Stringy Geometries in the Context of Double Field Theory

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String theory...

- string theory is a quantum gravity $\rightarrow$ spacetime is not fixed
- it should evolve from the theory itself

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"usual" implementations of string theory describe dynamic of strings in a certain **background** spacetime

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1. pick a spacetime compatible with string theory
2. use it as **background**
3. describe strings moving in the **background**

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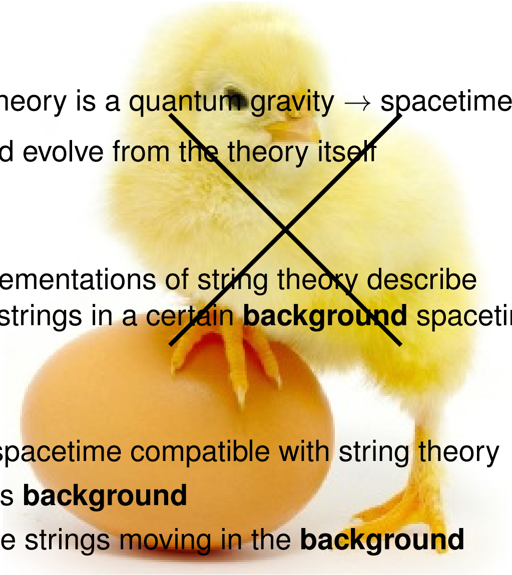

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…and the string theory landscape [1].

- How to choose such a background?
- Is (are) there one, ten, hunderts or billions of them?

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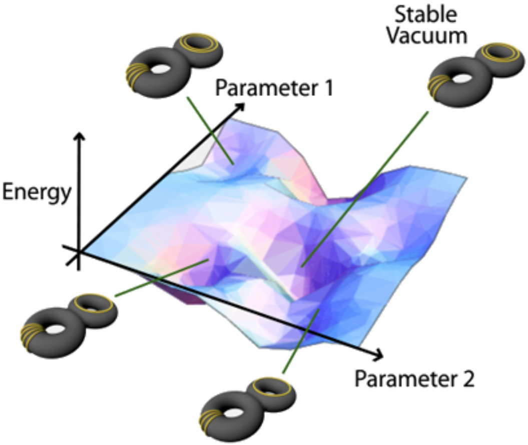
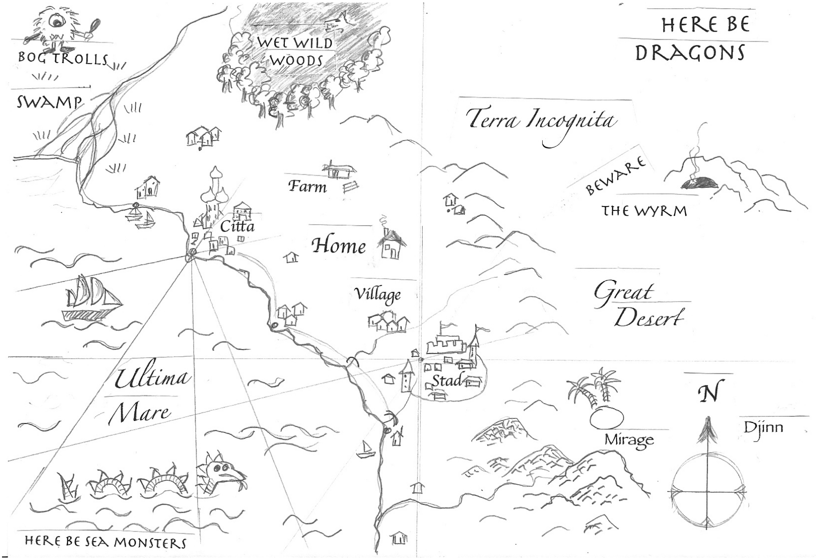


1. parameterize "shape" of background
2. assign energy to each background
3. find minima

10^{500} backgrounds [2, 3]

How we explore this landscape?



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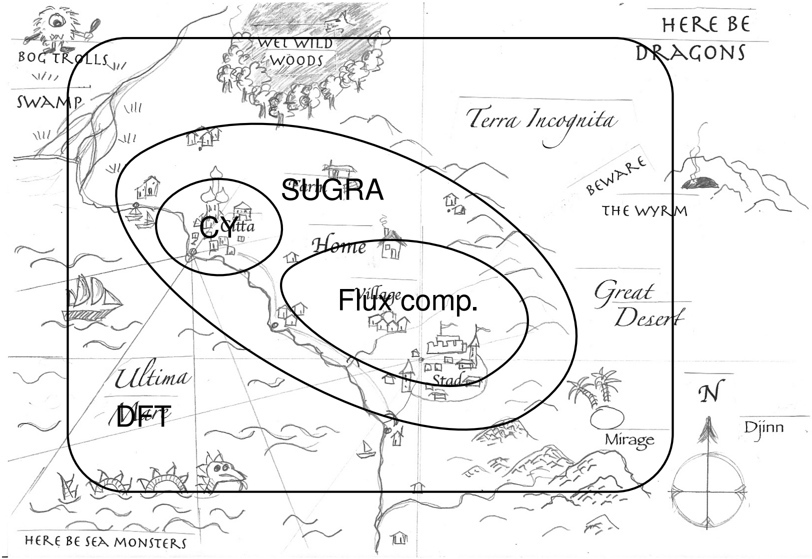


SUGRA in a nutshell

- low engery effective theory for (super) string theory
- here the NS/NS sector only

$$S_{\mathrm{NS}} = \int \mathrm{d}^D x \sqrt{g} e^{-2\phi} \left(\mathcal{R} + 4 \partial_\mu \phi \partial^\mu \phi - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} \right)$$

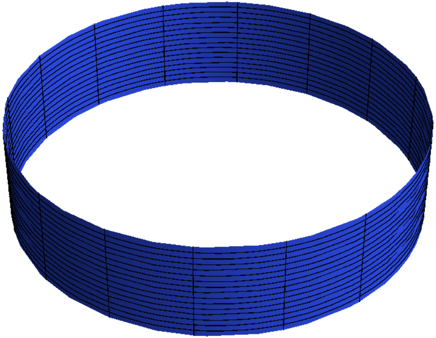
- Einstein-Hilbert like part = general relativity
- 2-form gauge field $B_{\mu\nu}$ with
- field strength $H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]}$

$\sim$ Einstein-Maxwell theory $\rightarrow$ point particles

- backgrounds solve S_{NS}'s field equations

Backgrounds “seen” by point particles

- general relativity: spacetime = smooth manifold



- geometric twists are possible
- B_{ij} fields connected by gauge transformations

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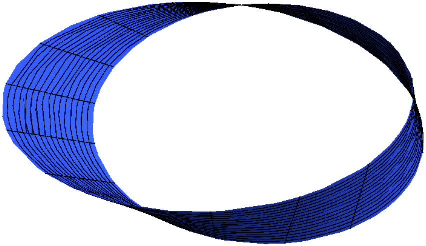

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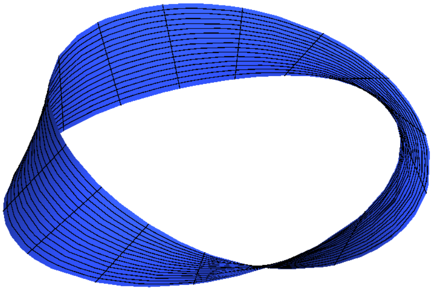

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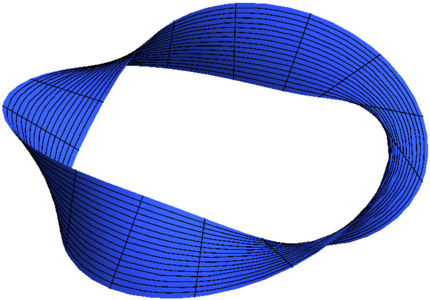

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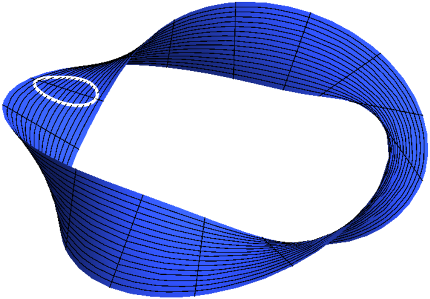

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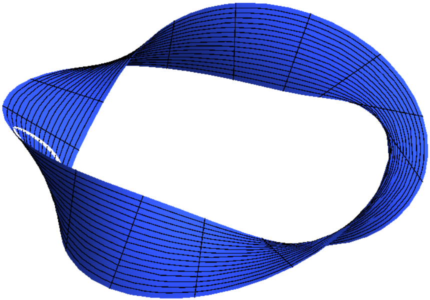

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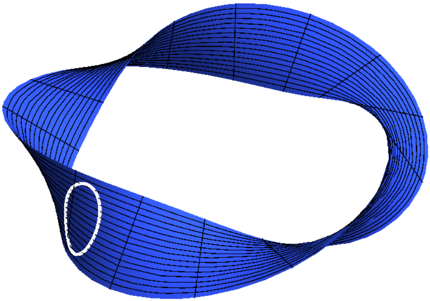

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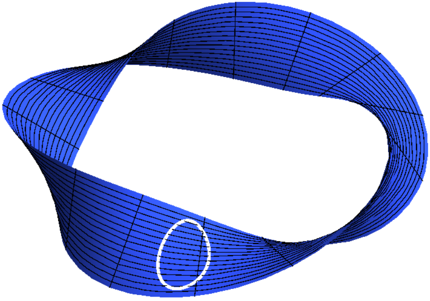

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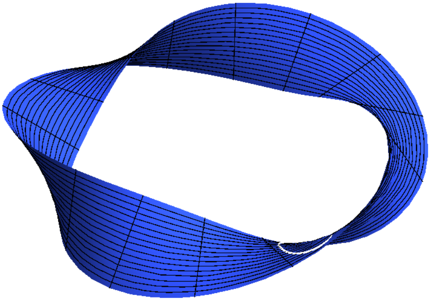

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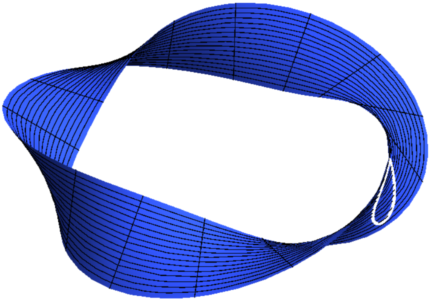

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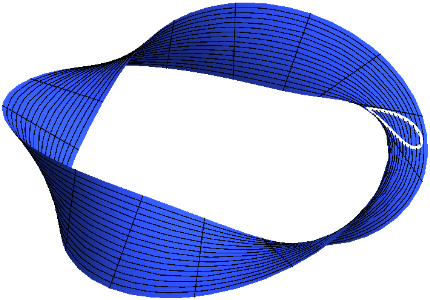

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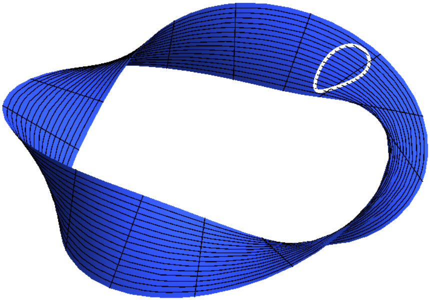

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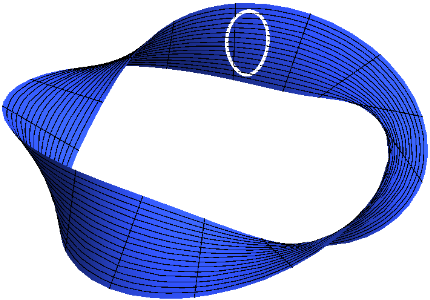

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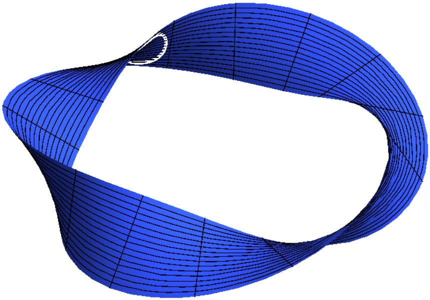

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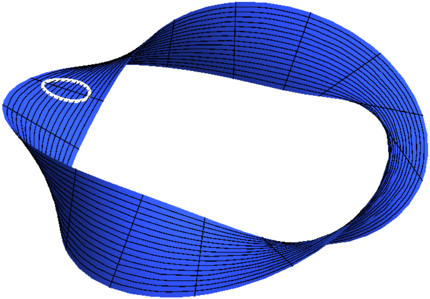

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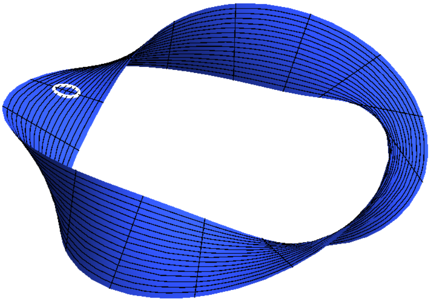

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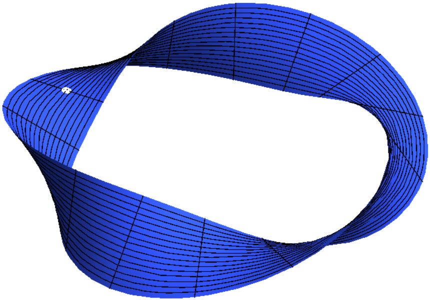

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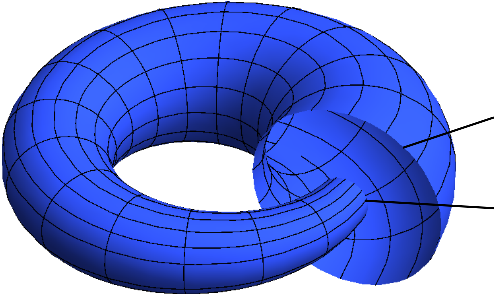
Strings have a different perspective [4]:

- closed strings also wind around the torus $\rightarrow$ T-duality



~~large volume limit~~

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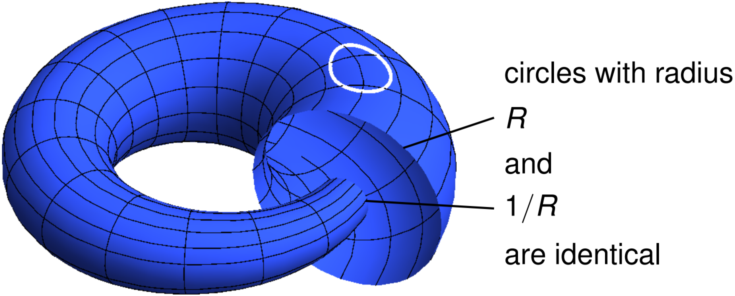

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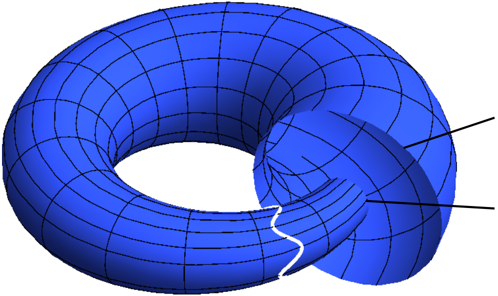


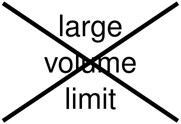
circles with radius

R

and

$1/R$

are identical



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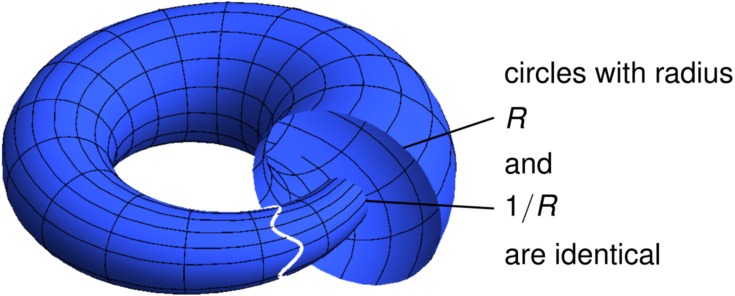

- new interesting properties like non-commutativity
- compactifications lead to gauged SUGRA
  - moduli stabilization
  - effective cosmological constant
  - spontaneous SUSY breaking

~~large volume limit~~

Double Field Theory [5, 6] in a nutshell

- considers both, winding and momentum mode of string
- doubling of coordinates $D \to 2D$

$$S_{\mathrm{DFT}} = \int \mathrm{d}^{2D}X \, e^{-2\phi'}\mathcal{R}$$

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$$X^M = \begin{pmatrix} \tilde{x}_i & x^i \end{pmatrix} \qquad \phi' = \phi - \tfrac{1}{2}\log\sqrt{g}$$

$$S_{\mathrm{DFT}} = \int \mathrm{d}^{2D}X\, e^{-2\phi'}\mathcal{R}$$

$$\mathcal{R} = 4\mathcal{H}^{MN}\partial_M\phi'\partial_N\phi' - \partial_M\partial_N\mathcal{H}^{MN} - 4\mathcal{H}^{MN}\partial_M\phi'\partial_N\phi' + 4\partial_M\mathcal{H}^{MN}\partial_N\phi' + \frac{1}{8}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_N\mathcal{H}_{KL} - \frac{1}{2}\mathcal{H}^{MN}\partial_N\mathcal{H}^{KL}\partial_L\mathcal{H}_{MK}$$

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$$+ \frac{1}{8} \mathcal{H}^{MN} \partial_M \mathcal{H}^{KL} \partial_N \mathcal{H}_{KL} - \frac{1}{2} \mathcal{H}^{MN} \partial_N \mathcal{H}^{KL} \partial_L \mathcal{H}_{MK}$$

$$\mathcal{H}^{MN} = \begin{pmatrix} g_{ij} - B_{ik} g^{kl} B_{lj} & -B_{ik} g^{kj} \\ g^{ik} B_{kj} & g^{ij} \end{pmatrix}$$

Gauge transformations and the strong constraint [7, 8]

- generalized Lie derivative combines
  1. diffeomorphisms
  2. *B*-field gauge transformations
  3. β-field gauge transformations

(1. and 2.: available in SUGRA)

$$\mathcal{L}_\xi \mathcal{H}^{MN} = \xi^P \partial_P \mathcal{H}^{MN} + (\partial^M \xi_P - \partial_P \xi^M)\mathcal{H}^{PN} + (\partial^N \xi_P - \partial_P \xi^N)\mathcal{H}^{MP}$$

$$\mathcal{L}_\xi \phi' = \xi^M \partial_M \phi' + \frac{1}{2}\partial_M \xi^M$$

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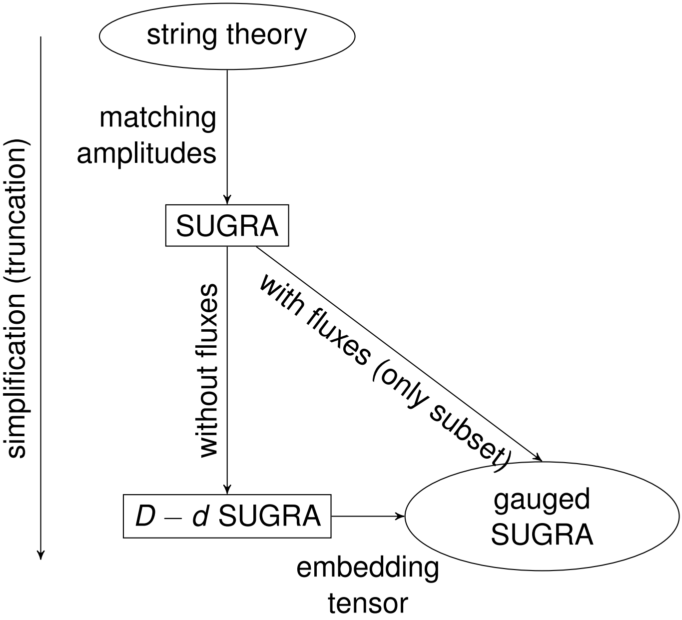
- closure of this algebra needs $\mathcal{L}_{\xi_1}\mathcal{L}_{\xi_2} - \mathcal{L}_{\xi_2}\mathcal{L}_{\xi_1} = \mathcal{L}_{\xi_3}$ with $\xi_3 = [\xi_1, \xi_2]_\mathrm{C}$ (C-bracket)
- only possible when strong constraint holds

$$\boxed{\partial_M \partial^M \cdot = 0}$$

Scherk-Schwarz compactification [9] or a tool to construct backgrounds



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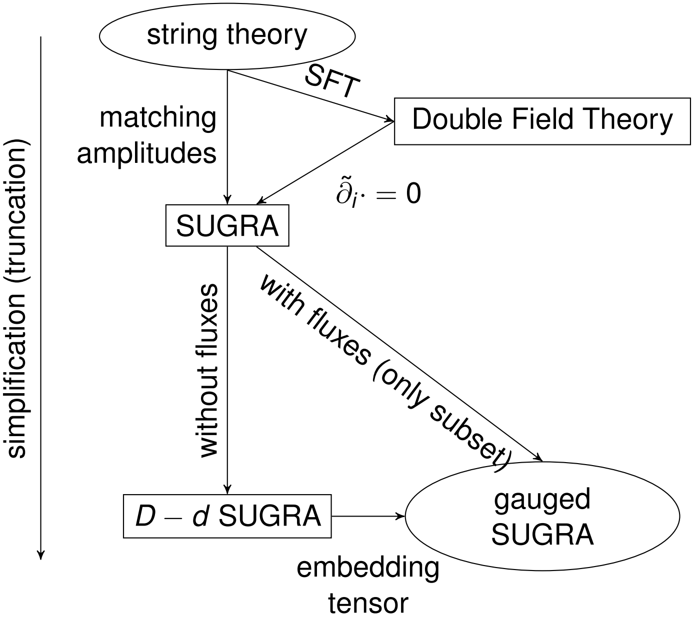

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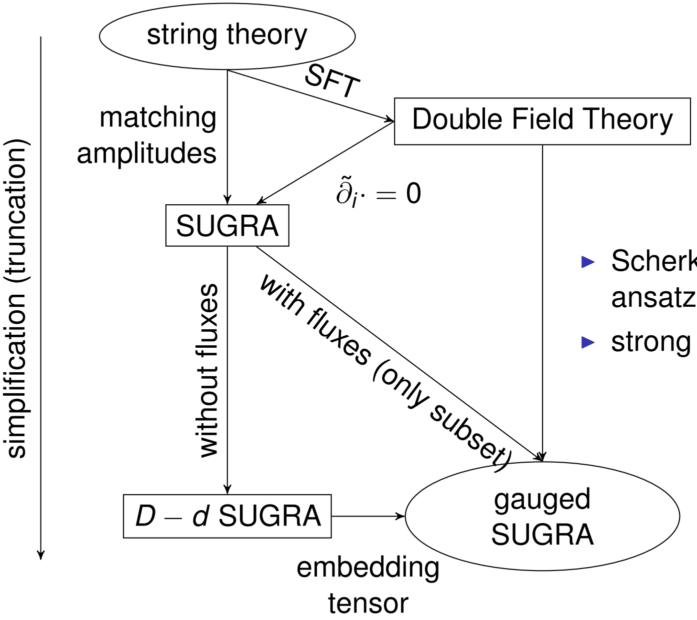

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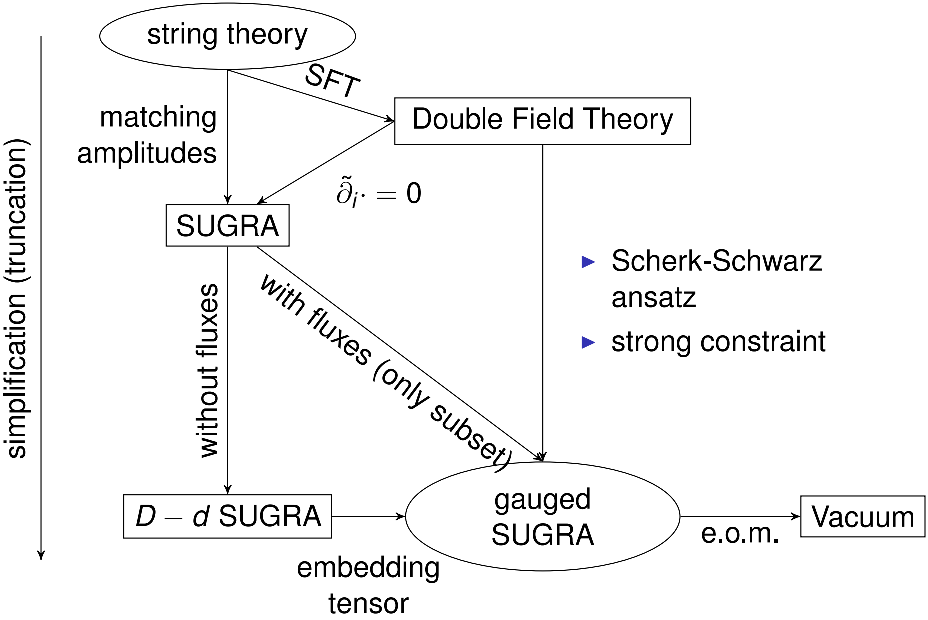

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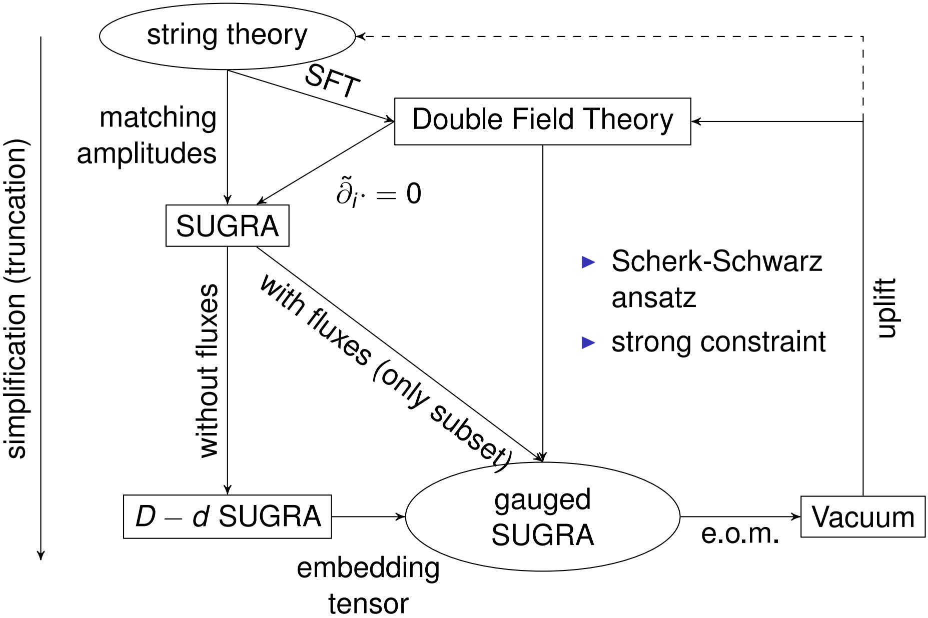

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Gauged SUGRA [10, 11] and its vaccua

- DFT action + Scherk-Schwarz ansatz gives rise to

$$S_{\text{eff}} = \int \mathrm{d}x^{(D-d)} \sqrt{-g} e^{-2\phi} \Big(\mathcal{R} + 4\partial_\mu \phi \partial^\mu \phi - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} - \frac{1}{4} \mathcal{H}_{MN} F^{M\mu\nu} F^{N}{}_{\mu\nu} + \frac{1}{8} D_\mu \mathcal{H}_{MN} D^\mu \mathcal{H}^{MN} - V \Big)$$

  with scalar potential

$$V = -\tfrac{1}{4} \mathcal{F}_I{}^{KL} \mathcal{F}_{JKL}\, \mathcal{H}^{IJ} + \tfrac{1}{12} \mathcal{F}_{IKM} \mathcal{F}_{JLN} \mathcal{H}^{IJ} \mathcal{H}^{KL} \mathcal{H}^{MN}$$

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- maximally symmetric vacuum = <u>Minkowski</u>, dS, or AdS
- for Minkowski e.o.m for vacuum reduce to

$$0 = \mathcal{R}_{\mu\nu}\,, \quad V = 0 \quad \text{and} \quad \mathcal{K}^{MN} = \frac{\delta V}{\delta\mathcal{H}_{MN}} \sim 0$$

- additional constraints on covariant fluxes $\mathcal{F}_{IJK}$

Covariant fluxes as classification tool

- covariant fluxes $\mathcal{F}_{IJK}$ combine
  1. geometric fluxes f and H-flux (known from SUGRA)
  2. non-geometric fluxes Q and R
- find fluxes which fulfill **all** constraint discussed so far
- solution for $D - d = 3$ (non-vanishing fluxes)

$$\boxed{H_{123} = Q_1^{23} = H \quad \text{and} \quad f_{31}^2 = f_{12}^3 = f}$$

- three different cases
  1. $H = 0$ and $f \neq 0$: Solvmanifold, known from SUGRA
  2. $H \neq 0$ and $f = 0$: T-dual version of 1.
  3. $H \neq 0$ and $f \neq 0$: genuinely non-geometric background, called double elliptic

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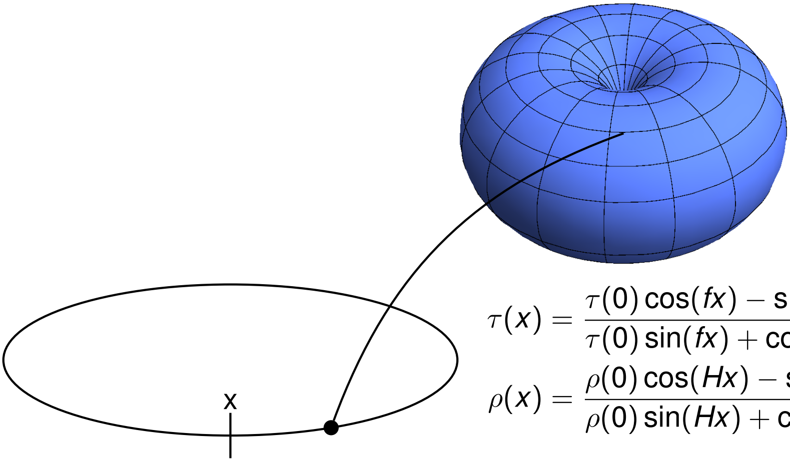
How do these backgrounds "look" like?

- fibration of T^2 over a S^1 base with coordinate x



$$\tau(x) = \frac{\tau(0)\cos(fx) - \sin(fx)}{\tau(0)\sin(fx) + \cos(fx)}$$

$$\rho(x) = \frac{\rho(0)\cos(Hx) - \sin(Hx)}{\rho(0)\sin(Hx) + \cos(Hx)}$$

- T^2 parameterized by ρ and τ (functions of x)
- fixed point of twist is $\rho(0) = \tau(0) = i$

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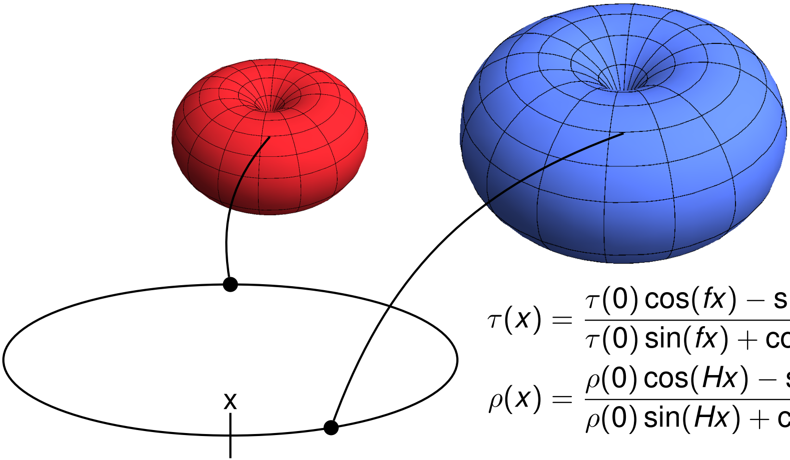

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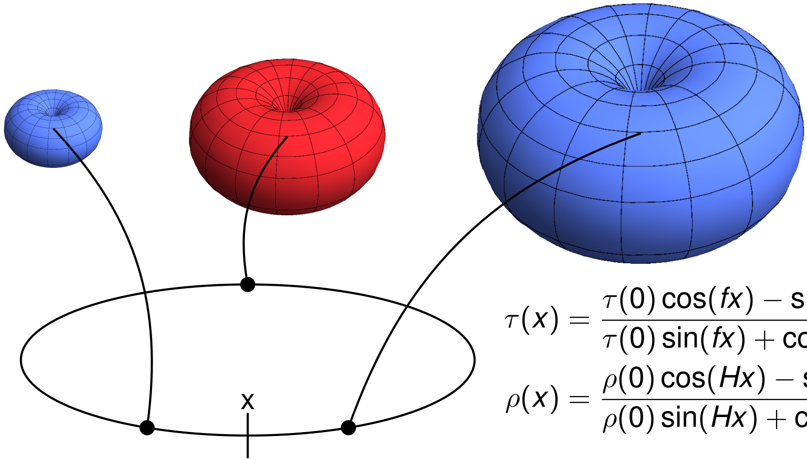

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Moduli stabilization

- scalar potential for fiber moduli $\rho(0) = \rho$ and $\tau(0) = \tau$
- minimum at fixed point of twist with $V_{\text{min}} = 0$ (Minkowski)
- mass terms for ρ and τ

modulus	ρ_R	ρ_I	τ_R	τ_I
mass	$2\lvert H\rvert$	$2\lvert H\rvert$	$2\lvert f\rvert$	$2\lvert f\rvert$

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Moduli stabilization

- scalar potential for fiber moduli $\rho(0) = \rho$ and $\tau(0) = \tau$
- minimum at fixed point of twist with $V_{\text{min}} = 0$ (Minkowski)
- mass terms for ρ and τ

modulus	ρ_R	ρ_I	τ_R	τ_I
mass	$2\|H\|$	$2\|H\|$	$2\|f\|$	$2\|f\|$

- H and $f \in \{1/6, 1/4, 1/3, 1/2\}$ are quantized
- volume ρ_I of fiber torus $\approx (l_s)^2$

$\rightarrow$ no large volume limit!

- closely related the asymmetric orbifold [12, 13]
- still 5 flat directions, e.g. radius of base R

A hidden violation of the strong constraint

We have found a background

- without large volume limit
- stabilizes additional moduli
- generalized metric fulfills the strong constraint

---

not in scope of SUGRA or generalized geometry

A hidden violation of the strong constraint

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---

not in scope of SUGRA or generalized geometry

BUT, looking more closely, we see

- one Killing vector which violates the strong constraint

$$K^I = \begin{pmatrix} 0 & -\frac{1}{2}(Hx^3 + f\tilde{x}^3) & \frac{1}{2}(Hx^2 + f\tilde{x}^2) & 1 & -\frac{1}{2}(fx^3 + H\tilde{x}^3) & \frac{1}{2}(fx^2 + H\tilde{x}^2) \end{pmatrix}$$

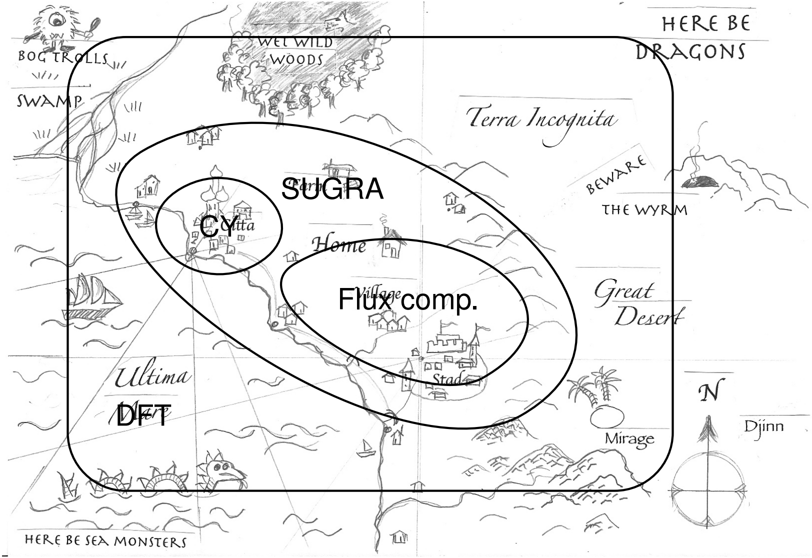
$\rightarrow$ patched by diffeomorphisms, B-field and β-transformations

- algebra of Killing vectors still closes

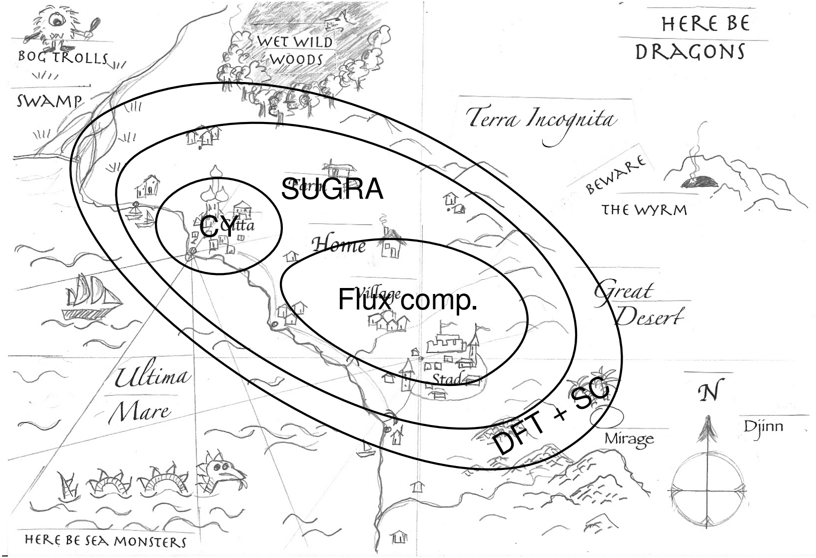
---

at the border of DFT's scope

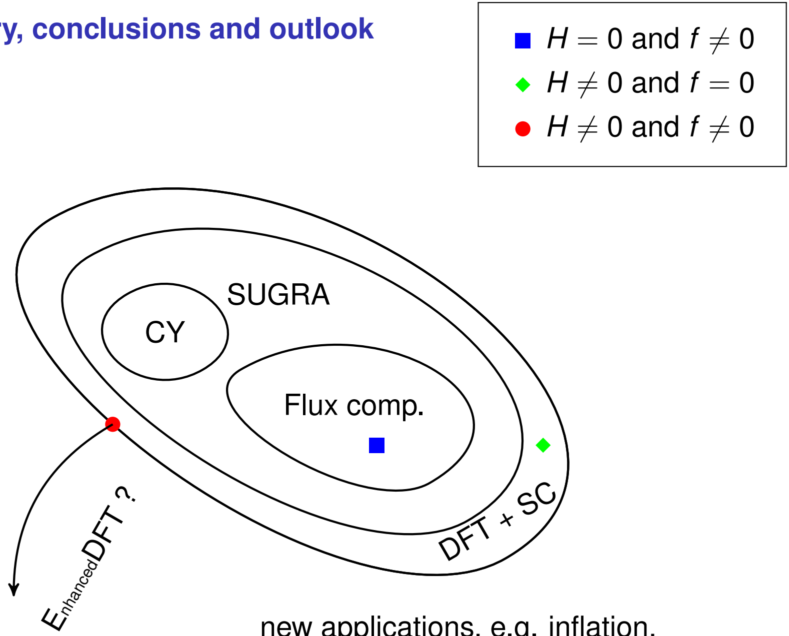
Summary, conclusions and outlook



Summary, conclusions and outlook



Summary, conclusions and outlook



new applications, e.g. inflation,
non-associative geometry[14] , . . .

Thank you for your attention.
Do you have any questions?

Group manifold = Scherk-Schwarz ansatz in doubled coordinates

1. Homogenious space in $2(D-d)$ dimensions

- space "looks" at every point the same
- $2(D-d)$ linear independent Killing vector ${K_I}^J$

$$\boxed{\mathcal{L}_{{K_I}^J}\mathcal{H}^{MN} = 0 \quad \text{and} \quad \mathcal{L}_{{K_I}^J}\phi' = 0}$$

- infinitesimal translations $\mathcal{L}_{{K_I}^J}$ form group G_L

2. Gauge transformations

- map space to itself by

$$\mathcal{L}_{{U_N}^M}\mathcal{H}^{IJ} = -\mathcal{F}_{IML}{U_N}^M\mathcal{H}^{LJ} - \mathcal{F}_{JML}{U_N}^M\mathcal{H}^{IL}$$

- infinitesimal translations $\mathcal{L}_{{U_N}^M}$ form group G_R
- structure coefficients $\mathcal{F}_{IJK}$ = covariant fluxes
- closure of $G_\mathrm{R} \rightarrow$ constraints on $\mathcal{F}_{IJK}$

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