MAX-PLANCK-GESELLSCHAFT

Max-Planck-Institut für Physik
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Recent advances in field theory dualities from string theory

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Dualities in field theory and string theory

Equivalence of two different descriptions of a physical system, either as exactly equivalent and complementary descriptions:

- Montonen-Olive duality in $\mathcal{N} = 4$ SYM
- T-duality (and mirror symmetry)
- AdS/CFT
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Goal

Understand better the strongly coupled dynamics of QFT.

String theory as a duality generator

Many of these dualities can be formulated purely in field theory, without knowing any string theory:

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But string theory provides a fantastic tool to **predict** and **analyze** field theory dualities.

Heiberg duality

Seiberg duality

Basic form of the duality

Given a theory A flowing to a non-trivial IR fixed point, there is a (different) theory B flowing to the same IR fixed point.

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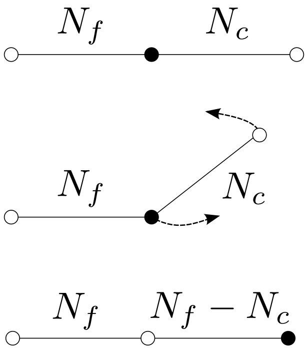
flows to the same IR fixed point as

SQCD with gauge group $SU(N_f - N_c)$, N_f flavors q, $\widetilde{q}$, a neutral meson M in the adjoint of the flavor group, and superpotential

$$W = \mathrm{Tr}\,(qM\widetilde{q})$$

Seiberg duality from string theory

The duality in string theory:



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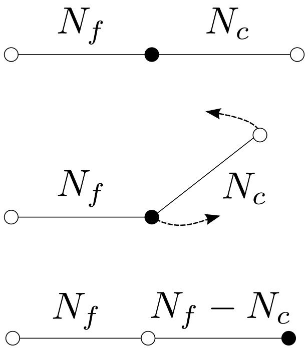

Very easy to generalize, and obtain other dual pairs.

Montonen-Olive duality

Montonen-Olive duality

Given a 4d $\mathcal{N}=4$ field theory with gauge group G and gauge coupling $\tau=\theta+i/g^2$, there is a completely equivalent description with gauge group $G^\vee$ and coupling $-1/\tau$ (for $\theta=0$ this is $g \leftrightarrow 1/g$). Examples:

G	$G^\vee$
$U(1)$	$U(1)$
$U(N)$	$U(N)$
$SU(N)$	$SU(N)/\mathbb{Z}_N$
$SO(2N+1)$	$Sp(2N)$

Very non-perturbative duality, exchanges **gauge bosons** with **monopoles**! (So, the usual field theory tools are not particularly illuminating here.)

Montonen-Olive duality

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Duality dictionary:

- Fundamental string $\leftrightsquigarrow$ D1 solitonic brane.
- D3 $\leftrightsquigarrow$ D3.
- $O3^{+} \leftrightsquigarrow \widetilde{O3^{-}}$.
- (p, q) 7-brane $\leftrightsquigarrow$ $(-q, p)$ 7-brane.
- ...

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Beautiful field theory insights follow trivially from the duality dictionary. For example, the gauge boson $\leftrightsquigarrow$ monopole map follows easily from the $F1 \leftrightsquigarrow D1$ duality dictionary entry.

Beyond $\mathcal{N} = 4$

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New $\mathcal{N} = 1$ dualities

Engineer certain $\mathcal{N} = 1$ theories in IIB, develop the S-duality dictionary as needed, and read the effect of strong/weak duality on $\mathcal{N} = 1$ theories.

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Work in collaboration with B. Heidenreich and T. Wrase
`arXiv:1210.7799, arXiv:1307.1701 and work to appear`

New $\mathcal{N}=1$ dualities

	$USp(\widetilde{N}+4)$	$SU(\widetilde{N})$	$SU(3)$	$U(1)_R$	$\mathbb{Z}_3$
A^i	$\overline{\square}$	$\square$	$\square$	$\frac{2}{3}-\frac{2}{\widetilde{N}}$	1
B^i	1	$\overline{\square\square}$	$\square$	$\frac{2}{3}+\frac{4}{\widetilde{N}}$	-2

(here $\widetilde{N}\in 2\mathbb{Z}$) is dual to

	$SO(N-4)$	$SU(N)$	$SU(3)$	$U(1)_R$	$\mathbb{Z}_3$
A^i	$\overline{\square}$	$\square$	$\square$	$\frac{2}{3}+\frac{2}{N}$	1
B^i	1	$\overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$	$\square$	$\frac{2}{3}-\frac{4}{N}$	-2

in both cases with $W=\frac{1}{2}\epsilon_{ijk}\mathrm{Tr}\,A^iA^jB^k$.

New $\mathcal{N} = 1$ dualities

The resulting dual pairs are very interesting

- Non-conformal
- Chiral
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A number of very non-trivial checks pass with flying colors

- Moduli space matching
- IR quantum dynamics match (when understood)
- 't Hooft anomaly matching
- **Superconformal index matching**

Dualities from Riemann surfaces

$\mathcal{N} = 2$ dualities from Riemann surfaces

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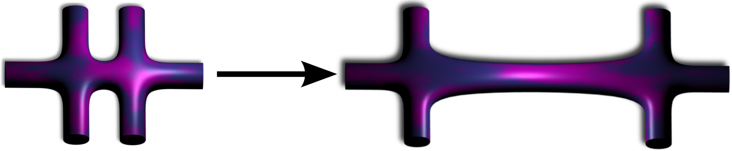

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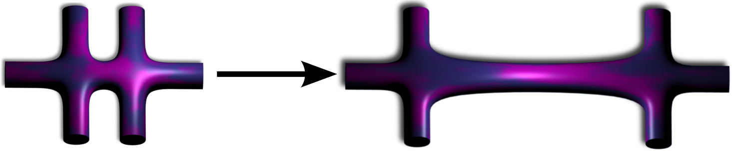

Deep relation between the 4d theory and the 2d surface. (Alday, Gaiotto, Tachikawa)

Conclusions and further directions

Main lesson

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If you see a beautiful field theory phenomenon, chances are that it becomes even prettier when formulated in string theory.

Further directions

Much to explore along these lines:

- Seiberg dualities for exceptional groups
- $\mathcal{N}=0$ "Montonen-Olive" dualities (Hook, Torroba)
- Confinement as a dual Meissner effect (Sugimoto)
- $\mathcal{N}=1$ generalizations of $\mathcal{N}=2$ dualities (Xie)
- ...