# Radiative Generation of Masses and Mixing Angles of the Standard Model

[arXiv:1403.2382]

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IMPRS Colloquium, May 2014

# Outline


Introduction

The 2HDM

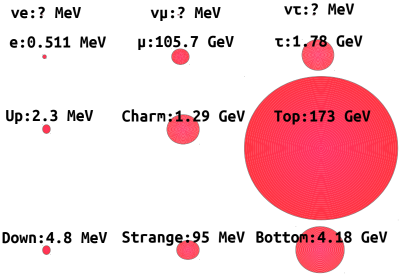
The Quark Sector

The Lepton Sector

Conclusions

# Introduction

- **Motivation:** Hierarchy for masses and mixing angles in the Standard Model. $\Rightarrow$ **New Physics?**

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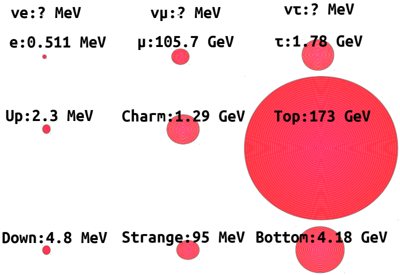


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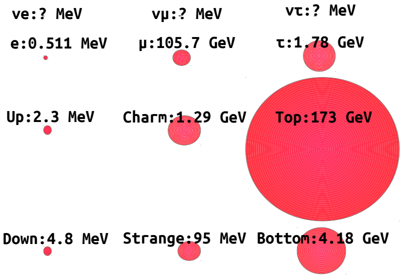


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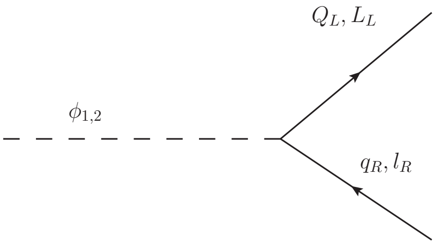

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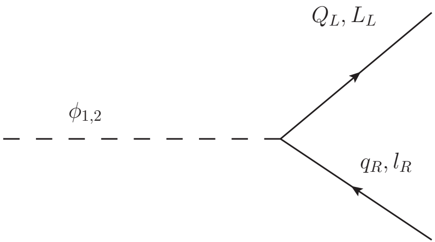


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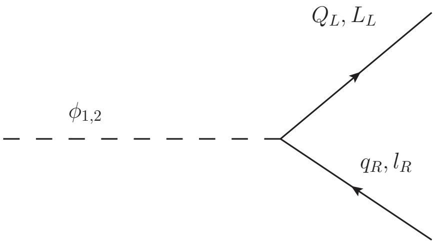


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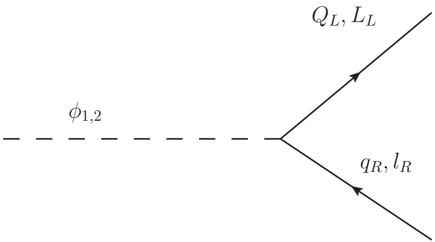


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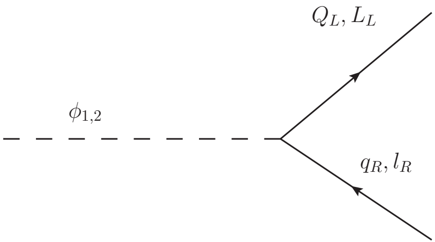


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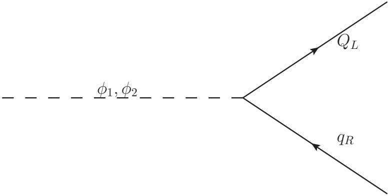

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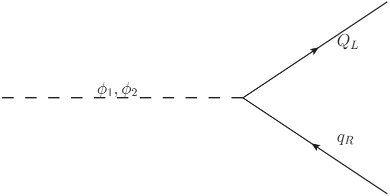


$$Y_u^{(1)}|_{\text{tree}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_u^{(1)} \end{pmatrix} , \quad Y_d^{(1)}|_{\text{tree}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \epsilon y_d^{(1)} \\ 0 & 0 & y_d^{(1)} \end{pmatrix} \to |V_{cb}| \ll 1 \Rightarrow \boldsymbol{\epsilon \to 0}$$

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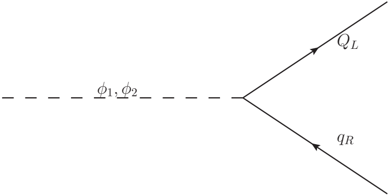


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- $Y_u^{(1)}|_{\text{tree}}$ & $Y_d^{(1)}|_{\text{tree}} \Rightarrow \begin{cases} m_t \\ m_b \end{cases}$ **@ tree level**

$$Y_u^{(2)}|_{\text{tree}} = U_L^\dagger \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_u^{(2)} \end{pmatrix} U_R \ , \quad Y_d^{(2)}|_{\text{tree}} = D_L^\dagger \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_d^{(2)} \end{pmatrix} D_R$$

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$$\begin{aligned} (U_L)_{31} &= e^{i\rho_{u_L}} \sin\theta_{u_L} \sin\omega_{u_L} \; , \\ (U_L)_{32} &= e^{i\xi_{u_L}} \sin\theta_{u_L} \cos\omega_{u_L} \; , \\ (U_L)_{33} &= \cos\theta_{u_L} \; , \end{aligned}$$

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- Neglect phases.
- Assume for simplicity $y_u^{(1)}, y_u^{(2)} >> y_d^{(1)}, y_d^{(2)}$

- 1-loop from $\beta$ function:

$$Y^{(1)}_{u,d}|_{1-\text{loop}} \simeq Y^{(1)}_{u,d}|_{\text{tree}} + \frac{1}{16\pi^2}\beta^{(1)}_{u,d}\log\frac{\Lambda}{M_H}$$

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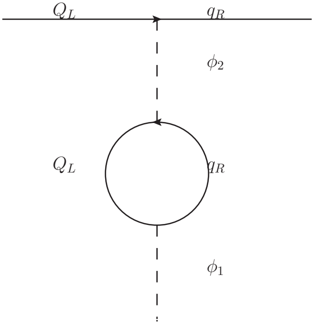
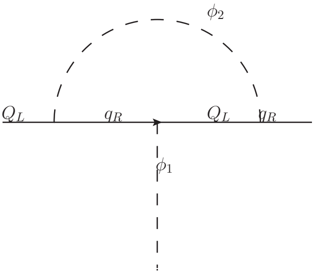
- 1-loop diagram (generate 2nd mass):



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Mass hierarchy for the quarks:

$$\frac{y_c}{y_t} \simeq \left( \frac{1}{16\pi^2} \log \frac{\Lambda}{M_H} \right) \frac{3}{4} (y_u^{(2)})^2 \times \text{mixing angles}$$

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- First generation massless.

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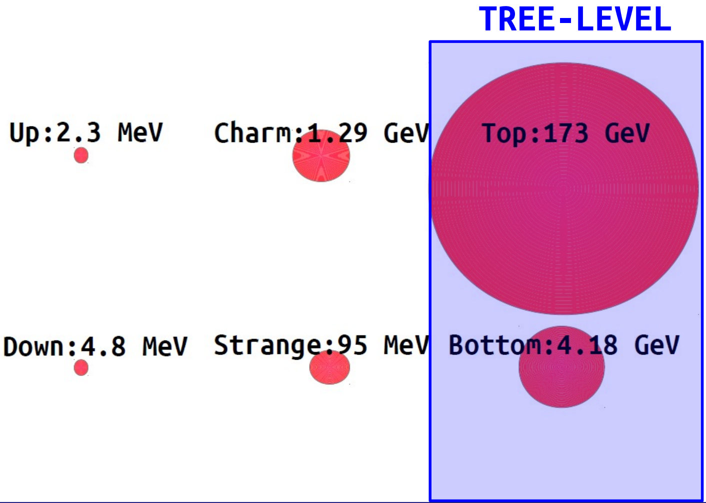

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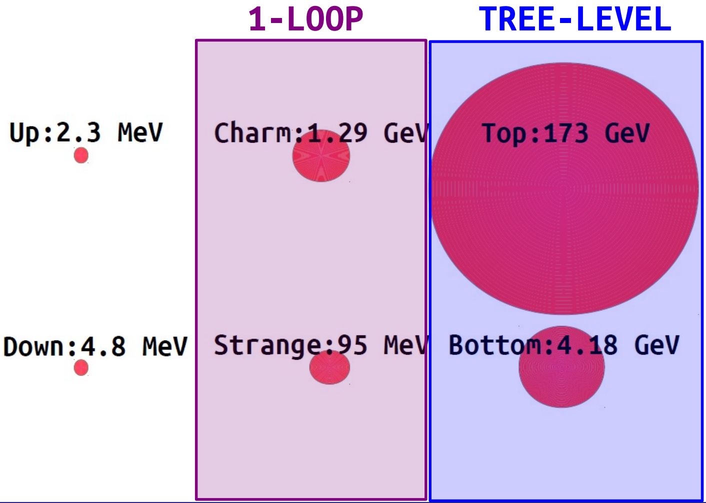

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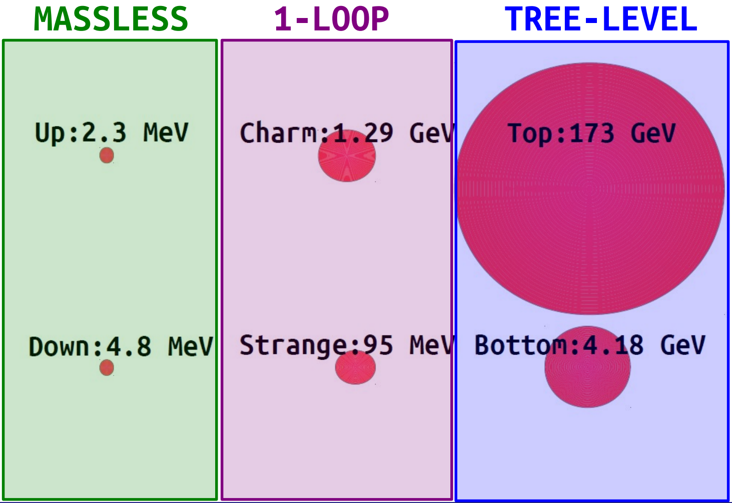

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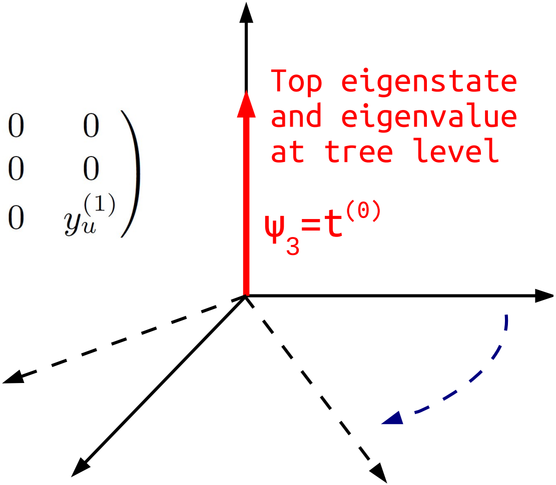

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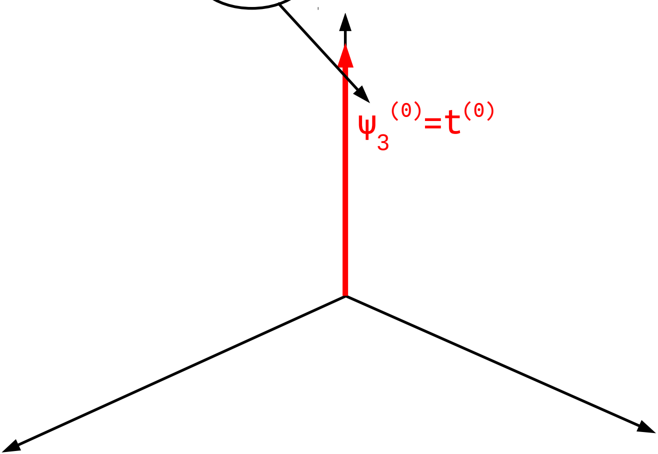

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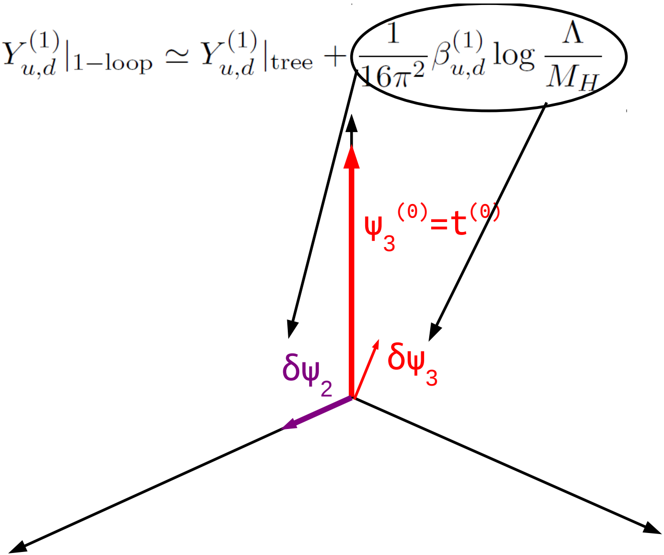

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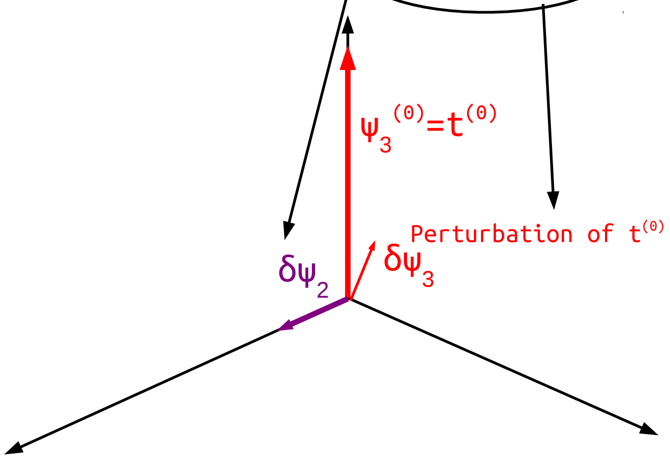

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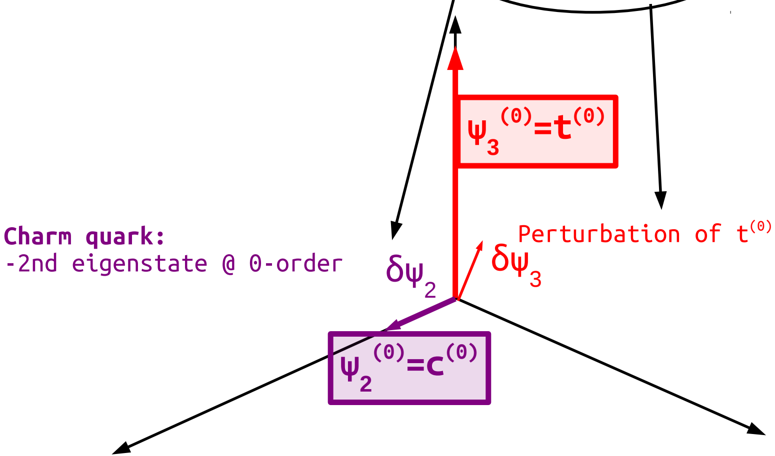

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- Cabibbo angle @ 0-order in perturbation theory:

$$V_{us} \simeq -V_{cd} \simeq \frac{3\sin\theta_{d_L}\cos\theta_{u_L}\sin(\omega_{d_L} - \omega_{u_L})}{N_d}$$

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- Remaining off-diagonal terms @ 1st order in perturbation theory:

$$V_{ub} \simeq \left(\frac{1}{16\pi^2}\log\frac{\Lambda}{M_H}\right)\frac{3y_u^{(1)}y_u^{(2)}y_d^{(2)}}{y_d^{(1)}} \times \text{mixing angles}$$

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- Constraints from masses and CKM matrix:

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$$\Rightarrow \theta_{u_R} \approx 0.16, \ \theta_{d_R} \approx 1.06$$

- Degeneracy for other parameters:
  - ✓ All masses and angles can be reproduced (example: $y_u^{(2)} \approx 1.04$, $y_d^{(2)} \approx 0.02$, $\theta_{d_L} \approx 0.61$, $\theta_{u_L} \approx 0.51$, $\omega_{d_L} - \omega_{u_L} \approx 0.10$)

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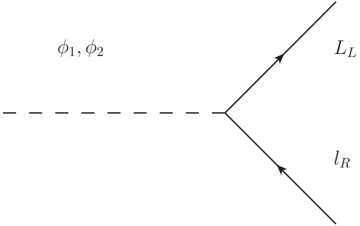

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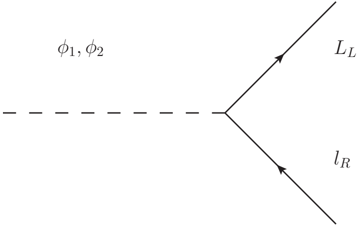


$$Y_e^{(1)}|_{\text{tree}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_e^{(1)} \end{pmatrix} , \quad Y_e^{(2)}|_{\text{tree}} = U_{Le}^T \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_e^{(2)} \end{pmatrix} U_{Re}$$

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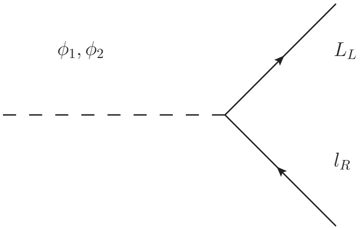


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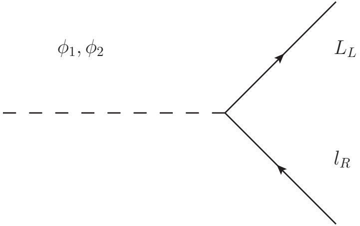


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$$Y_\nu^{(1)}|_{\text{tree}} = y_\nu^{(1)}(0, \sin\alpha, \cos\alpha)^T$$
$$Y_\nu^{(2)}|_{\text{tree}} = y_\nu^{(2)}(\sin\theta_\nu \sin\omega_\nu, \sin\theta_\nu \cos\omega_\nu, \cos\theta_\nu)^T.$$

- Parametrize $U_{Le3i}$ (idem for $U_{eR}$):

$$\begin{aligned}(U_{Le})_{31} &= e^{i\rho_{e_L}} \sin\theta_{e_L} \sin\omega_{e_L} \ , \\ (U_{Le})_{32} &= e^{i\xi_{e_L}} \sin\theta_{e_L} \cos\omega_{e_L} \ , \\ (U_{Le})_{33} &= \cos\theta_{e_L} \ ,\end{aligned}$$

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- $Y_e^{(1)}|_{\text{tree}}$ & $Y_\nu^{(1)}|_{\text{tree}} \Rightarrow \left\{ \begin{array}{c} m_\tau \\ m_{\nu 3} \end{array} \right.$ **@ tree level**

*Neutrino sector studied: Ibarra, Simonetto [arXiv:1107.2386 ]*

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Muon mass @ 1-loop:

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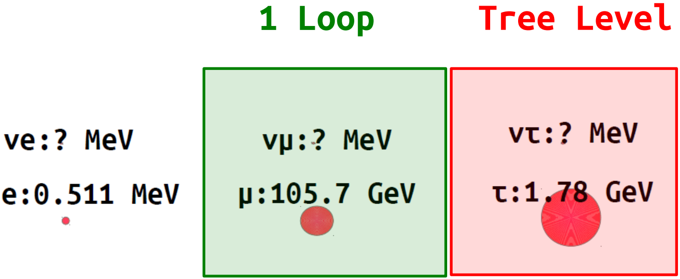
- First generation massless.

# Lepton Masses

**Tree Level**

| | | |
|---|---|---|
| νe:? MeV | νμ:? MeV | ντ:? MeV |
| e:0.511 MeV | μ:105.7 GeV | τ:1.78 GeV |

# Lepton Masses



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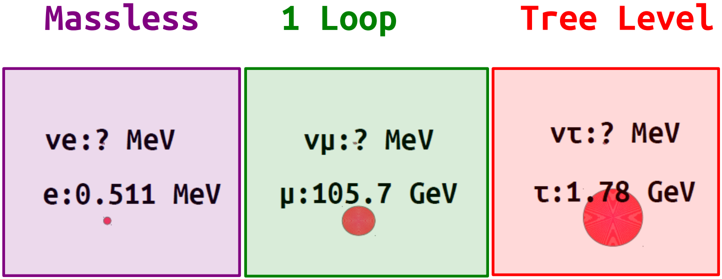

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- All other terms @ 0-th order in perturbation theory:

$$U_{13}^{PMNS} = -\sin\alpha\sin\tilde{\theta}_e = \text{mixing angles}$$

# PMNS Matrix

$$\left|U^{PMNS}\right| = \begin{pmatrix} 0.822 & 0.574 & 0.156 \\ 0.355 & 0.704 & 0.614 \\ 0.443 & 0.452 & \boxed{0.774} \end{pmatrix}$$

Tree-Level

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0-Order

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- Eigenstates and eigenvalues of $\lambda \mathrm{W}^{(n)}$ are eigenstates (0-order) and eigenvalues (1st-order) of H from the subspace $\perp |t^{(0)}\rangle$.