

Radiative Generation of Masses and Mixing Angles of the Standard Model

[arXiv:1403.2382]

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Outline

Introduction

The 2HDM

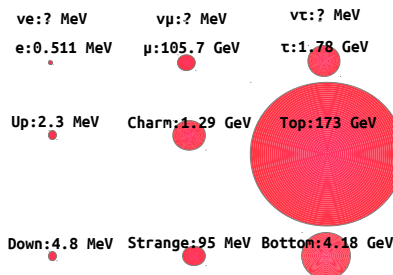
The Quark Sector

The Lepton Sector

Conclusions

Introduction

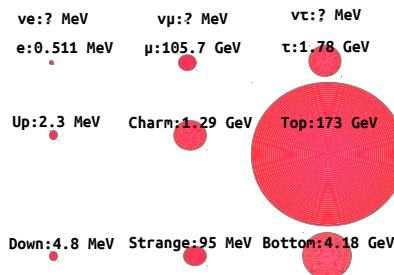
- **Motivation:** Hierarchy for masses and mixing angles in the Standard Model. $\Rightarrow$ **New Physics?**



$$V_{CKM} = \begin{pmatrix} 0.974 & 0.225 & 0.0035 \\ 0.225 & 0.973 & 0.041 \\ 0.0087 & 0.04 & 0.999 \end{pmatrix}$$

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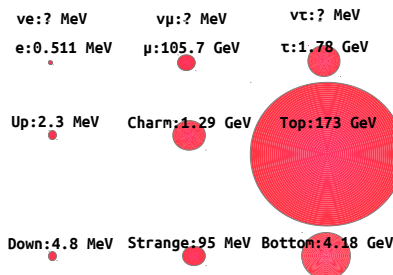


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- **How?:** 2HDM

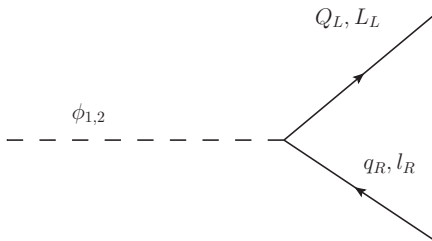
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- ▶ Standard Model + Higgs doublet.

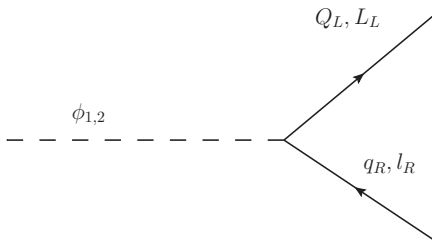
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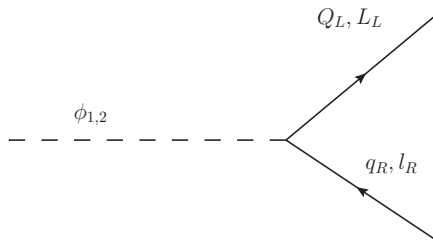
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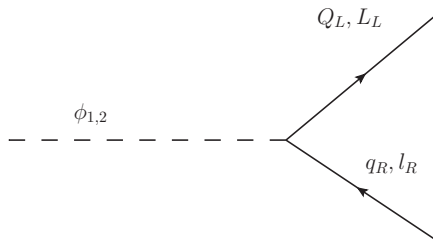
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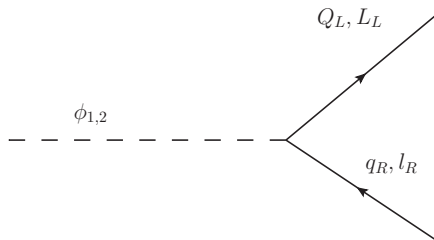
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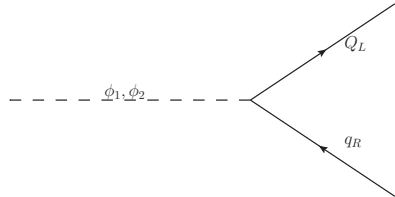
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The Quark Sector

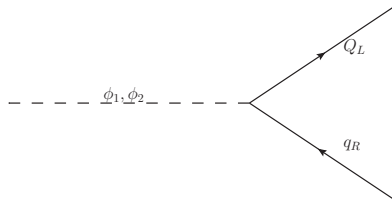
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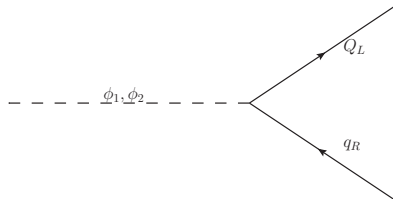
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$$Y_u^{(1)}|_{\text{tree}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_u^{(1)} \end{pmatrix}, \quad Y_d^{(1)}|_{\text{tree}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \epsilon y_d^{(1)} \\ 0 & 0 & y_d^{(1)} \end{pmatrix} \rightarrow |V_{cb}| \ll 1 \Rightarrow \epsilon \rightarrow 0$$

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- ▶ $Y_u^{(1)}|_{\text{tree}} \ \& \ Y_d^{(1)}|_{\text{tree}} \Rightarrow \begin{cases} m_t \\ m_b \end{cases} \quad \text{@ tree level}$

$$Y_u^{(2)}|_{\text{tree}} = U_L^\dagger \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_u^{(2)} \end{pmatrix} U_R, \quad Y_d^{(2)}|_{\text{tree}} = D_L^\dagger \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_d^{(2)} \end{pmatrix} D_R$$

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- $Y_{u,d}^{(2)}|_{\text{tree}}$ just depend on $U_{L3i}, U_{R3i}, D_{L3i}, D_{R3i}$. Parametrize:

$$\begin{aligned} (U_L)_{31} &= e^{i\rho_{uL}} \sin \theta_{uL} \sin \omega_{uL}, \\ (U_L)_{32} &= e^{i\xi_{uL}} \sin \theta_{uL} \cos \omega_{uL}, \\ (U_L)_{33} &= \cos \theta_{uL}, \end{aligned}$$

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- ▶ Neglect phases.
- ▶ Assume for simplicity $y_u^{(1)}, y_u^{(2)} \gg y_d^{(1)}, y_d^{(2)}$

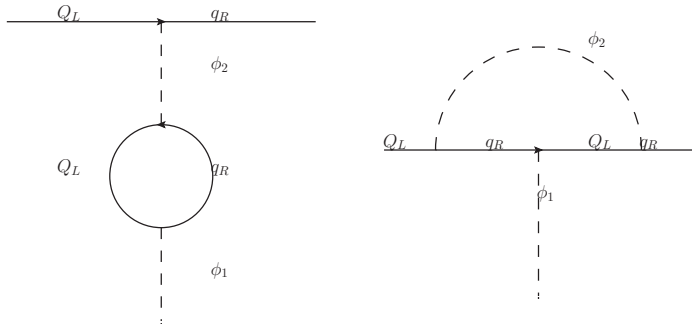
- ▶ 1-loop from β function:

$$Y_{u,d}^{(1)}|_{1\text{-loop}} \simeq Y_{u,d}^{(1)}|_{\text{tree}} + \frac{1}{16\pi^2} \beta_{u,d}^{(1)} \log \frac{\Lambda}{M_H}$$

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- 1-loop diagram (generate 2nd mass):



Quark Masses

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Mass hierarchy for the quarks:

$$\frac{y_c}{y_t} \simeq \left(\frac{1}{16\pi^2} \log \frac{\Lambda}{M_H} \right) \frac{3}{4} (y_u^{(2)})^2 \times \text{mixing angles}$$

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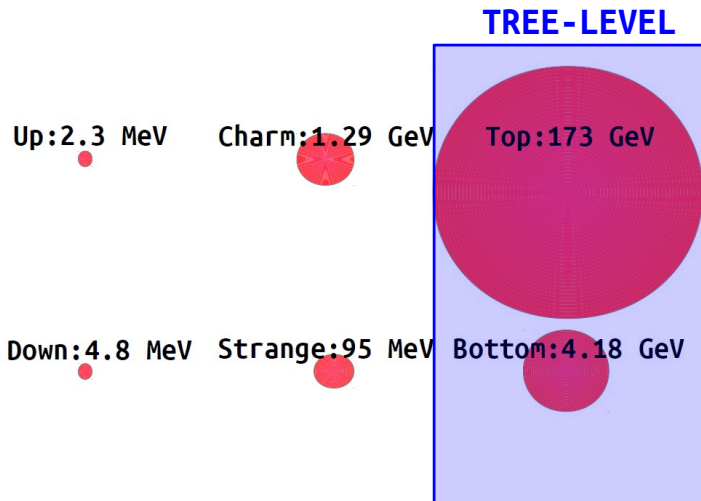
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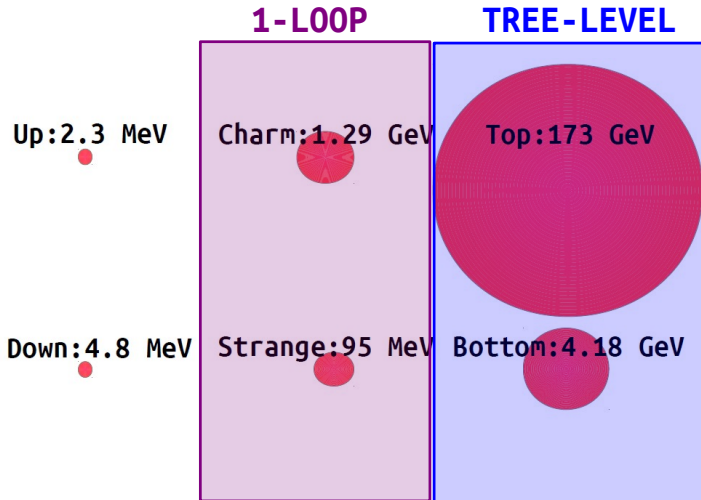
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- First generation massless.

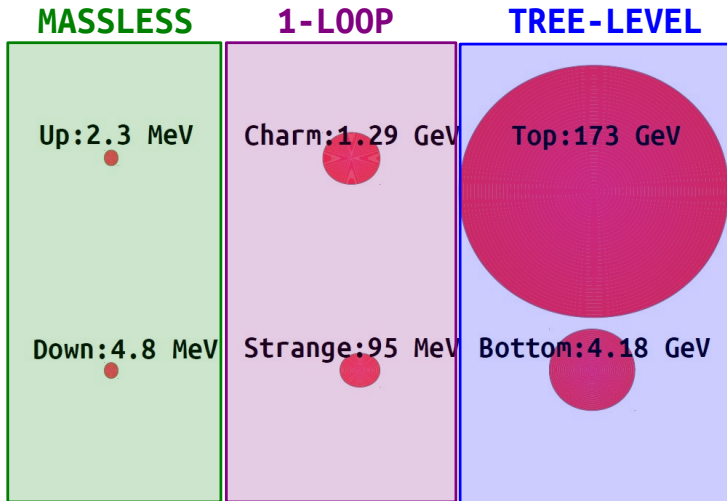
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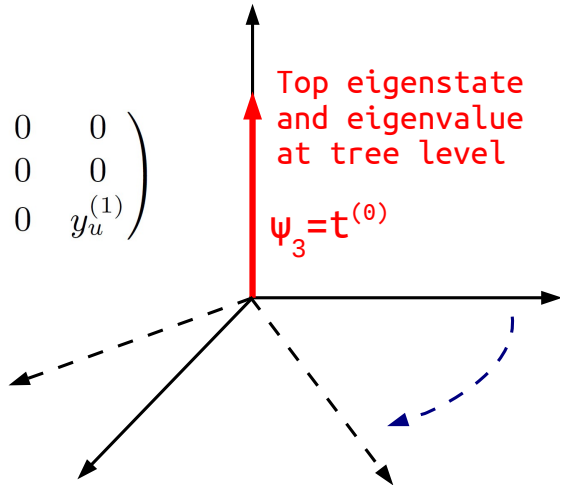
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$$Y_{u,d}^{(1)}|_{1\text{-loop}} \simeq \underbrace{Y_{u,d}^{(1)}|_{\text{tree}}}_{\text{red arrow}} + \frac{1}{16\pi^2} \beta_{u,d}^{(1)} \log \frac{\Lambda}{M_H}$$

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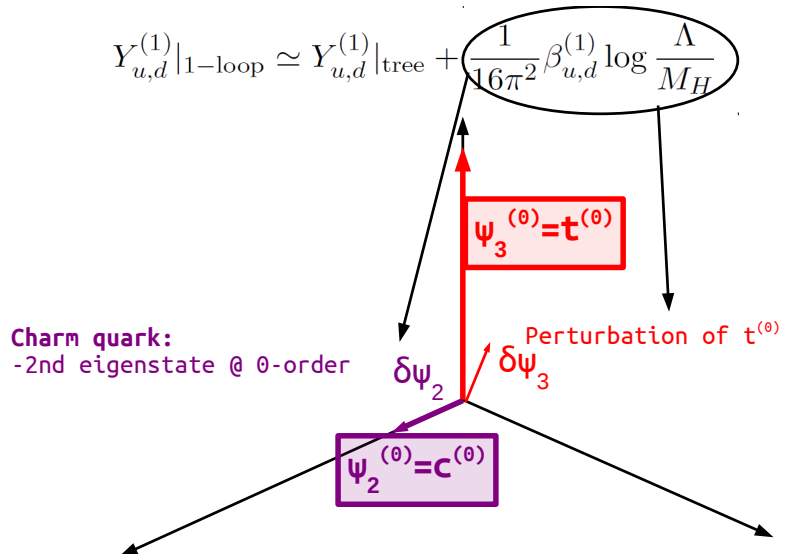
Diagram illustrating the relationship between the 1-loop Yukawa coupling $Y_{u,d}^{(1)}|_{1\text{-loop}}$ and the tree-level Yukawa coupling $Y_{u,d}^{(1)}|_{\text{tree}}$. The equation shows a correction term involving the beta function $\beta_{u,d}^{(1)}$ and the logarithm of the ratio of the UV cutoff Λ to the Higgs mass M_H . The diagram also shows the dependence of the Yukawa coupling on the Higgs mass M_H and the mixing angle ψ_3 .

Key parameters and variables shown in the diagram:

- $\psi_3^{(\theta)} = t^{(\theta)}$ (Red text)
- $\delta\psi_2$ (Purple text)
- $\delta\psi_3$ (Red text)

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The diagram illustrates the relationship between the 1-loop Yukawa coupling $Y_{u,d}^{(1)}|_{1\text{-loop}}$ and the tree-level Yukawa coupling $Y_{u,d}^{(1)}|_{\text{tree}}$. The equation is shown at the top, with the term $\frac{1}{16\pi^2} \beta_{u,d}^{(1)} \log \frac{\Lambda}{M_H}$ enclosed in an oval. Arrows point from this oval to the tree-level term and to the text "Perturbation of $t^{(\theta)}$ ". A red arrow labeled $\psi_3^{(\theta)} = t^{(\theta)}$ points upwards from the origin. A purple arrow labeled $\delta\psi_2$ points downwards and to the left. A red arrow labeled $\delta\psi_3$ points upwards and to the right. The text "Perturbation of $t^{(\theta)}$ " is written in red.



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- ▶ Cabibbo angle @ 0-order in perturbation theory:

$$V_{us} \simeq -V_{cd} \simeq \frac{3 \sin \theta_{d_L} \cos \theta_{u_L} \sin(\omega_{d_L} - \omega_{u_L})}{N_d}$$

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$$N_d = [9 \sin^2 \theta_{d_L} \cos^2 \theta_{u_L} + 4 \cos^2 \theta_{d_L} \sin^2 \theta_{u_L} - 3 \sin 2\theta_{d_L} \sin 2\theta_{u_L} \cos(\omega_{d_L} - \omega_{u_L})]^{1/2}$$

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- ▶ Remaining off-diagonal terms @ 1st order in perturbation theory:

$$V_{ub} \simeq \left(\frac{1}{16\pi^2} \log \frac{\Lambda}{M_H} \right) \frac{3y_u^{(1)} y_u^{(2)} y_d^{(2)}}{y_d^{(1)}} \times \text{mixing angles}$$

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 **Tree-level**

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0th Order
 ↓
 (Green box highlights the top-left 2x2 submatrix)
 (Purple box highlights the bottom-left 2x2 submatrix)
 (Red box highlights the bottom-right 1x1 element)
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1st Order **Tree-level**

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- Constraints from masses and CKM matrix:

$$\frac{y_s}{y_b} \frac{V_{us}}{V_{ub}} \simeq \tan \theta_{d_R} ,$$
$$\frac{y_c}{y_t} \frac{V_{us}}{V_{td}} \simeq \frac{3 \sin 2\theta_{u_R}}{2 + 3 \cos 2\theta_{u_R}}$$

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$$\Rightarrow \theta_{u_R} \approx 0.16, \theta_{d_R} \approx 1.06$$

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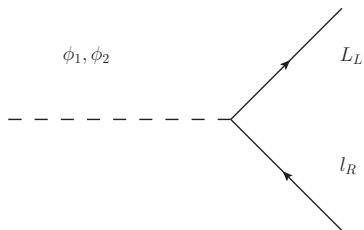
✓ All masses and angles can be reproduced

(example: $y_u^{(2)} \approx 1.04$, $y_d^{(2)} \approx 0.02$, $\theta_{d_L} \approx 0.61$, $\theta_{u_L} \approx 0.51$,
 $\omega_{d_L} - \omega_{u_L} \approx 0.10$)

The Lepton Sector

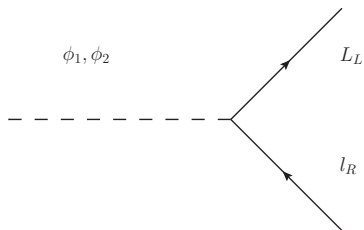
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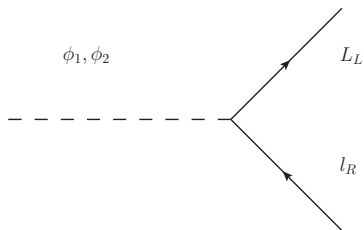
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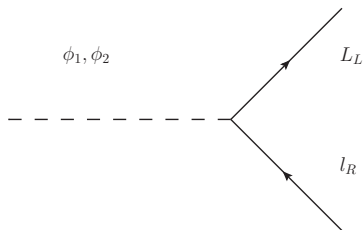


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$$Y_\nu^{(1)}|_{\text{tree}} = y_\nu^{(1)}(0, \sin \alpha, \cos \alpha)^T$$

$$Y_\nu^{(2)}|_{\text{tree}} = y_\nu^{(2)}(\sin \theta_\nu \sin \omega_\nu, \sin \theta_\nu \cos \omega_\nu, \cos \theta_\nu)^T.$$

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- ▶ $Y_e^{(1)}|_{\text{tree}} \ \& \ Y_\nu^{(1)}|_{\text{tree}} \Rightarrow \left\{ \begin{array}{l} m_\tau \\ m_{\nu 3} \end{array} \right. \quad \textbf{@ tree level}$

Neutrino sector studied: Ibarra, Simonetto [arXiv:1107.2386]

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- First generation massless.

Lepton Masses

Tree Level

$\nu_e: ? \text{ MeV}$

$e: 0.511 \text{ MeV}$



$\nu_\mu: ? \text{ MeV}$

$\mu: 105.7 \text{ GeV}$

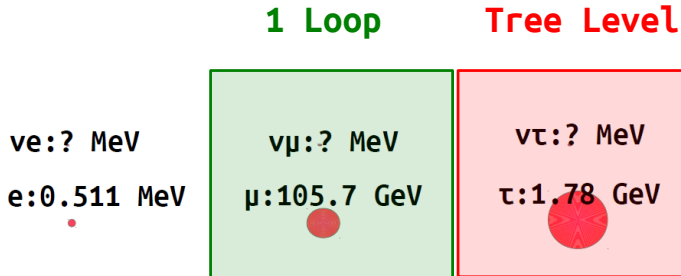


$\nu_\tau: ? \text{ MeV}$

$\tau: 1.78 \text{ GeV}$

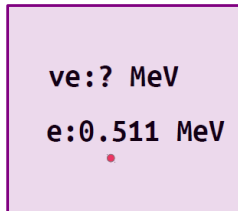


Lepton Masses

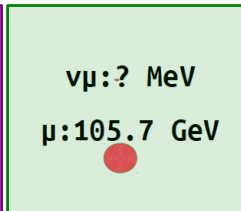


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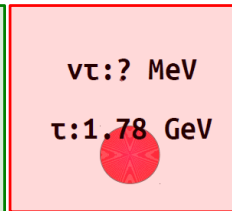
Massless



1 Loop



Tree Level



Lepton Mixing Angles

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- ▶ All other terms @ 0-th order in perturbation theory:

$$U_{13}^{PMNS} = -\sin \alpha \sin \tilde{\theta}_e = \text{mixing angles}$$

PMNS Matrix

$$|U^{PMNS}| = \begin{pmatrix} 0.822 & 0.574 & 0.156 \\ 0.355 & 0.704 & 0.614 \\ 0.443 & 0.452 & 0.774 \end{pmatrix}$$

Tree-Level

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0-Order Tree-Level

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- ▶ Eigenstates and eigenvalues of $\lambda W^{(n)}$ are eigenstates (0-order) and eigenvalues (1st-order) of H from the subspace $\perp |t^{(0)}\rangle$.