Lessons from String Amplitudes

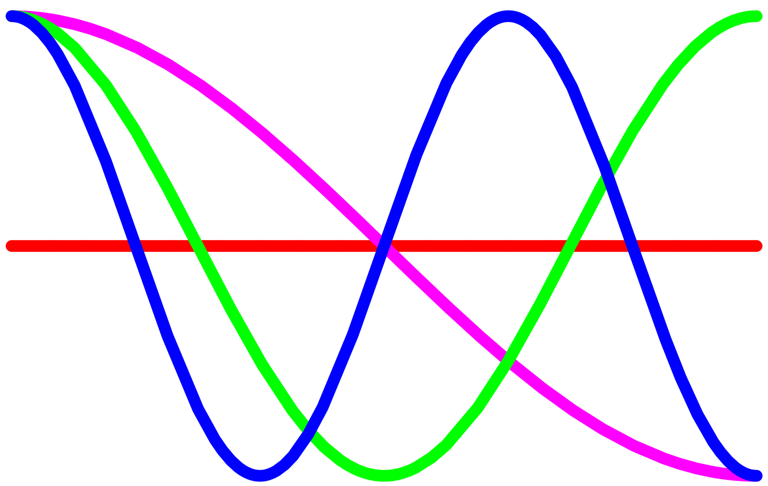
Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

Stephan Stieberger, MPP München

- *Impact of a low string scale to string amplitudes at colliders*
- *High energy behaviour of string amplitudes*
- *String amplitudes as tools for field-theory amplitudes*
- *Unified description of tree-level superstring & field-theory amplitudes*

String amplitudes at Colliders

Strings:



- massless modes (m=0): graviton, gauge boson, …
- massive modes

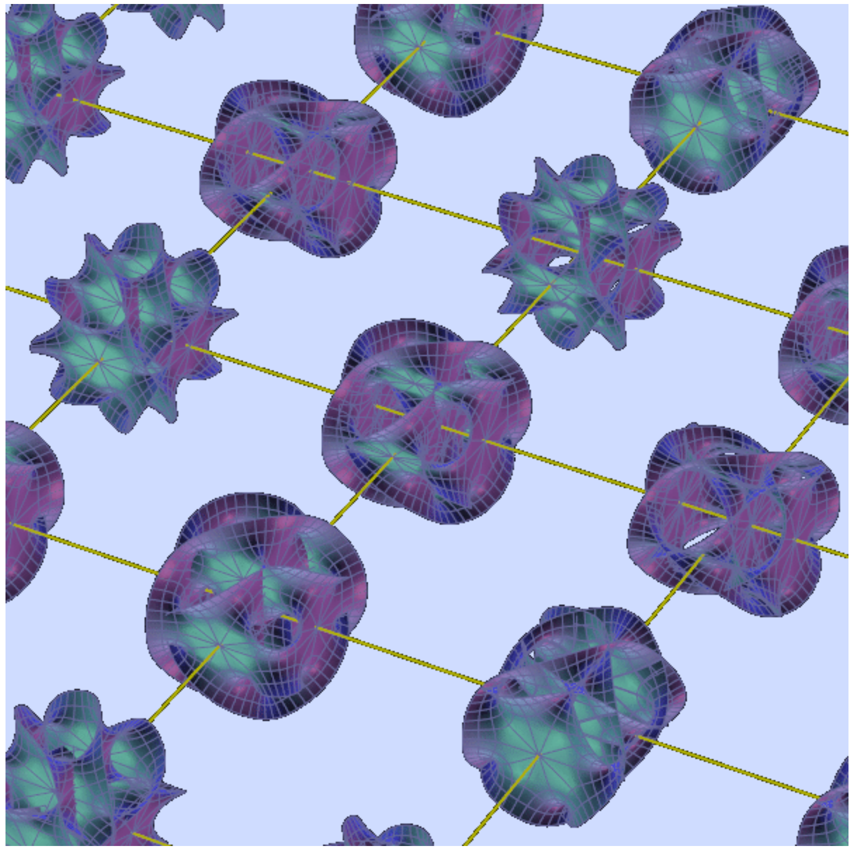
$$m \sim M_{\text{string}} \sim \frac{1}{\sqrt{\alpha'}}$$

String theory is only consistent in D=10

$\Longrightarrow$ Compactification on manifold X_6

$$\text{space} - \text{time}: \qquad M_4 \times X_6$$

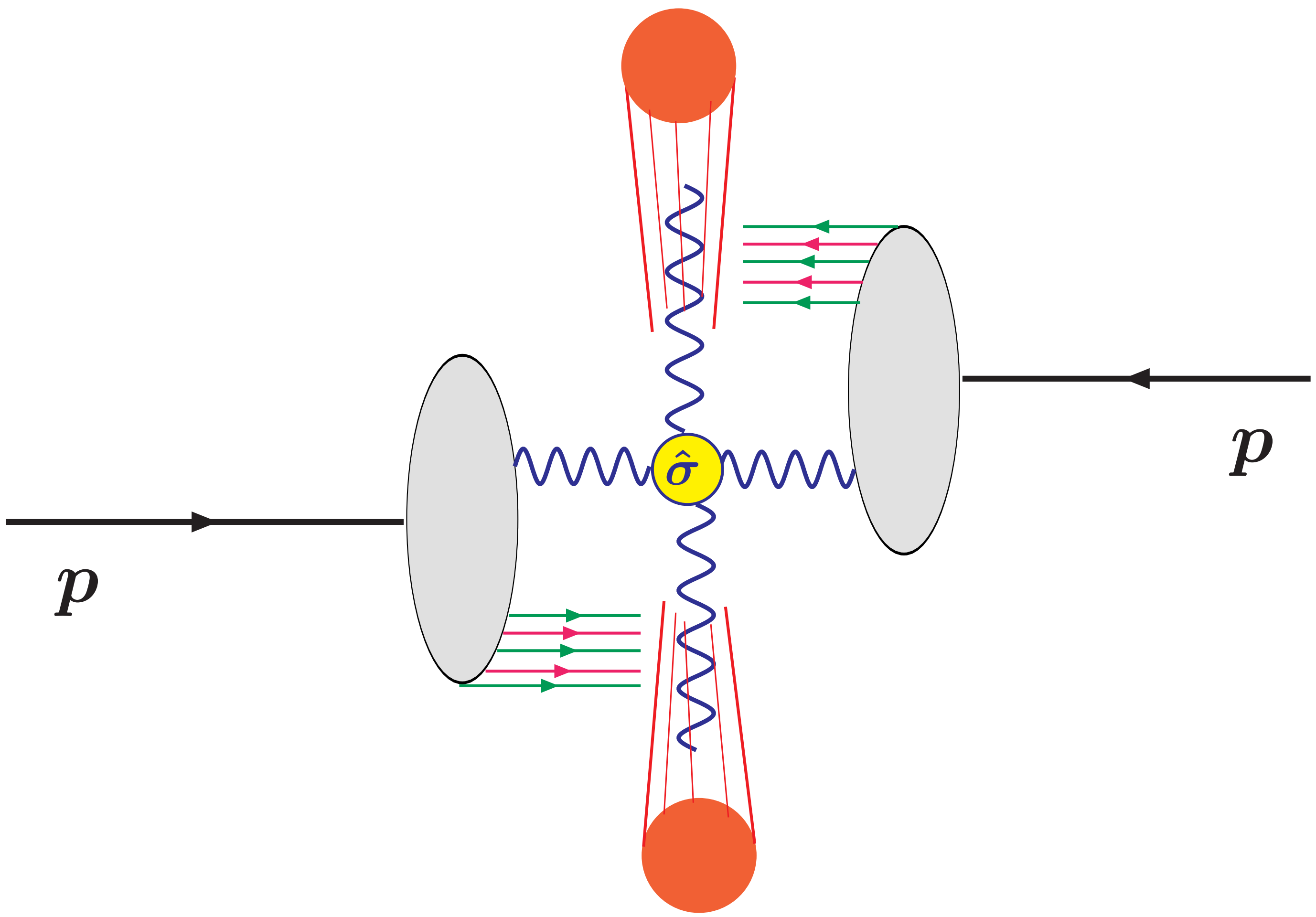
$$\begin{pmatrix} y^i \\ x^\mu \end{pmatrix}, \; i = 4, \ldots, 9 \quad , \quad \mu = 0, \ldots, 3$$



many possible manifolds X_6

$\Longrightarrow$ huge number of D=4 string vacua

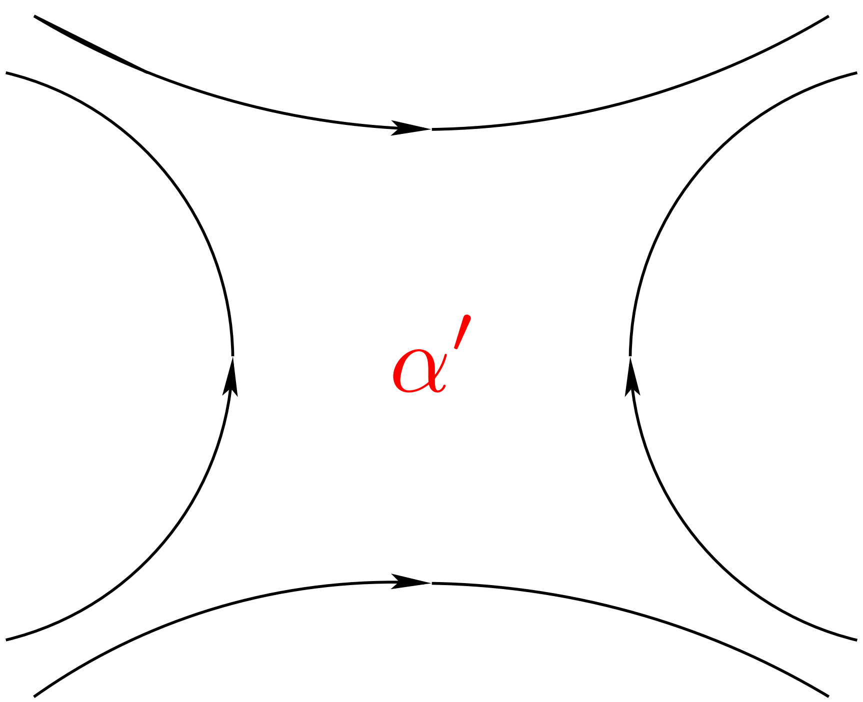
Question: *Can we make model-independent low-energy string predictions from parton amplitudes in superstring theory ?*



LHC: Multijet production is dominated by tree-level QCD-scattering

Parton amplitudes are important for (collider) phenomenology

String signatures at LHC ?

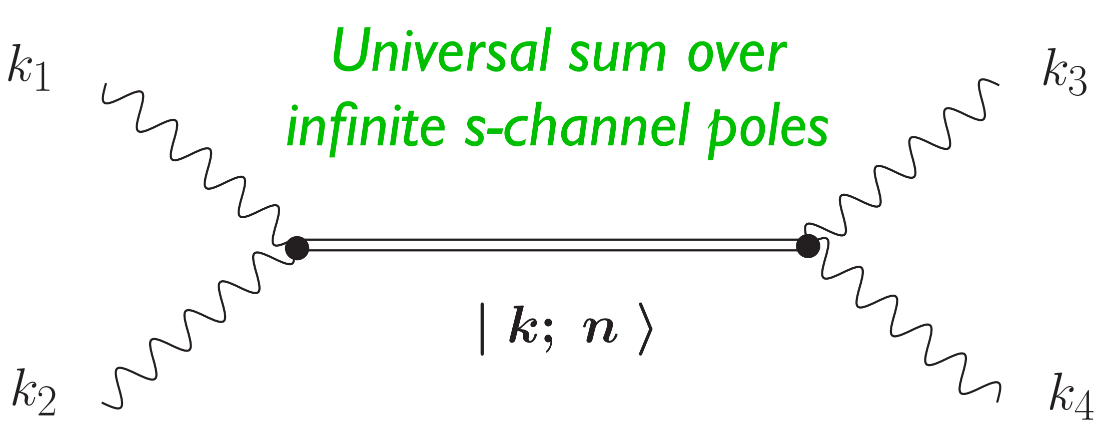


$$\left.\begin{array}{l}\mathcal{A}(g^{a_1}, g^{a_2}, g^{a_3}, g^{a_4}) \\ \mathcal{A}(\chi^{a_1}, \overline{\chi}^{a_2}, g^{a_3}, g^{a_4}) \\ \mathcal{A}(\psi^{a_1}, \overline{\psi}^{a_2}, g^{a_3}, g^{a_4}) \\ \mathcal{A}(\phi^{a_1}, \overline{\phi}^{a_2}, g^{a_3}, g^{a_4})\end{array}\right\}$$

- *completely model independent*
- *for any string compactification*
- *any number of supersymmetries*
- *even with broken supersymmetry*

$g =$ gluon, $\chi =$ gaugino, $\psi =$ fermion, $\phi =$ scalar

$$\mathcal{A}(k_1, k_2, k_3, k_4; \alpha') \sim \frac{\Gamma(-s)\ \Gamma(1-u)}{\Gamma(-s-u)} = \sum_{n=0}^{\infty} \frac{\gamma(n)}{(k_1+k_2)^2 - M_n^2}$$



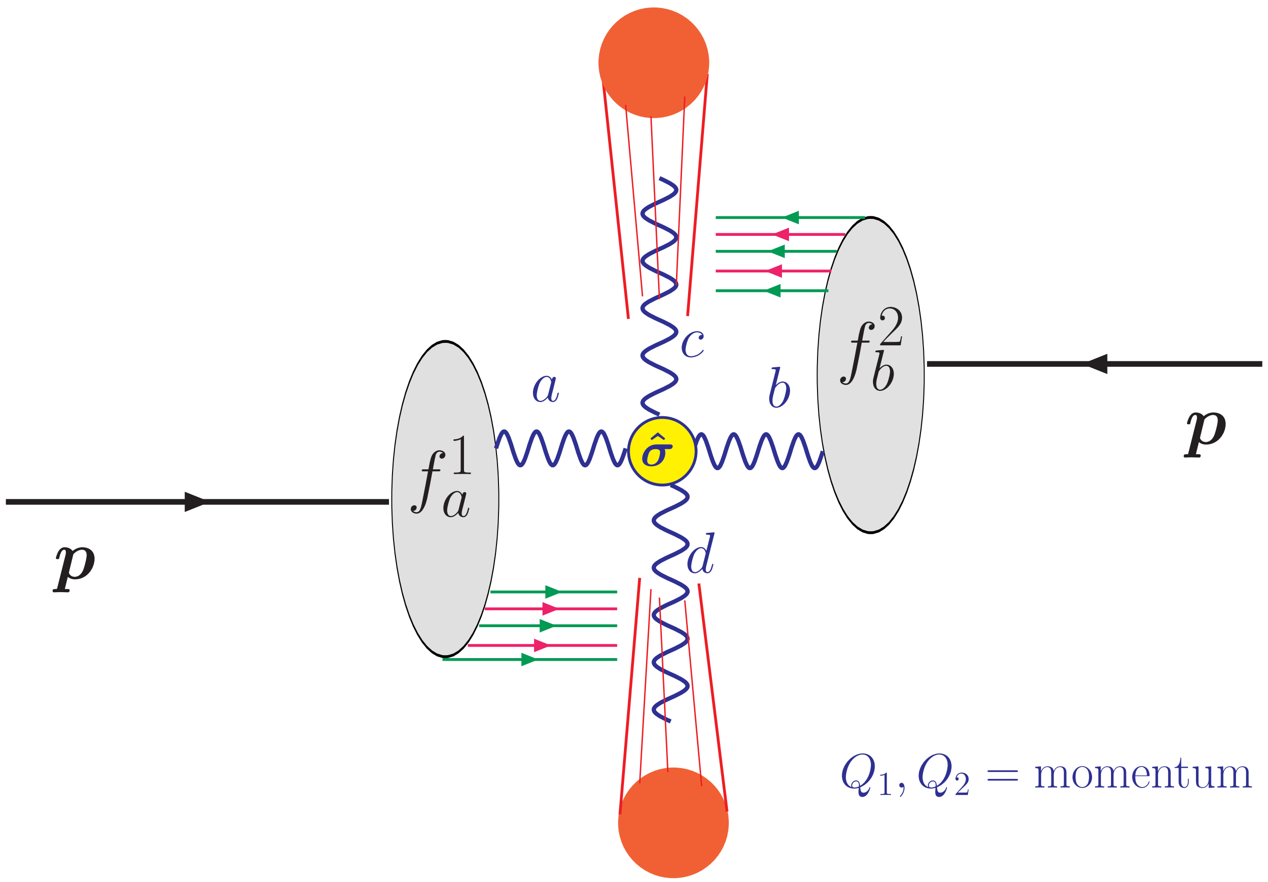
intermediate mass: $M_n^2 = M_{\text{string}}^2\ n$

residua: $\gamma(n) \sim u^n$

$s = -\alpha'(k_1+k_2)^2\ , \qquad u = -\alpha'(k_1+k_4)^2\ ,$

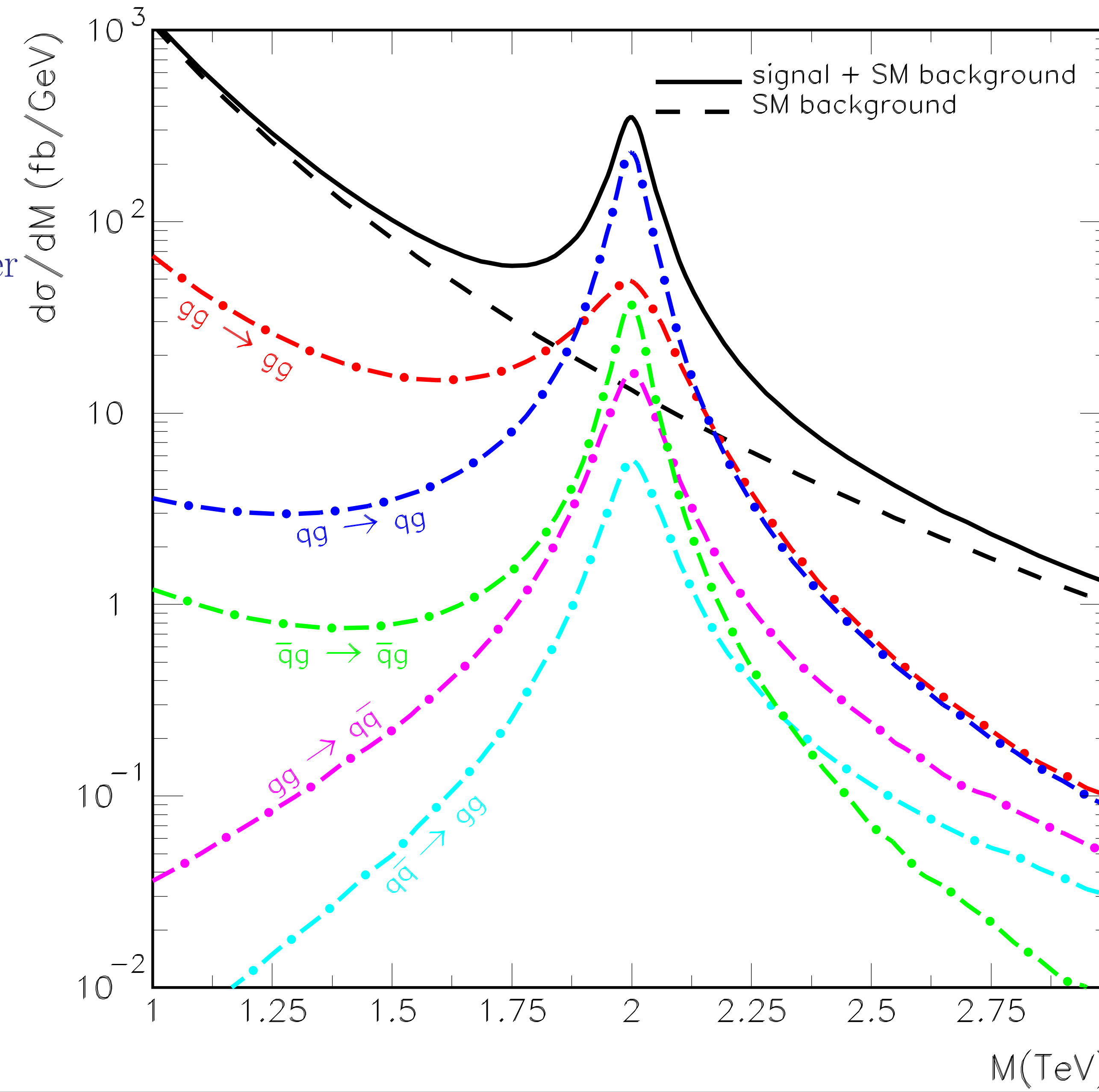
$t = -\alpha'(k_1+k_3)^2\ , \qquad s+t+u = 0$

$$\sigma(pp \to 2 \text{ jets}) = \sum_{a,b,c,d} \int dx_1 \, dx_2 \, f_a^1(x_1; Q_1^2) \, f_b^2(x_2; Q_2^2) \, \hat{\sigma}_{ab\to cd}(\underbrace{sx_1x_2}_{\hat{s}}; \underbrace{Q_1^2, Q_2^2}_{Q_1^2=Q_2^2=\hat{t}}, \alpha')$$



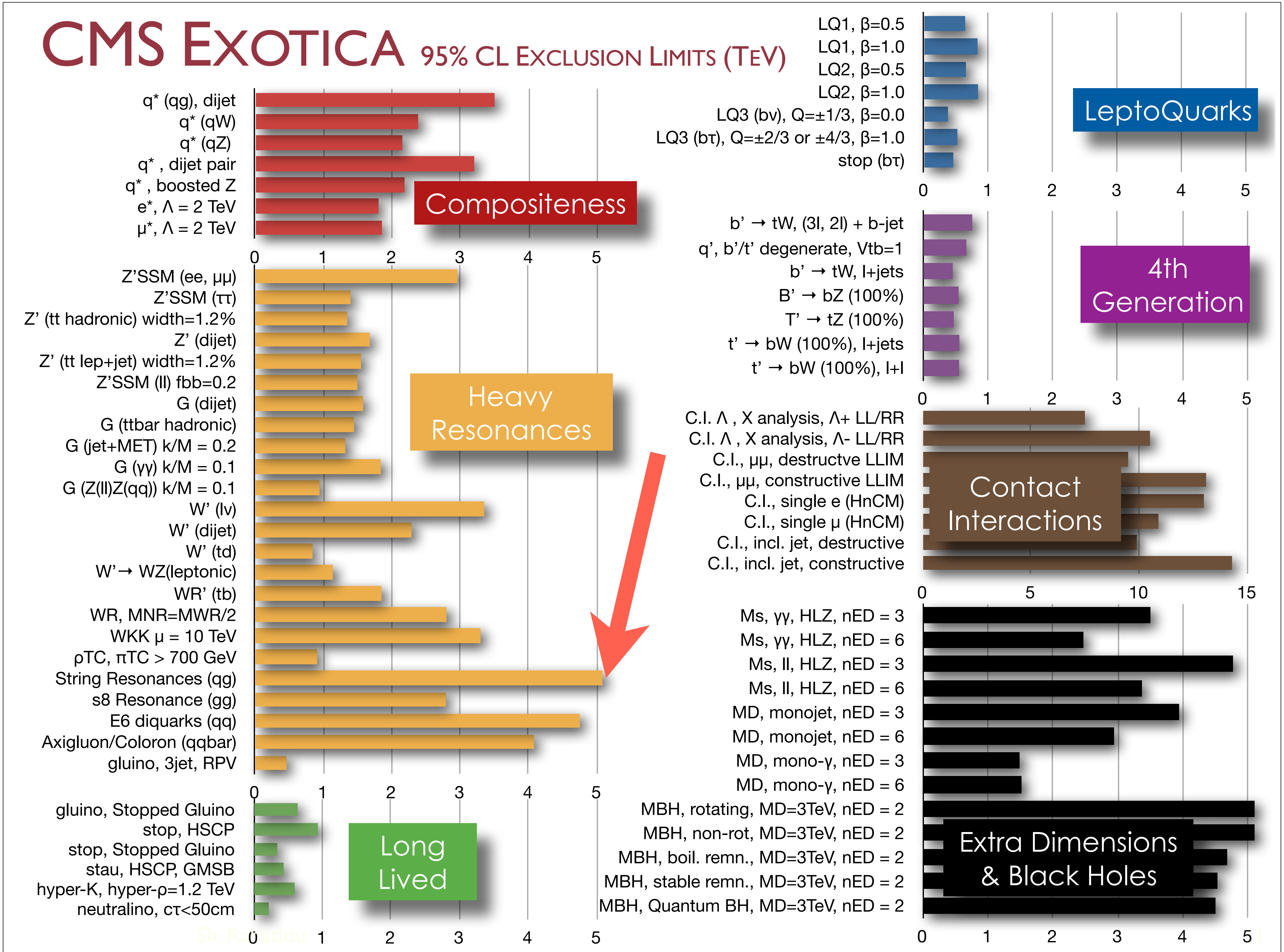
$Q_1, Q_2 =$ momentum transfer

Anchordoqui, Goldberg, Lüst, Nawata, Stieberger, Taylor



Universal deviation from SM in jet distribution

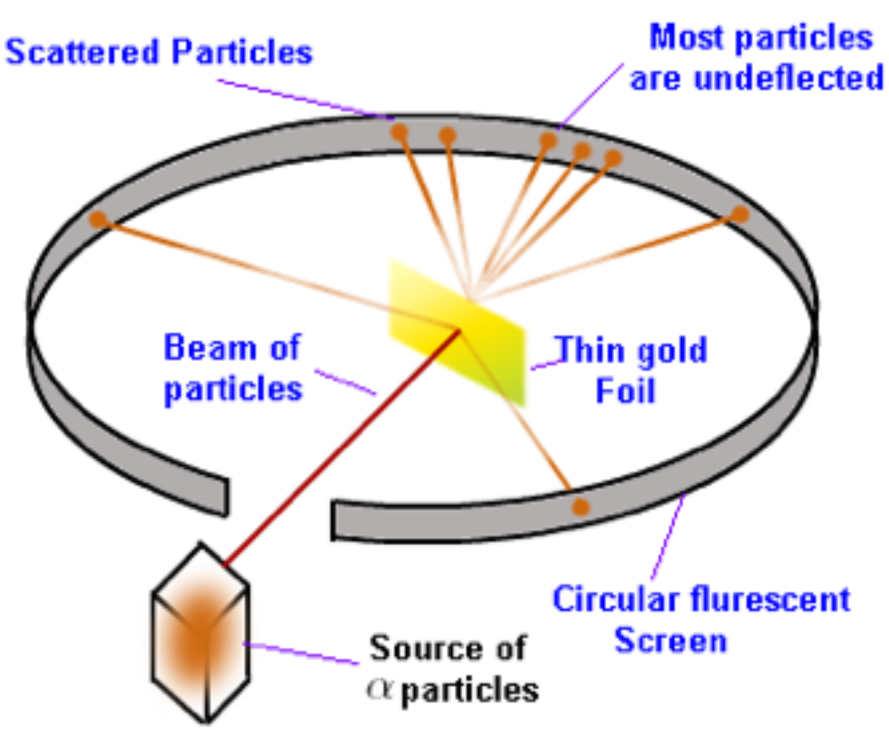
Any superstring theory with low M_{string} and g_{string}<1


CMS Exotica 95% CL Exclusion Limits (TeV)
Compositeness
q* (qg), dijet
q* (qW)
q* (qZ)
q* , dijet pair
q* , boosted Z
e*, Λ = 2 TeV
μ*, Λ = 2 TeV
0 1 2 3 4 5
Heavy Resonances
Z'SSM (ee, μμ)
Z'SSM (ττ)
Z' (tt hadronic) width=1.2%
Z' (dijet)
Z' (tt lep+jet) width=1.2%
Z'SSM (ll) fbb=0.2
G (dijet)
G (ttbar hadronic)
G (jet+MET) k/M = 0.2
G (γγ) k/M = 0.1
G (Z(ll)Z(qq)) k/M = 0.1
W' (lν)
W' (dijet)
W' (td)
W'→ WZ(leptonic)
WR' (tb)
WR, MNR=MWR/2
WKK μ = 10 TeV
ρTC, πTC > 700 GeV
String Resonances (qg)
s8 Resonance (gg)
E6 diquarks (qq)
Axigluon/Coloron (qqbar)
gluino, 3jet, RPV
0 1 2 3 4 5
Long Lived
gluino, Stopped Gluino
stop, HSCP
stop, Stopped Gluino
stau, HSCP, GMSB
hyper-K, hyper-ρ=1.2 TeV
neutralino, cτ<50cm
0 1 2 3 4 5
LeptoQuarks
LQ1, β=0.5
LQ1, β=1.0
LQ2, β=0.5
LQ2, β=1.0
LQ3 (bν), Q=±1/3, β=0.0
LQ3 (bτ), Q=±2/3 or ±4/3, β=1.0
stop (bτ)
0 1 2 3 4 5
4th Generation
b' → tW, (3l, 2l) + b-jet
q', b'/t' degenerate, Vtb=1
b' → tW, l+jets
B' → bZ (100%)
T' → tZ (100%)
t' → bW (100%), l+jets
t' → bW (100%), l+l
0 1 2 3 4 5
Contact Interactions
C.I. Λ , X analysis, Λ+ LL/RR
C.I. Λ , X analysis, Λ- LL/RR
C.I., μμ, destructve LLIM
C.I., μμ, constructive LLIM
C.I., single e (HnCM)
C.I., single μ (HnCM)
C.I., incl. jet, destructive
C.I., incl. jet, constructive
0 5 10 15
Extra Dimensions & Black Holes
Ms, γγ, HLZ, nED = 3
Ms, γγ, HLZ, nED = 6
Ms, ll, HLZ, nED = 3
Ms, ll, HLZ, nED = 6
MD, monojet, nED = 3
MD, monojet, nED = 6
MD, mono-γ, nED = 3
MD, mono-γ, nED = 6
MBH, rotating, MD=3TeV, nED = 2
MBH, non-rot, MD=3TeV, nED = 2
MBH, boil. remn., MD=3TeV, nED = 2
MBH, stable remn., MD=3TeV, nED = 2
MBH, Quantum BH, MD=3TeV, nED = 2
0 1 2 3 4 5

Future Projects

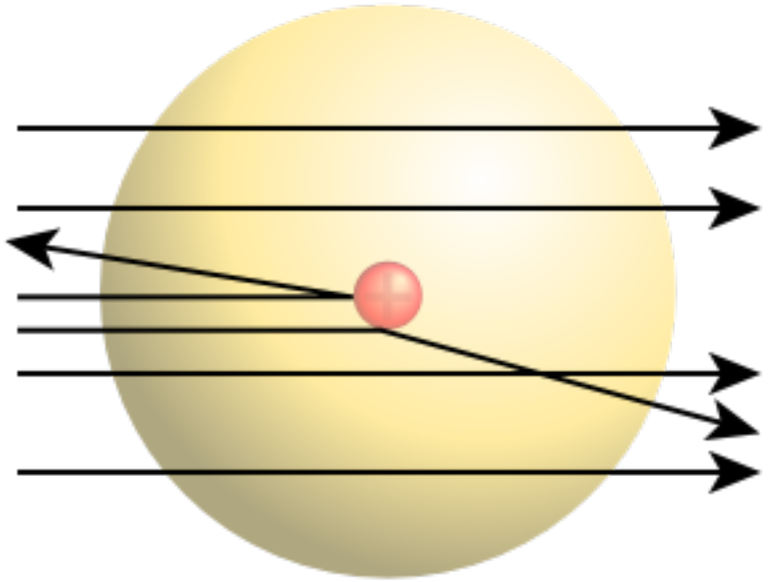
- Update computations and adjust to recent LHC data
- Multi-jet processes: spin-dependence, ...
- one-loop corrections
- prepare for future colliders: linear collider, e.g. processes with Z'

High energy behaviour of string amplitudes



Hard scattering (Rutherford scattering) probes the internal structure of the objects being scattered

$$\frac{d\sigma}{d\Omega} \sim \frac{1}{E^2} \frac{1}{\sin^4\left(\frac{\theta}{2}\right)}$$



$$s, t \to \infty \quad , \quad \frac{t}{s} = fixed$$

fixed angle θ

$$\begin{aligned} s &= \frac{E^2}{M^2_{\rm string}} \\ t &= -\frac{E^2}{M^2_{\rm string}} \sin^2\frac{\theta}{2} \\ u &= -\frac{E^2}{M^2_{\rm string}} \cos^2\frac{\theta}{2} \end{aligned}$$

In string theory:

$$\mathcal{A}_4 \sim \exp\left\{E^2 \left(\sin^2\frac{\theta}{2}\ln\sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2}\ln\cos^2\frac{\theta}{2}\right)\right\}$$

Obtained from a saddle point approximation w.r.t. string world-sheet coordinates

$$\mathcal{A}_4 \sim \int_0^1 dx\; x^s\; (1-x)^u = \int_0^1 dx\;\; \exp\left\{s \ln x + u \ln(1-x)\right\}$$

Saddle point approximation: $s,t,u \to \infty$ $\qquad x_0 = \dfrac{s}{s+u}$ *(Laplace method)*

Generalization to N>4:

$$\mathcal{A}_N \sim \int_{z_1<\ldots<z_N} \left(\prod_{l=1}^{N} dz_l\right) \prod_{i<j}^{N} |z_i - z_j|^{s_{ij}}\; \delta\left(\sum_{b\neq a}^{N} \frac{s_{ab}}{z_a - z_b}\right)$$

Scattering equations (Witten):

$$s_{ij} = \alpha'\;(k_i + k_j)^2$$

(N-3)! solutions

$$\sum_{b\neq a} \frac{s_{ab}}{z_a - z_b} = 0 \quad , \quad a \in \{1,\ldots,N\}$$

related to discussions with:
Dieter, Gia
and Isermann ?

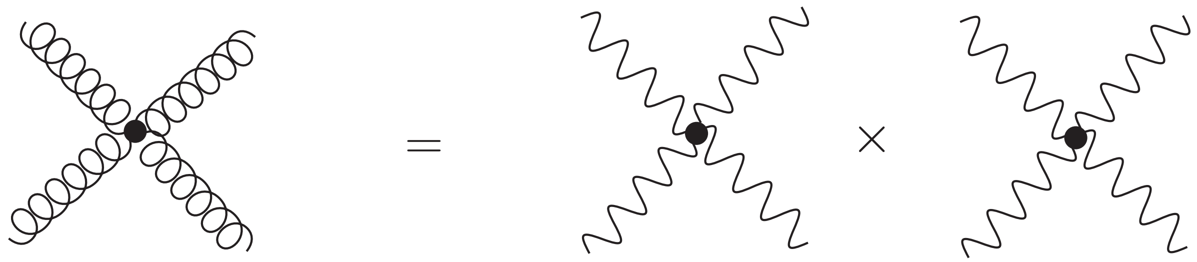
Equations describe also Yang-Mills theory

Link between YM and string theory at high energies !

Sign, that SYM and string theory have same underlying principles and descriptions ?

String amplitudes as tools for field-theory amplitudes

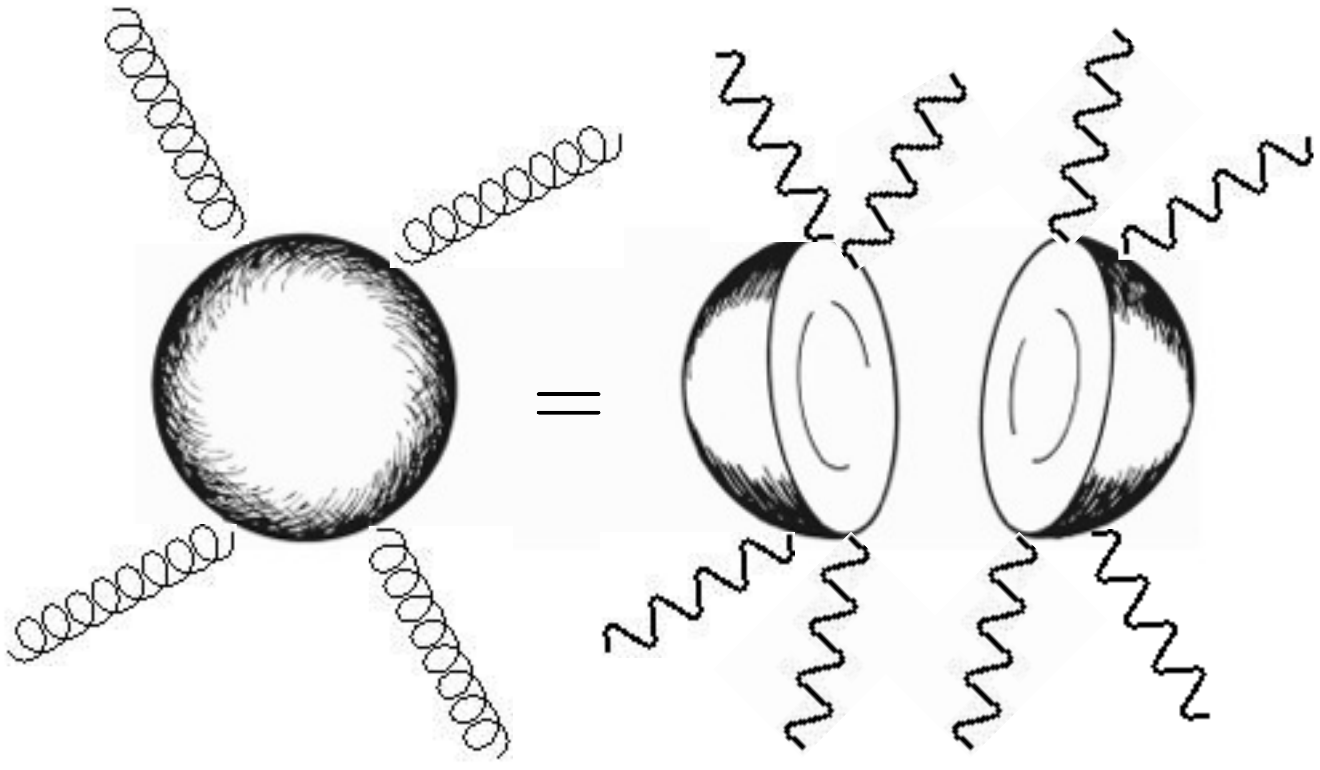
The structure of string amplitudes shows remarkable impact on the form and organization of field-theory amplitudes



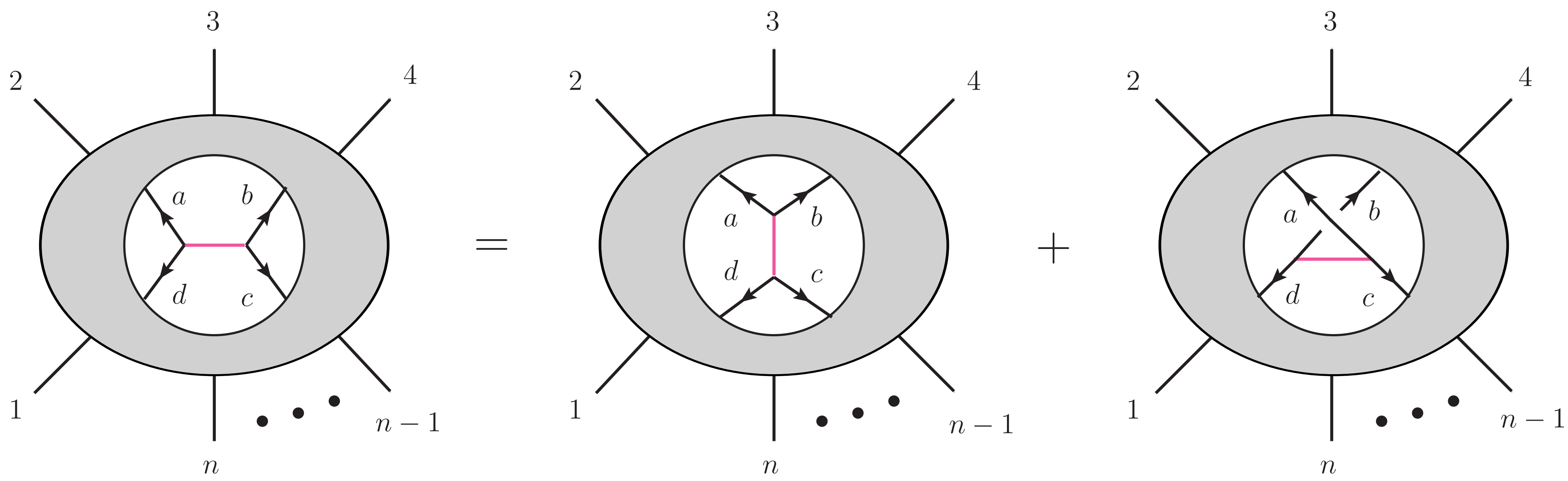
$$M_{FT}(1,2,3,4) = s_{12}\, A_{FT}(1,2,3,4)\, \tilde{A}_{FT}(1,2,4,3)$$

$$\text{graviton amplitudes} = (\text{gauge amplitudes}) \times (\text{gauge amplitudes})$$

KLT relations



BCJ relations at work



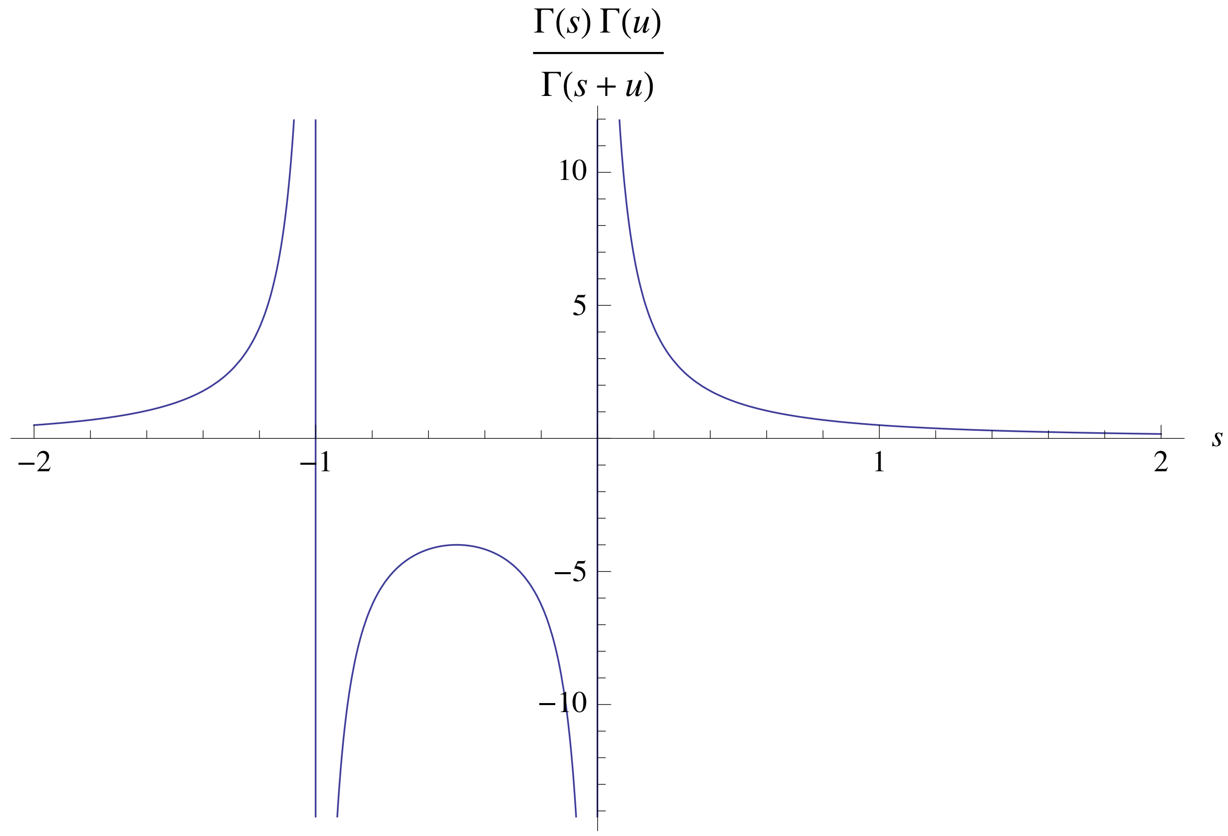
Monodromy properties of the string world-sheet yield BCJ relations

Stieberger

Many more ***concepts*** *and* ***structures***
of field-theory amplitudes ***following*** *from*
mathematical aspects *(motives, symbols, coproduct, ...)*
of ***string amplitudes***
to efficiently compute Feynman diagrams

Properties of scattering amplitudes in both gauge and gravity theories suggest a deeper understanding from string theory

Euler Betafunction



(Inverse) double Mellin transformation:

$$\frac{1}{(2\pi i)^2}\int_{-i\infty+c}^{+i\infty+c} ds \int_{-i\infty+c}^{+i\infty+c} du\; x^{-s}\; y^{-u}\; \frac{\Gamma(s)\;\Gamma(u)}{\Gamma(s+u)}$$

$$= \delta(1-x-y)\;\theta(1-x)\;\theta(1-y)$$

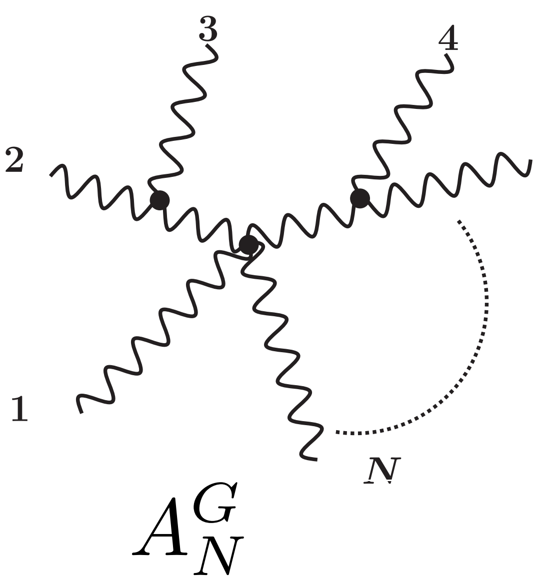
Correlation functions from delta-functions and residua integrals

Multiple (inverse) Mellin transforms trivialize tree-level string amplitudes

Stieberger, Taylor

Superstring/supergravity Mellin correspondence

A unified description of superstring and supergravity amplitudes

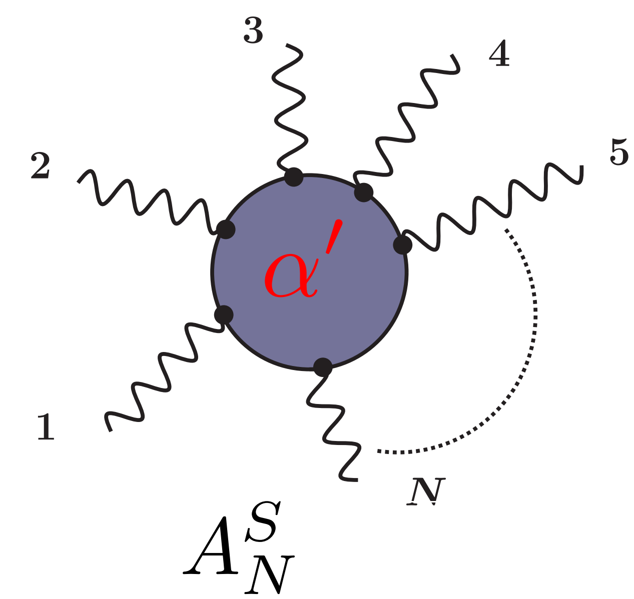


$$A_N^G = \int dz\ \delta(z,\lambda) \int dz'\ \delta(z',\lambda)\ H_N(z,z')$$

$$A_N^S = \int d\mu(z) \int dz'\ \delta(z',\lambda)\ H_N(z,z')$$

↓ multiple Mellin transform

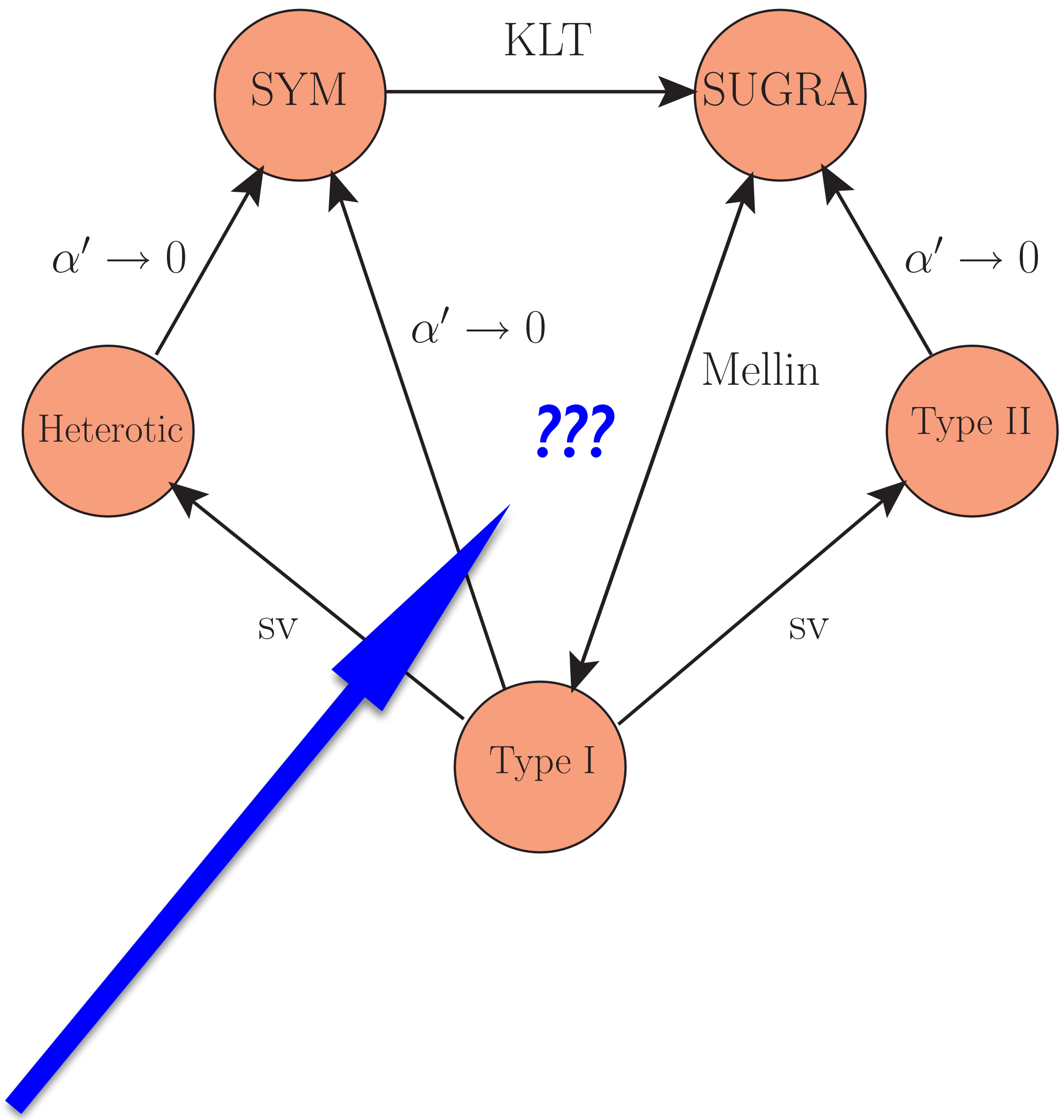
↓ Hodges' determinant



- Striking match between supergravity and open superstring tree-level amplitude communicated by Hodges' determinant
- Mellin transform from string world-sheet into dual space of kinematic invariants thus bypassing space-time
- yields possible profound connection towards a dual description of perturbative string theory

Unity of tree-level field-theory and superstring couplings

Amplitude space =
space of physical observables



Unexpected connections between
field- and string theory amplitudes !

Amplitudes in non-trivial background,
e.g.: warped geometries, AdS_5,...

Future Projects

- String amplitudes serve as models for understanding field-theory aspects (e.g. high energy behaviour, ...)
- String amplitudes serve as tools for deriving properties of field-theory amplitudes (e.g. KLT, BCJ, ...)
- Understand web of amplitudes in view of string dualities or within M-theory => dual description of perturbative string theory

SPRING SCHOOL on SUPERSTRING THEORY and RELATED TOPICS

31 March - 8 April 2014

Miramare, Trieste, Italy

TOPICS:

- **Dynamics of Supersymmetric Theories**
- **Entanglement Entropy and Holography**
- **Integrability in N=4 Yang-Mills Theory**
- **Higher Spins and Holography**
- **Physics of Black-Holes**
- **Scattering Amplitudes in Field- and String-Theory**

The Aim of the activity is to provide pedagogical treatment of these subjects in the form of a series of lectures by individual speakers. The activity is intended for theoretical physicists or mathematicians with knowledge of quantum field theory, general relativity and string theory.

PARTICIPATION

Scientists and students from all countries which are members of the United Nations, UNESCO or IAEA may attend the School. As it will be conducted in English, participants should have an adequate working knowledge of this language. Although the main purpose of the Centre is to help research workers from developing countries, through a programme of training activities within a framework of international cooperation, students and post-doctoral scientists from advanced countries are also welcome to attend.

As a rule, travel and subsistence expenses of the participants should be borne by the home institution. Every effort should be made by candidates to secure support for their fare (or at least half-fare). However,

ORGANIZERS:

N. ARKANI-HAMED
(IAS, Princeton)

E. GAVA
(INFN, Trieste)

R. GOPAKUMAR
(HRI, Allahabad)

K.S. NARAIN
(ICTP, Trieste)

S. RANDJBAR-DAEMI
(ICTP, Trieste)

LECTURERS INCLUDE:

N. ARKANI-HAMED
(IAS, Princeton)

M. GABERDIEL
(ETH, Zurich)

J. MALDACENA
(IAS Princeton)

N. SEIBERG
(IAS Princeton)

S. STIEBERGER
(Max-Planck-Institut, Munich)

T. TAKAYANAGI
(Kyoto University)