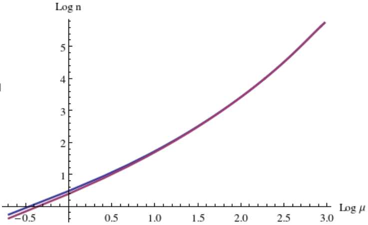
Put machinery to work!

1. Choose your set-up.
2. Numerize
3. Get results: functions, perturbations,...
4. Look for interpretation



Interpretation





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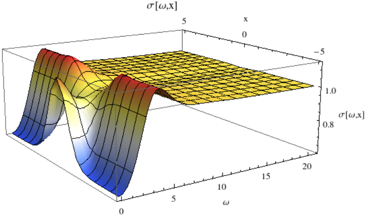

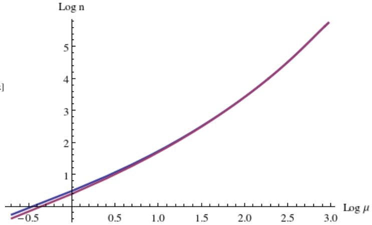

Now try to find and find analogies between your model and some other!
This is how analogies and possibly new insights arise:

- Heavy ion collisions
- Superconductor physics (high Tc)
- Superfluidity
- Interface Physics
- Topological insulators


"That's all Folks!"