UNIVERSITY OF BARI
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ANISOTROPIC UNIVERSE AND POLARIZATION OF COSMIC MICROWAVE BACKGROUND

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SUMMARY

1. *Cosmological Principle and Standard Model;*

2. *Bianchi models and Ellipsoidal Universe;*

3. *CMB Anisotropies;*

4. *CMB Polarization;*

5. *CMB Polarization on* ***large scales*** *induced by the anisotropy in the* ***primordial space-time geometry****;*

6. *Conclusions and future developments.*

Cosmological Principle and FLRW geometry

"On large scales, all the observers, wherever they are, measure the same density of galaxies along every direction".

ISOTROPY **HOMOGENEITY**

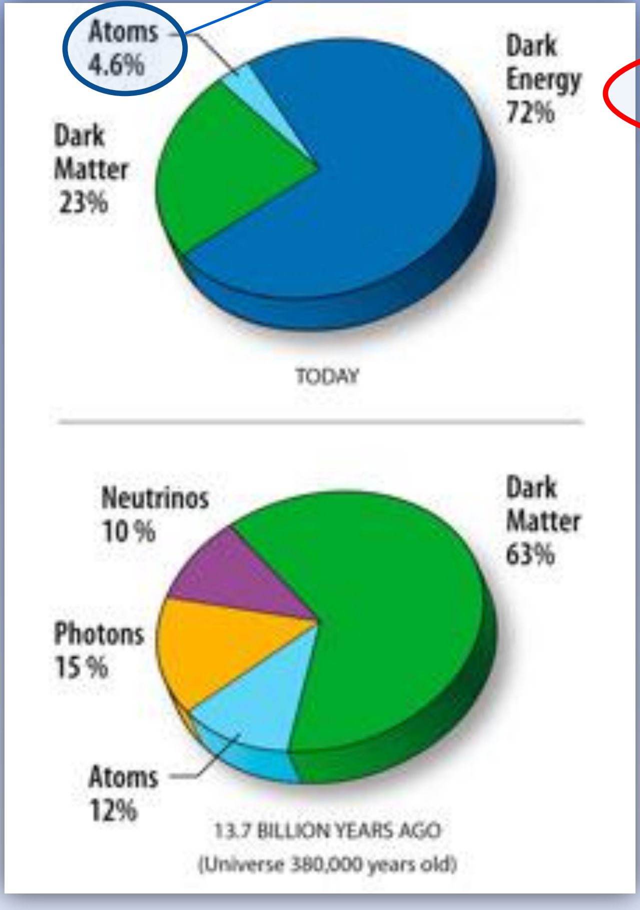
Friedmann, Lemaître, Robertson e Walker (FLRW)

$$ds^2 = N^2(t')dt'^2 - a^2(t')\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right)$$

SCM problems: current era

Phase dominated by matter

1. ***Dark Matter.*** $\Omega_m + \Omega_k = 1 \Rightarrow \Omega_{cdm} + \Omega_b \equiv \Omega_m \simeq 1$



$$\Omega_k \equiv -k/a^2H^2 \leq 0.012$$

Lack of direct experimental check!

2. ***Dark Energy.***

Accelerated expansion factor:

$$\Omega_m + \Omega_\Lambda + \Omega_k = 1$$

Cosmological constant era??

MCS PROBLEMS: EARLY UNIVERSE

3. Flatness.

$$r(t) = \frac{L_H}{L_k} \sim \frac{\text{spatial curvature}}{\text{space-time curvature}}$$

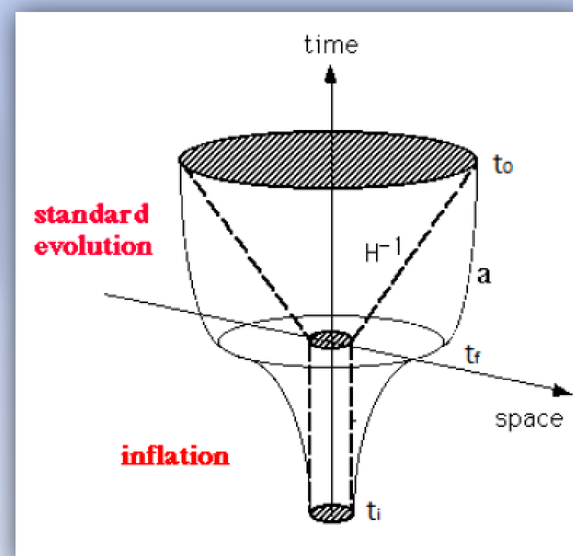
Early state of Universe with a spatial curvature strongly suppressed than space-time curvature: *fine tuning problem*.

*Early stage in which the ratio **decreases in time**, from **"more natural"** conditions (about 1), to values suitable for the standard initial conditions:*
INFLATION

4. Horizon.

$$r(t) = \frac{H^{-1}}{a} \sim \frac{\text{horizon radius}}{\text{part of space radius}}$$

Causal connection problem



Bianchi models

*Models of **anisotropic** space-time,*
*due to **Bianchi** (1897)*

*Catalogue of different types of geometry connected to different groups of **isometries**,*
*identified by a roman number from **I** to **IX***

***Generalized Minkowski metric**, which introduces an anisotropy, but preserves homogeneity:*

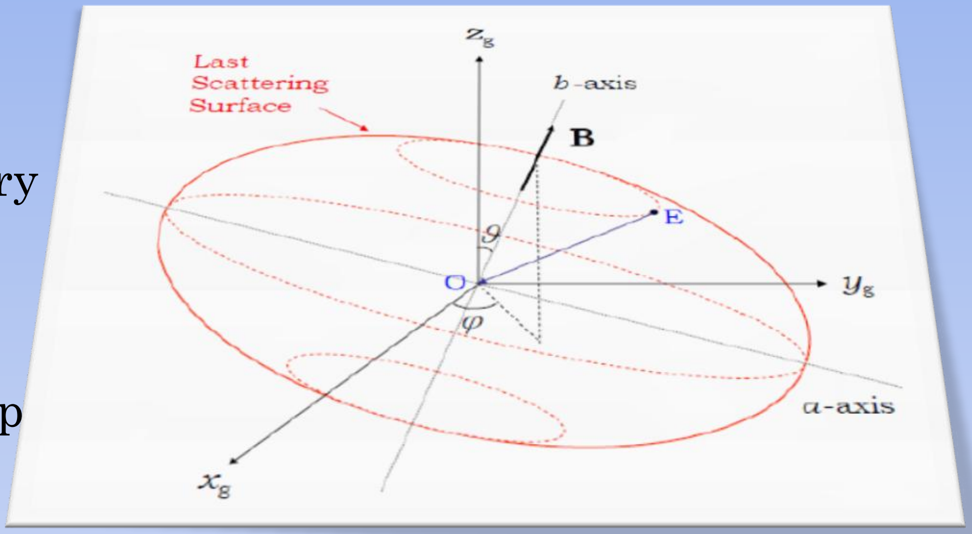
$$ds^2 = c^2 dt^2 - \sum_{i=1}^{d} a_i^2(t) dx_i^2$$

Planar symmetry: Ellipsoidal Universe

Bianchi I

✓ the spatial part of geometry allows 3D translations as a isometry group;

✓3-parameters abelian group



$$ds^2 = dt^2 - a^2(t)\left(dx^2 + dy^2\right) - b^2(t)dz^2$$

Sources: barotropic ideal fluid
homogeneous but anisotropic

$$T^{\mu}{}_{\nu} = diag\left(\rho, -p_{\parallel}, -p_{\parallel}, -p_{\perp}\right)$$

Planar symmetry: Ellipsoidal Universe (2)

Anisotropy sources:

1. *Primordial magnetic field*
2. *Domain wall*
3. *Cosmic string*

Eccentricity

$$e \equiv \sqrt{1 - \left(\frac{b}{a}\right)^2}$$

Einstein equations *(e-dependent):*

$$\frac{d}{dt}\left(\frac{e\dot{e}}{1-e^2}\right) + 3H_e\left(\frac{e\dot{e}}{1-e^2}\right) = \pm 8\pi G\left(p_{\parallel}^A - p_{\perp}^A\right)$$

Solution in Magnetic case

$$e^2 = 8\Omega_B(t_0)\left(1 - 3a^{-1} + 2a^{-3/2}\right)$$

Eccentricity at *decoupling*

$$e_{dec} \simeq 0.64 \cdot 10^{-2}$$

Cosmic Microwave Background

✓*CMB (**Cosmic Microwave Background**) is a "snapshot" of Universe at decoupling, 380000 years after the Big Bang, when Universe had a temperature* $T = 3000K$

✓ *Its lucky discovery was due to Arno Penzias and Robert Wilson in1965, during the calibration of an antenna at Bell Laboratories*

✓ *CMB is composed by photons thermally distributed according to the black-body law, with mean temperature of* $T_0 = 2.73K$

✓ *Photons that managed to reach us today offer a "pure" sight of the **last scattering surface**, in which we had the radiation-matter decoupling*

✓ *CMB has some anisotropies in temperature of the order* $\frac{\Delta T}{T_0} \approx 10^{-5}$

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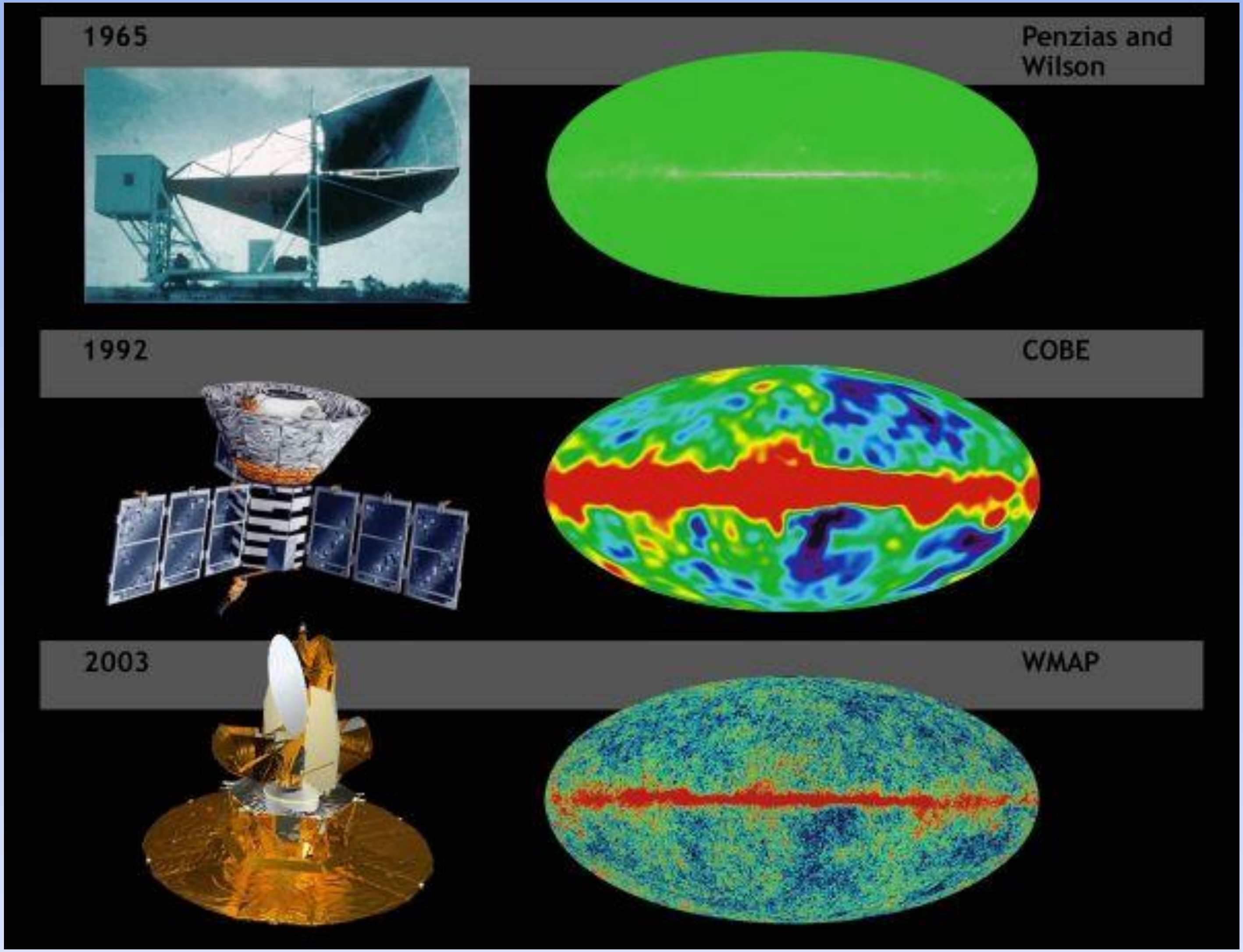
Cosmic Microwave Background (2)



CMB ANISOTROPIES

At decoupling, ***perturbations*** *strengthened by inflation had produced a* ***redshift*** *of CMB photons that we measure today: this effect is revealed by* ***temperature fluctuations*** *depending on their direction.*

First-type Anisotropies	**Second-type Anisotropies**
• Density of matter and gravity: ***Sachs-Wolfe effect***; • Speed: ***Doppler effect***; • ***Acoustic oscillations***.	•***Integrated Sachs-Wolfe (ISW)*** *(early ISW, late ISW, Rees-Sciama)* • ***Weak lensing.***
$z \approx 10^3$	$0 \le z \le 10^3$

CMB Power spectrum

In order to study the directional dependence of temperature fluctuations, we expand anisotropies in ***spherical harmonics*** *extended to the whole sky:*

$$\frac{\Delta T}{T}(\theta, \varphi) = \sum_{lm} a_{lm} Y_{lm}(\theta, \varphi)$$

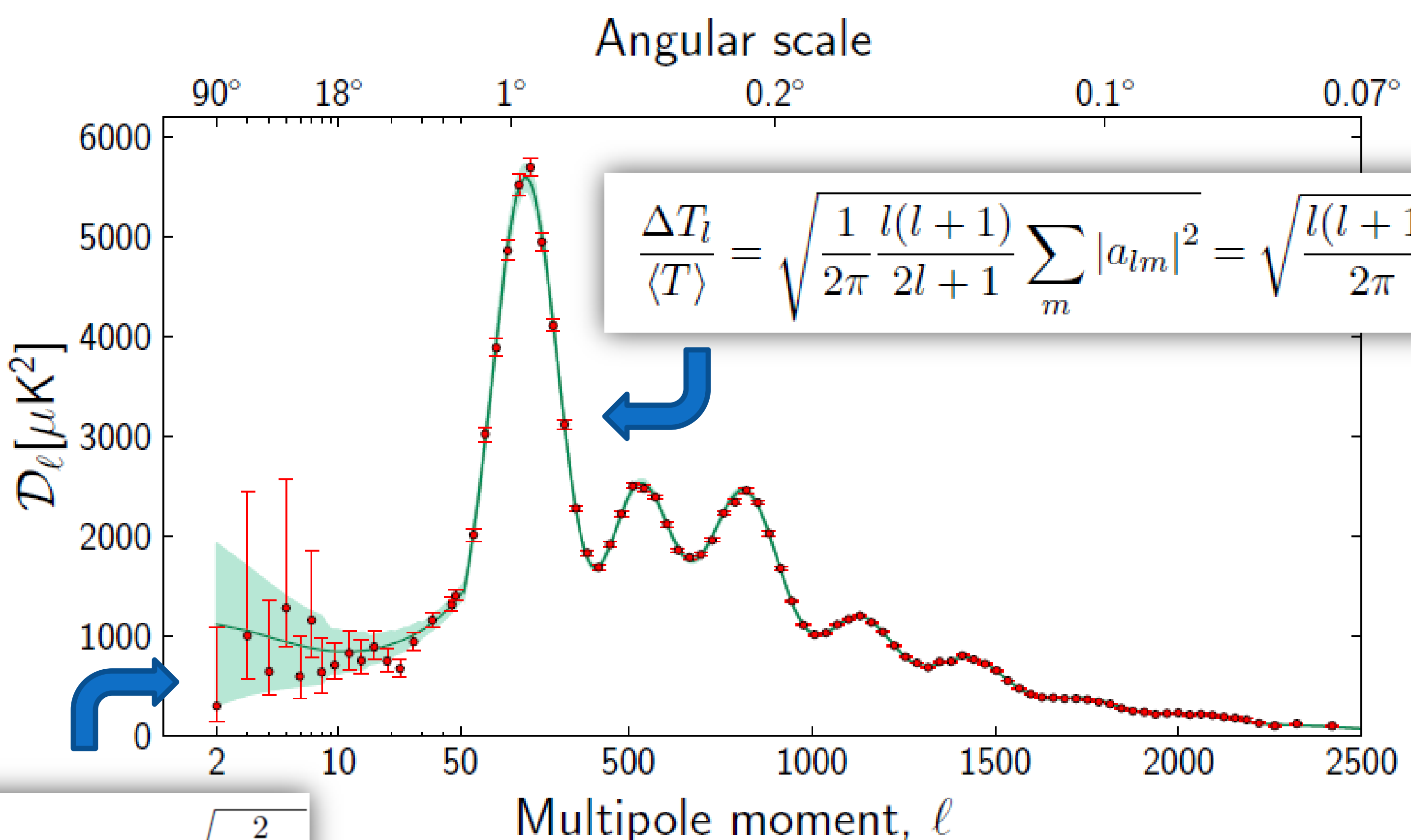
where $l \sim \pi/\theta$ *is the* ***multipole order*** *and the* a_{lm} *the* ***multipole momenta****, characterized by zero mean and non zero variance.*

We define the ***spectral coefficients***

$$C_l = \frac{1}{2l+1} \sum_{m=-l}^{l} |a_{lm}|^2$$

whose distribution with respect to multipole momenta gives the ***thermal power spectrum*** *of CMB.*

CMB Power spectrum (2)

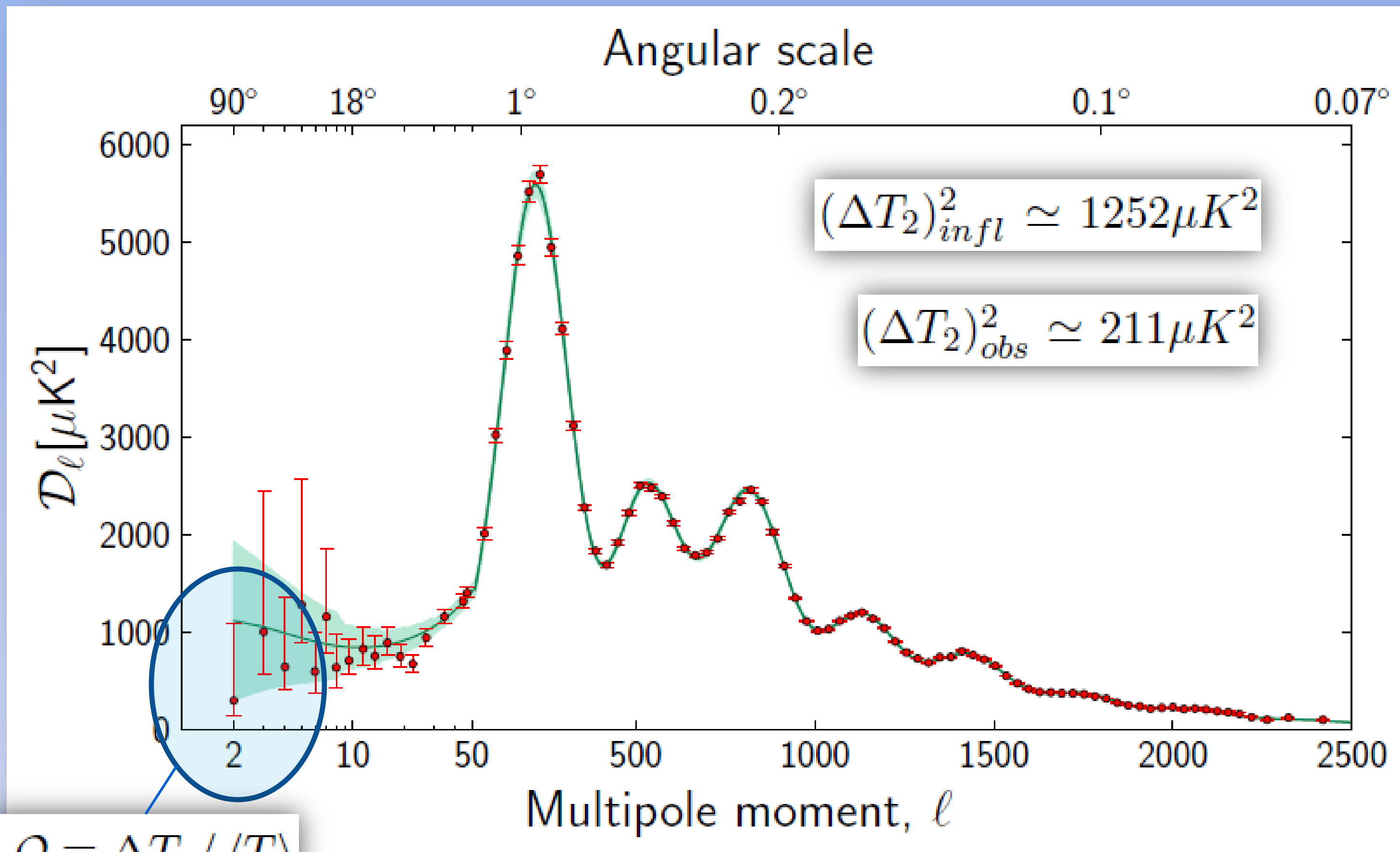


$$\frac{\Delta T_l}{\langle T \rangle} = \sqrt{\frac{1}{2\pi} \frac{l(l+1)}{2l+1} \sum_m |a_{lm}|^2} = \sqrt{\frac{l(l+1)}{2\pi} C_l}$$

$$\left(\frac{\Delta C_l}{C_l}\right)_{\text{cosmic variance}} = \sqrt{\frac{2}{2l+1}}$$

A cosmological model, through theory constraints, can predict form, position and height of spectrum peaks.

QUADRUPOLE ANOMALY



$\mathcal{Q} \equiv \Delta T_2 / \langle T \rangle$

The quadrupole anomaly could hide a non trivial topology in the primordial space-time geometry on large scales.

Quadrupole anomaly (2)

*Assuming that the spatial geometry on large scale follows a **planar symmetry**, Bianchi I-type, with an eccentricity at decoupling:*

$$e_{dec} \simeq 0.64 \cdot 10^{-2}$$

*we can **reduce dramatically** the spectrum at quadrupole, without any modification of the higher orders. We suppose*

$$\Delta T = \Delta T_A + \Delta T_{infl}, \quad \text{with} \quad a_{lm} = a_{lm}^{A} + a_{lm}^{infl}$$

we can prove that

$$\frac{\Delta T}{\langle T \rangle} = -\frac{1}{2} e_{dec}^2 n_3^2$$

from which:

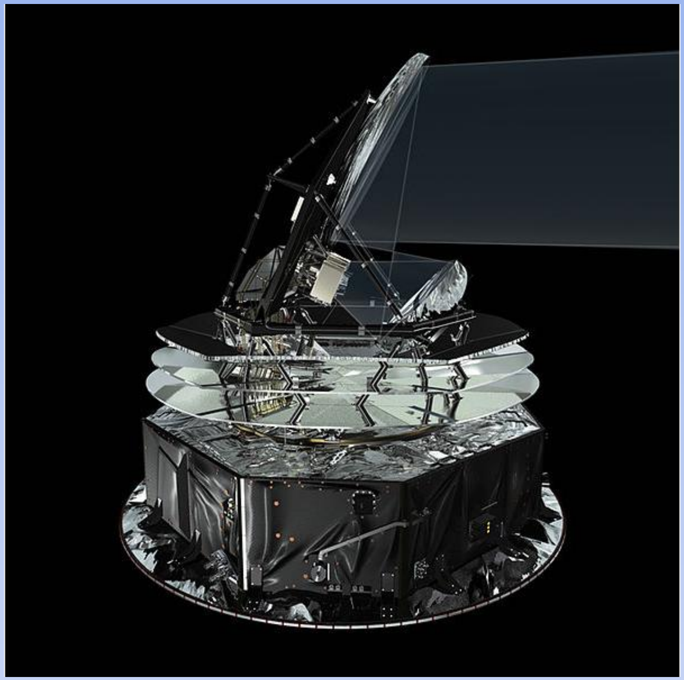
$$Q_A = \frac{2}{5\sqrt{3}} e_{dec}^2$$

with

$$Q^2 = Q_A^2 + Q_{infl}^2 - 2 f Q_A Q_{infl}$$

Furthermore, with this model, we can estimate the proper direction of anisotropy, called "axis of evil".

PLANCK MISSION



Space mission from **ESA**
(there is also the NASA collaboration for the cooling equipments)

Launch: May 14th, 2009
together with *Ariane 5* of the *Herschel Space Observatory*

It reached the L2 point at about **1.5 millions of Km** from Earth, in the opposite position of the Sun.

It's the coldest object in Universe known: -230 °C
(the equipments reach the temperature of -273.05 °C)

Significant increase of **angular resolution** and **sensitivity** with respect to COBE and WMAP

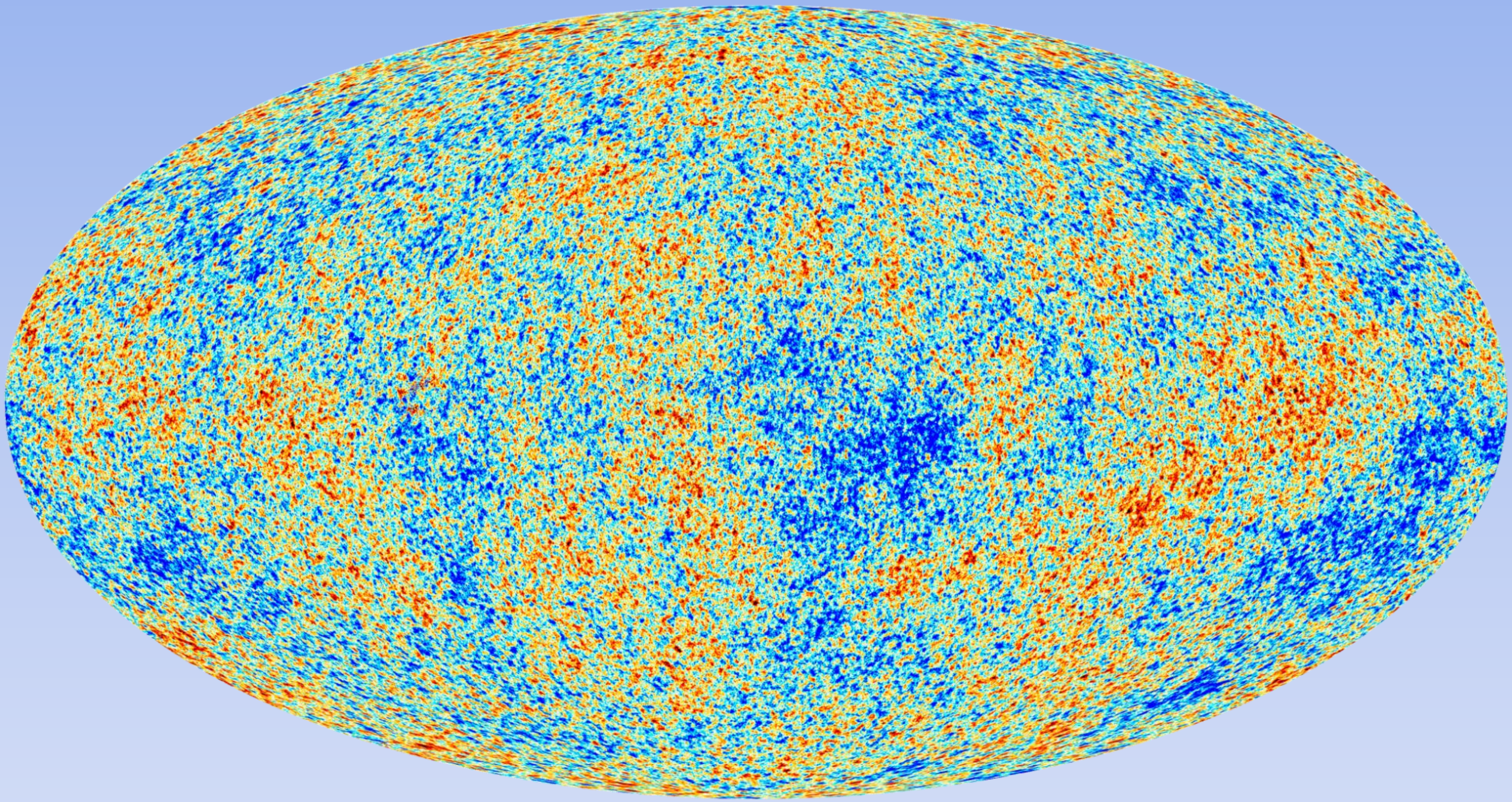
Spectrum: from **30 GHz** of **LFI** radiometers to **857 GHz** of **HFI** for the *foregrounds subtraction.*

PLANCK MISSION (2)

Main goals:

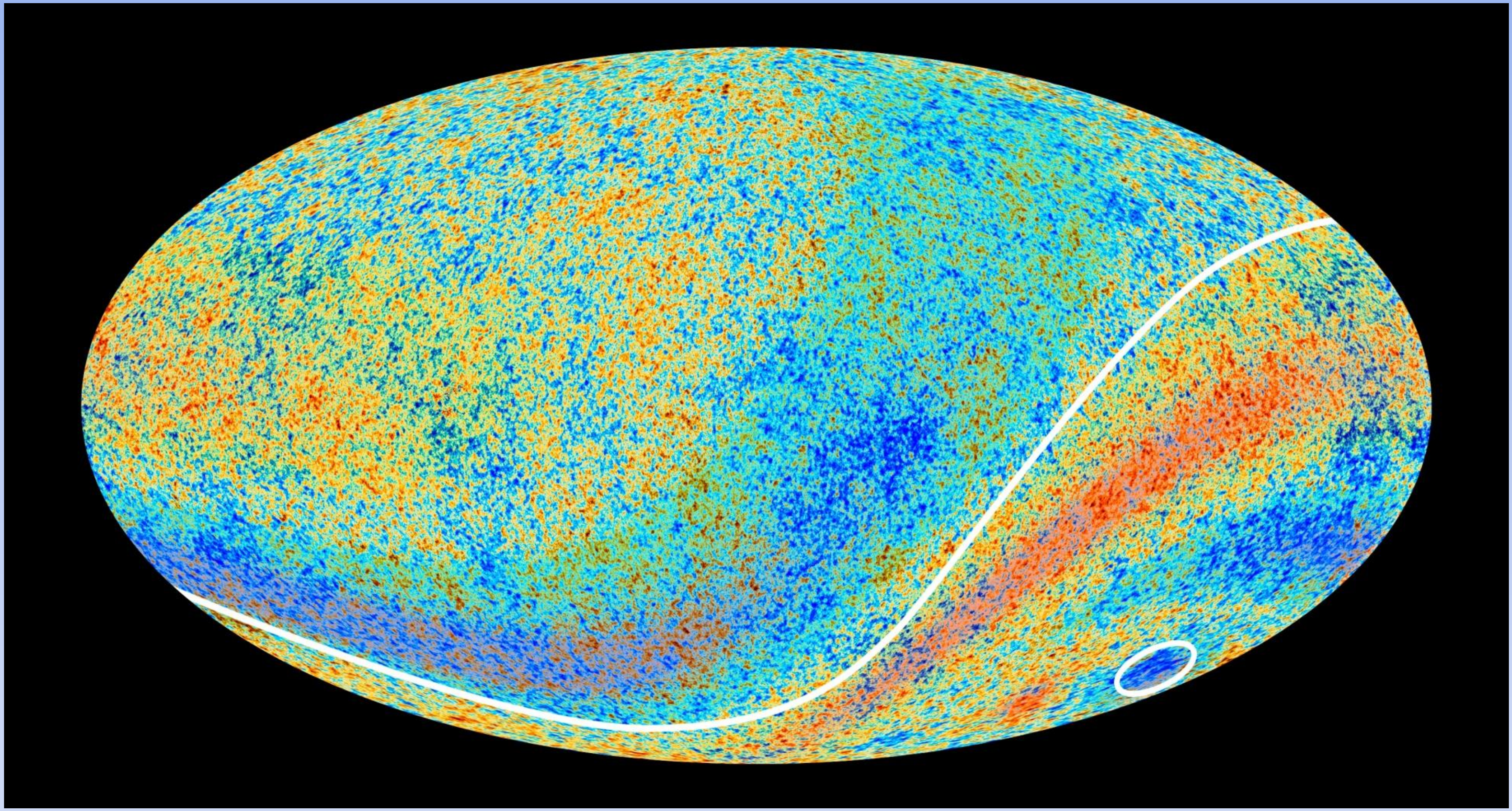
- *measure of CMB* ***polarization;***
- *test of* ***inflationary model;***
- *more accuracy in estimating* ***cosmological parameters;***
- *study of clusters of galaxies through the detection of the* ***Sunyaev-Zel'dovich effect;***
- *study of* ***interstellar medium;***
- *detection of* ***B-mode*** *polarization and* ***gravitational waves.***

CMB MAP SEEN BY PLANCK



General corroboration of SCM, but also new **challenges**:
• crucial confirmation of **quadrupole anomaly**;
• **thermal asymmetry** in the opposite hemispheres of the sky;
• some **"cold" zones** are larger than expected.

CMB MAP SEEN BY PLANCK (2)



General corroboration of SCM, but also new **challenges**:
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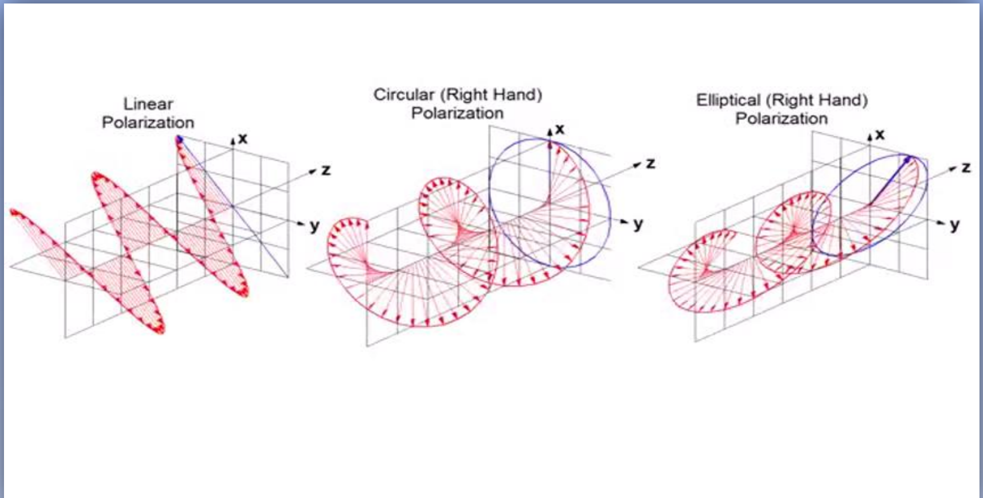
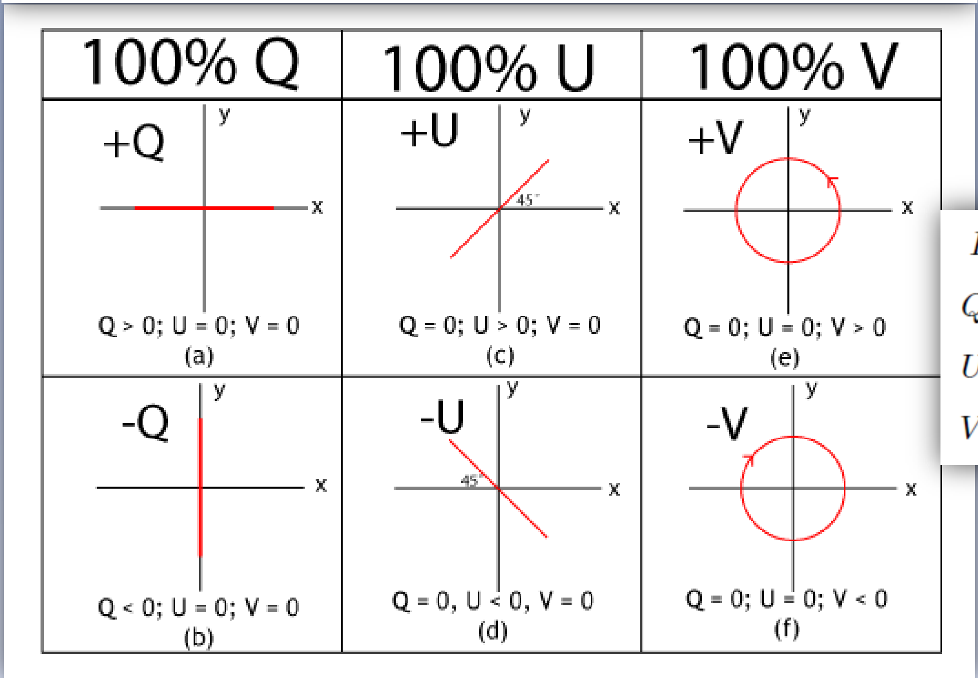
POLARIZATION



Types of polarization of electromagnetic radiation



Stokes parameters

$$I = (E_{0z})^2 + (E_{0y})^2 = I_z + I_y$$

$$Q = (E_{0z})^2 - (E_{0y})^2 = I\cos(2\beta)\cos(2\chi) = I_z - I_y$$

$$U = 2E_{0z}E_{0y}\cos(\phi_1 - \phi_2) = I\cos(2\beta)\sin(2\chi)$$

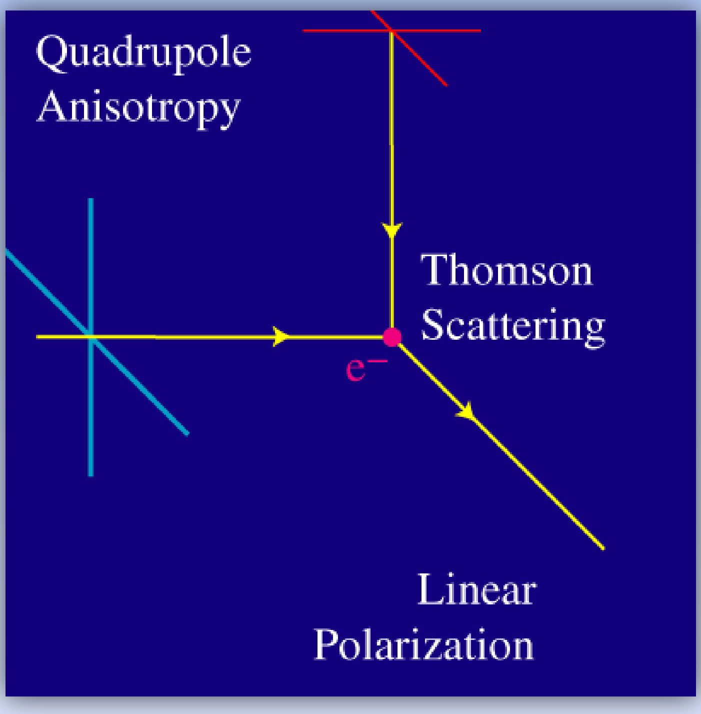
$$V = 2E_{0z}E_{0y}\sin(\phi_1 - \phi_2) = I\sin(2\beta).$$

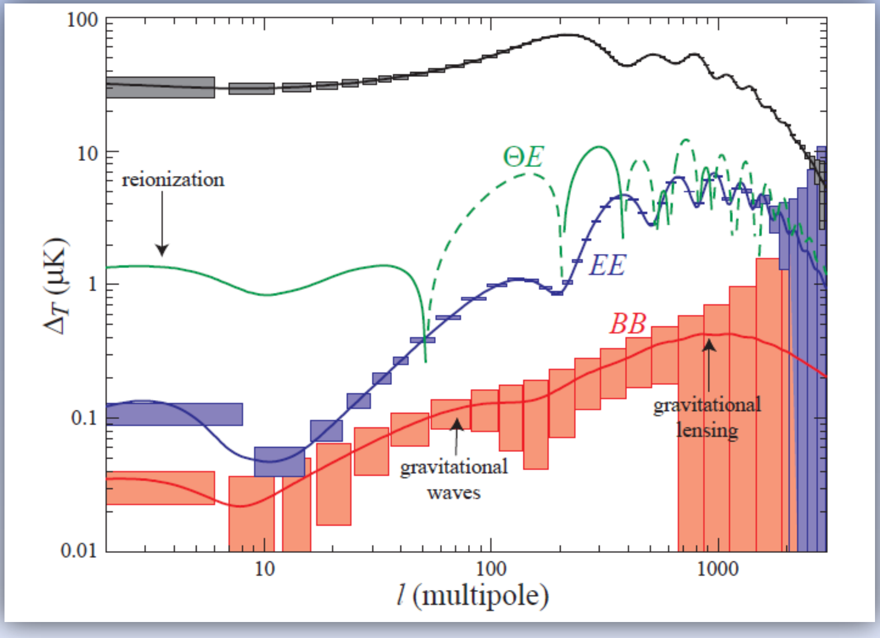
CMB Polarization

*It's the **most efficient probe** suitable to look at the Universe during recombination.*

*It is produced only by **Thomson scattering** at decoupling and iff the incoming radiation changes at 90° (**quadrupole**)*

The quadrupole anisotropy is generated by **photon scattering**: the polarized part of anisotropy amounts to approx. **10%** since diffusion starts just near the **end of recombination**.





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CMB POLARIZATION(2)

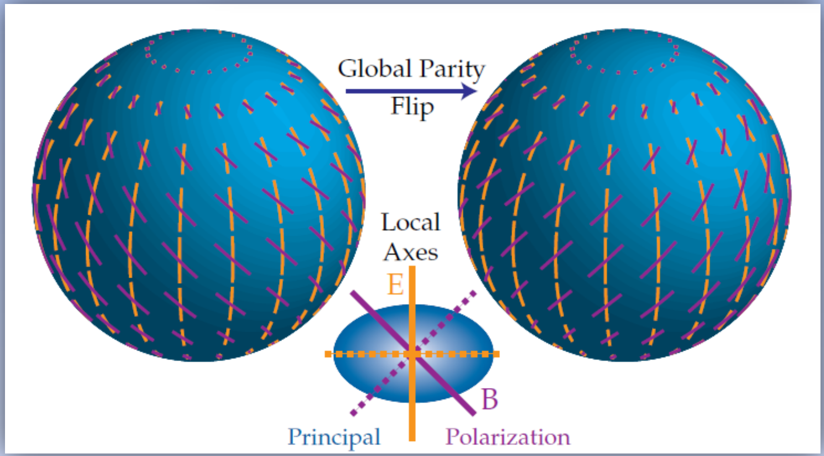
We distinguish two different ***pattern of polarization*** according to the transformations properties under ***parity*** of some linear combination of ***Q*** and ***U***

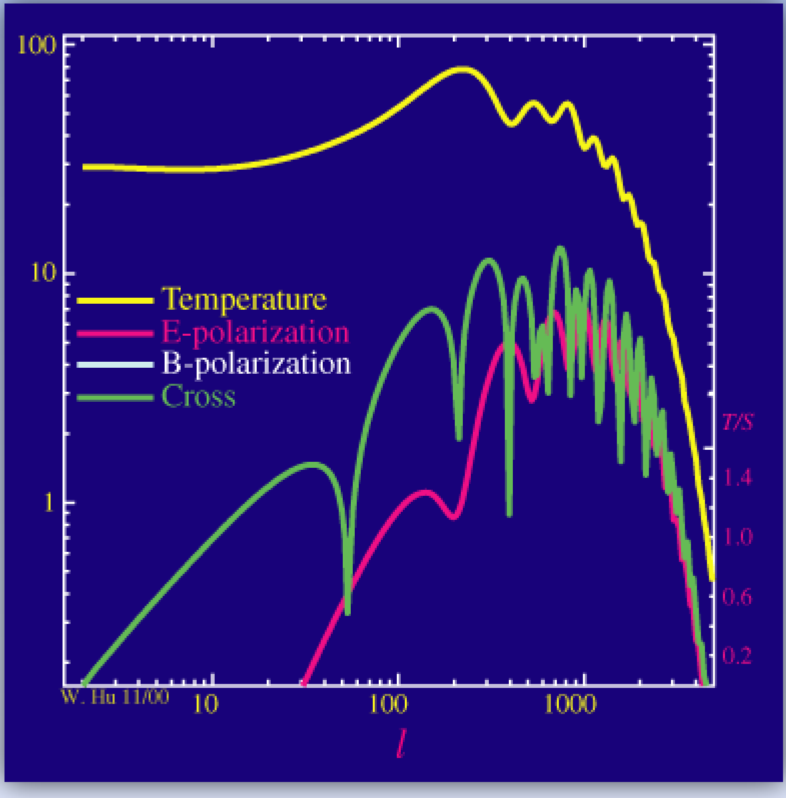
$$E(\vec{l}) \equiv \tilde{Q}(\vec{l})\cos(2\phi_l) + \tilde{U}(2\phi_l)\sin(2\phi_l)$$

$$B(\vec{l}) \equiv -\tilde{Q}(\vec{l})\sin(2\phi_l) + \tilde{U}(2\phi_l)\cos(2\phi_l)$$

E-mode

B-mode





Ratio Tensor/Scalar:

Gravitational waves

Reionization

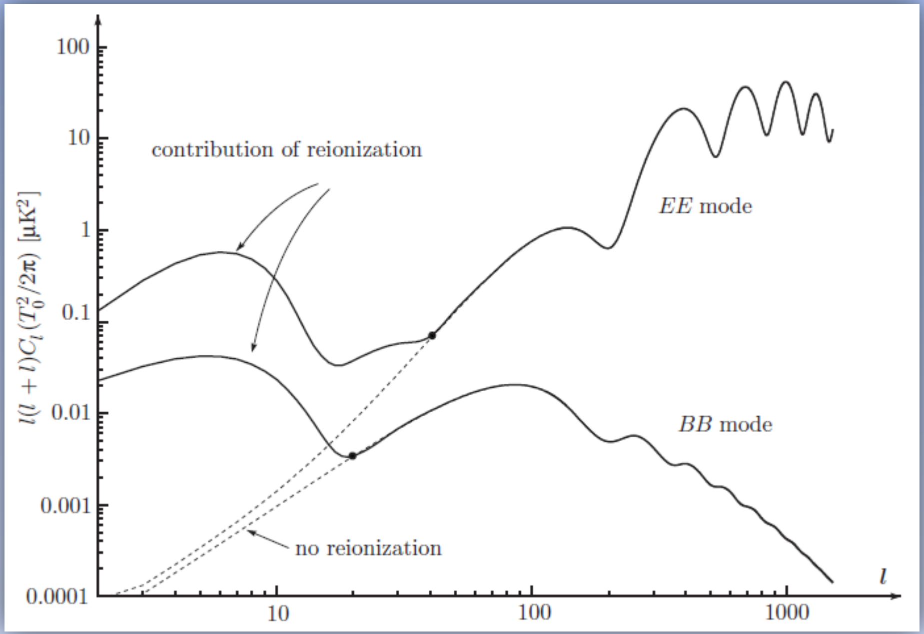
*The excess of signal on large scales was explained as a trace of a possible **reionization**:*

Structures on large scales **radiate** towards near interstellar gases

Hydrogen and helium atoms are **reionized**

CMB photons are **coupled** to free electrons again via Thomson scattering

Radiation is not completely hidden, but has inside the trace of the repeated interactions: ***Polarization***



Universe is now **much less dense** than at *decoupling*

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Experimental framework and Reionization

$$\tau(\eta) \equiv \int_{\eta}^{\eta_0} d\eta' n_e \sigma_T a(\eta)$$

	Optical depth	**Reionization**
WMAP 1 *(2003)*	$\tau \simeq 0.17$	$z_{re} \simeq 17$
WMAP 5 *(2009)*	$\tau = 0.084$	$z_{re} = 10 \pm 1.4$

Sloan Digital Sky Survey *(2001)*

Gunn-Peterson Effect

$z \simeq 6$

Universe ***neutral*** *at* $z \geq 10$

Reionization is shifted time after time to later epochs: **unsatisfactory explanation!**

POLARIZATION DATA

In the CMB polarization map on *large scales* given by **WMAP 5** we have an ***E-mode*** *signal*:

$$\frac{l(l+1)}{2\pi} C_{l=2}^{EE} = 0.15^{+0.427}_{-0.125} \mu K^2$$

where the cosmic variance is already taken in account.

So we obtain a **polarization anisotropy**:

$$\frac{(\Delta T)_{pol}}{T_0} = 0.145^{+0.207}_{-0.059} \cdot 10^{-6}$$

This value will be compared with the one obtained from the analytical calculation in a planar Bianchi I geometry.

Boltzmann Equation for CMB photons

$f(\vec{x}, t)$ *Photon distribution function (black-body)*

We evaluate the evolution of this function, **strongly coupled** to electrons via *Thomson scattering*

Boltzmann Equation

$$\frac{df}{dt} = C[f]$$

where $C[f]$ is the collision term, while the metric used

is a Bianchi I metric $ds^2 = dt^2 - a^2(t)(\delta_{ij} + h_{ij})dx^i dx^j$

with the anisotropy term
(we are interested in its effects) $h_{ij} = -e^2(t)\delta_{i3}\delta_{j3}$

Boltzmann Equation for CMB photons (2)

Expanding all functional **dependencies** of the distribution function:

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x^i} \cdot \frac{dx^i}{dt} + \frac{\partial f}{\partial p} \cdot \frac{dp}{dt} + \frac{\partial f}{\partial \hat{p}^i} \cdot \frac{d\hat{p}^i}{dt}$$

$$\frac{dx^k}{dt} = -\frac{\hat{p}^k}{a(t)}\left(1 - \frac{1}{2}h_{ij}\hat{p}^i\hat{p}^j\right)$$

$$P^k = -p\frac{\hat{p}^k}{a(t)}\left(1 - \frac{1}{2}h_{ij}\hat{p}^i\hat{p}^j\right)$$

from geodesic eq.

$$\frac{dP^0}{d\lambda} = \Gamma_{\alpha\beta}{}^0 P^\alpha P^\beta$$

$$\frac{dp}{dt} = -p\left(H + \frac{1}{2}\dot{h}_{ij}\hat{p}^i\hat{p}^j\right)$$

second order

The resulting **collisionless** equation is:

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\hat{p}^k}{a(t)}\left(1 - \frac{1}{2}h_{ij}\hat{p}^i\hat{p}^j\right)\frac{\partial f}{\partial x^k} - p\frac{\partial f}{\partial p}\left(H + \frac{1}{2}\dot{h}_{ij}\hat{p}^i\hat{p}^j\right)$$

Boltzmann Equation for CMB photons (3)

Expanding the distribution near the zeroth order (**Bose-Einstein**)

$$f_0(p,t) = \frac{1}{e^{p/T(t)} - 1}$$

we have: $$f(x^k, p, \hat{p}^k, t) \simeq f_0(p,t) + f_1(x^k, p, \hat{p}^k, t)$$

$$\frac{\partial f_1}{\partial t} + \frac{p^k}{a(t)}\frac{\partial f_1}{\partial x^k} - \left(pH\frac{\partial f_1}{\partial p} - \frac{1}{2}\dot{h}_{ij}\hat{p}^i\hat{p}^j\right) = C[f]$$

Zeroth order $$\frac{\partial f_0}{\partial t} - p\frac{\partial f_0}{\partial p}H = 0$$ → $$T \sim a^{-1}$$

as predicted by Standard Cosmological Model

Boltzmann Equation for CMB photons (4)

First order

$$\frac{\partial f_1}{\partial t} + \frac{p^k}{a(t)}\frac{\partial f_1}{\partial x^k} - \left(pH\frac{\partial f_1}{\partial p} + e(t)\dot{e}(t)\mu^2\right) = C[f]$$

where we used the Rayleigh-Jeans approximation

$$\partial f_0/\partial \log p = p(\partial f_0/\partial p) \simeq -1$$

where we have defined

$$\mu \equiv \hat{p}_3 = \cos\theta_{\vec{p}\cdot\hat{n}}$$

and

$$\dot{h}_{ij}\hat{p}^i\hat{p}^j = -2e(t)\dot{e}(t)\mu^2$$

Polarization signal computation

In order to determine the CMB polarization, we need the ***polarized distribution function****, parameterized by a two component* ***Stokes vector***
(axial symmetry with no circular polarization)

$$\vec{f}_0(p,t) = f_0(p,t)\begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad \vec{f}_1(x^k,p,\hat{p}^k,t) = \begin{pmatrix} \xi_1(x^k,p,\hat{p}^k,t) \\ \xi_2(x^k,p,\hat{p}^k,t) \end{pmatrix}$$

Following *Chandrasekhar (1960),* the collision term can be written as the following:

$$C[f] = -n_e\sigma_T\left[\vec{f}_1(x,\mu,t) - \frac{3}{8}\int_{-1}^{1} d\mu' \begin{pmatrix} 2(1-\mu^2)(1-\mu'^2)+\mu^2\mu'^2 & \mu^2 \\ \mu'^2 & 1 \end{pmatrix} \vec{f}_1(x,\mu',t)\right]$$

knowing that

$$\frac{dp}{dt} = -pH - pe(t)\dot{e}(t)\mu^2$$

Polarization signal computation (2)

1. *Substituting:*
the expression for $\frac{dp}{dt}$ *and for the collision term;*

2. *Ignoring:*
- *the* ***spatial derivatives*** *(long wavelengths),*
- *higher order terms (greater than first);*

3. *Passing to* ***conformal time****;*

4. *Using an* ***appropriate normalization*** *for black-body distribution;*

$$\frac{\partial \vec{f}_1}{\partial \eta} - e(\eta)e'(\eta)\left(\mu^2 - \frac{1}{3}\right)\begin{pmatrix} 1 \\ 1 \end{pmatrix} =$$

$$= -n_e\sigma_T a(\eta)\left[\begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} - \frac{3}{8}\int_{-1}^{1} d\mu' \begin{pmatrix} 2(1-\mu^2)(1-\mu'^2)+\mu^2\mu'^2 & \mu^2 \\ \mu'^2 & 1 \end{pmatrix}\begin{pmatrix} \xi_1' \\ \xi_2' \end{pmatrix}\right]$$

Polarization signal computation (3)

Following *Basko and Polnarev (1980)*, we can write the solution in the form:

$$\vec{f}_1(x,\mu,t) = \theta_a(\eta)\left(\mu^2 - \frac{1}{3}\right)\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \theta_p(\eta)(1-\mu^2)\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$\theta_a(\eta)$

degree of anisotropy

$\theta_p(\eta)$

degree of polarization

Polarization signal computation (4)

We arrive at a system of **coupled differential equations** of the first order:

$$\frac{3\,n_e\sigma_T}{a^3(\eta)}\,[3\,\theta_a(\eta)+2\,\theta_p(\eta)]-10\,\left[e(\eta)e'(\eta)+\theta'_a(\eta)\right]=0$$
$$\frac{n_e\sigma_T}{a^3(\eta)}\,[\theta_a(\eta)+4\,\theta_p(\eta)]+10\,\theta'_p(\eta)=0,$$

where the μ dependence was factorized.

We assume that the anisotropy and polarization terms are **quasi-stationary**: their values are fixed at *decoupling* until today, an hypotesis that is reasonable in a **scenario without reionization.**

- Polarization remains fixed at *decoupling* without any *rescattering*;
- Anisotropy remains equal to $-e^2_{dec}/2$.

Polarization signal Computation (5)

Final result for the ***magnetic case****:*

$$\theta_a = -\frac{1}{3H_0^2 n_{e,0}\sigma_T \eta_{dec}}\Big[(e_0^2 - e_{dec}^2)H_0^2(2+3H_0\eta_{dec}) + 64G\pi(2+H_0\eta_{dec})\rho_{B,0} + \\ -64\cdot 2^{2/3}G\pi(2+3H_0\eta_{dec})^{1/3}\rho_{B,0}\Big]$$

$$\theta_p = \frac{1}{12H_0^2 n_{e,0}\sigma_T \eta_{dec}}\Big[(e_0^2 - e_{dec}^2)H_0^2(2+3H_0\eta_{dec}) + 64G\pi(2+H_0\eta_{dec})\rho_{B,0} + \\ -64\cdot 2^{2/3}G\pi(2+3H_0\eta_{dec})^{1/3}\rho_{B,0}\Big]$$

Similarly we performed the calculation for domain walls and cosmic strings.

Polarization signal Computation (6)

Numerical results:

$$\theta_a = \frac{(\Delta T)_{anis}}{T_0} \simeq 0.27 \cdot 10^{-5}$$

in excellent agree with the predicted value

$$\theta_p = \frac{(\Delta T)_{pol}}{T_0} \simeq 0.65 \cdot 10^{-6}$$

about two times greater than the measured value by WMAP

CONCLUSIONS

✓ ***Overcoming*** *of the Cosmological Principle:* ***non trivial anisotropy*** *in primordial geometry.* ***Ellipsoidal Universe****;*

✓ ***Solution of the quadrupole anomaly****, without any modification of higher orders;*

✓***Boltzmann Equation*** *in a Bianchi I Universe;*

✓ *Possible* ***explanation of CMB polarization on large scales*** *(waiting for new* ***PLANCK*** *data).*

Possible future developments

- **Computation** of the polarization signal in case of anisotropy induced by an ***anisotropic dark energy***;

- **Check** of the **eccentricity** value and its evolution;

- **Detection** of ***B-mode*** *polarization:* ***gravitational waves***;

- **Study** of the effects caused by the **action of a primordial magnetic field** on CMB photons and their polarization.

Backup Slides

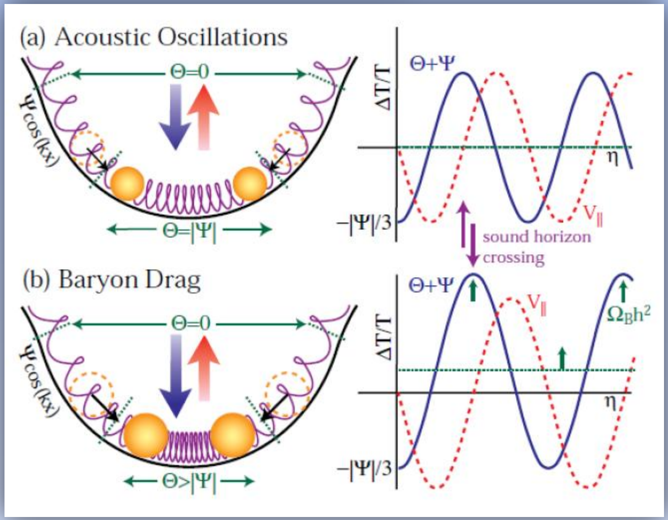
CMB Anisotropies

First type

Sachs-Wolfe effect

$$\left(\frac{\Delta T}{T}\right)_o \simeq \frac{\phi_e}{3}$$

Acoustic oscillations



Doppler effect

due to the movement of the Earth with respect to the system in which we have CMB isotropy

CMB ANISOTROPIES (2)

Second type

Integrated Sachs-Wolfe effect

$$\frac{\delta T}{T} = \int dt \phi' \left(r(t), T \right)$$

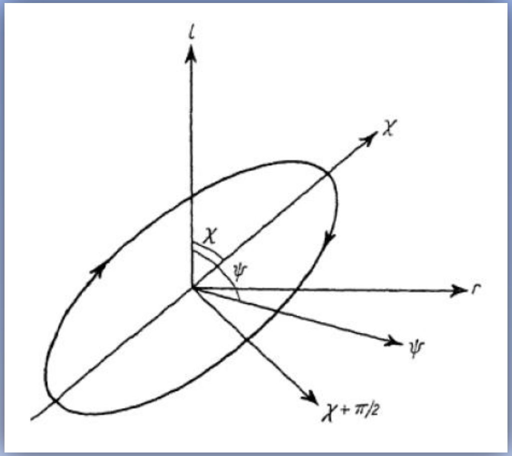
where $\phi' \neq 0$ in the following situations:

1. *Early ISW*. We have a contribution from radiation shortly after recombination: linear decrease of the potential due to CMB photons scattering.
2. *Late ISW*. Cosmological constant contribution.
3. *Rees-Sciama effect*. Escape of photons from potential wells, which grows in a non-linear way: redshift.

Reionization

Sunyaev-Zel'dovich effect. Kinetic and thermal (high temperature and peculiar velocities): we have *redshift* induced by the presence of clusters of galaxies.

Stokes parameters



$$E_z = E_{0z}\cos(\omega t - \phi_1)$$
$$E_y = E_{0y}\cos(\omega t - \phi_2)$$

$$E_\chi = E^{(0)}\cos\beta\sin(\omega t)$$
$$E_{\chi+\pi/2} = E^{(0)}\sin\beta\cos(\omega t)$$

β tangent is equal to the ratio between the ellipse axes

$$E_{0z} = E^{(0)}(\cos^2\beta\cos^2\chi + \sin^2\beta\sin^2\chi)^{1/2}$$
$$E_{0y} = E^{(0)}(\cos^2\beta\sin^2\chi + \sin^2\beta\cos^2\chi)^{1/2}$$
$$\tan\phi_1 = \tan\beta\tan\chi \qquad \tan\phi_2 = -\tan\beta\cot\chi$$

$$I_z = (E_{0z})^2 = I(\cos^2\beta\cos^2\chi + \sin^2\beta\sin^2\chi)^{1/2}$$
$$I_y = (E_{0y})^2 = I(\cos^2\beta\sin^2\chi + \sin^2\beta\cos^2\chi)^{1/2}$$

Stokes parameters (2)

$$I^2 = Q^2 + U^2 + V^2$$

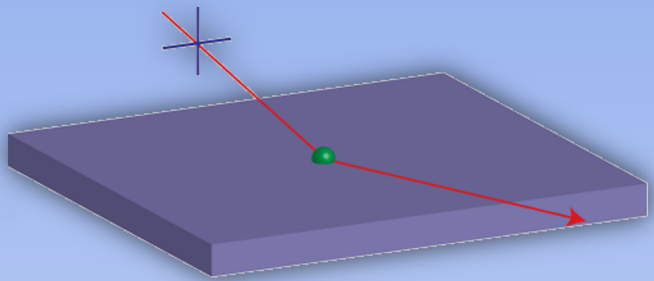
$$\tan(2\chi) = \frac{U}{Q}$$

$$\begin{pmatrix} 1 \\ 0 \\ \pm 1 \\ 0 \end{pmatrix} \equiv \texttt{linear polarization} \quad : 45^\circ\ (+),\ -45^\circ\ (-)$$

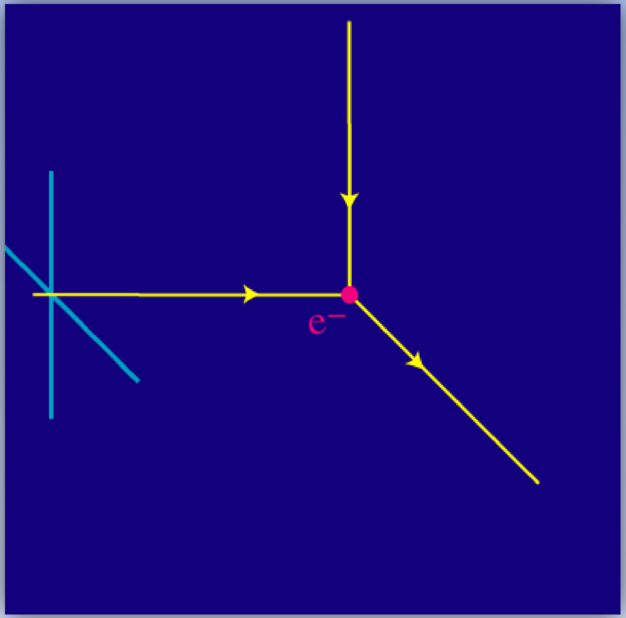
$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ \pm 1 \end{pmatrix} \equiv \texttt{circular polarization} \quad : \quad \texttt{left} \quad (+),\quad \texttt{right} \quad (-)$$

$$\begin{pmatrix} 1 \\ \pm 1 \\ 0 \\ 0 \end{pmatrix} \equiv \texttt{linear polarization} \quad : \texttt{horizontal}\ (+),\ \texttt{vertical}\ (-)$$

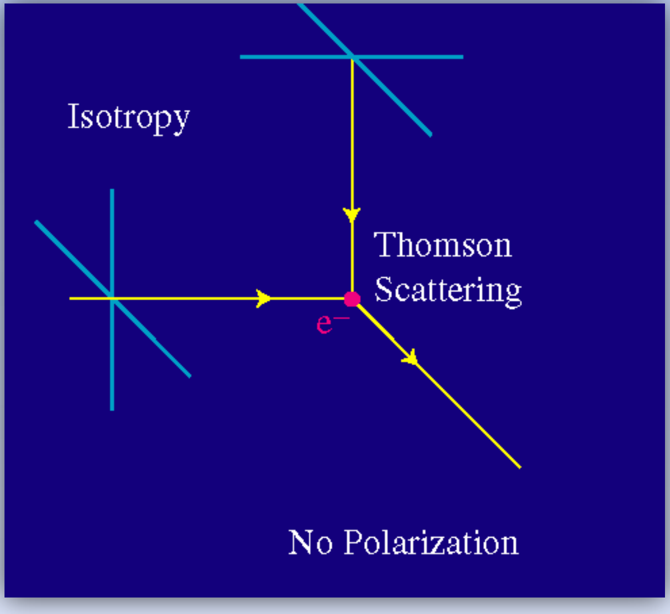
Polarization from Thomson Scattering



Polarization induced by simple reflection



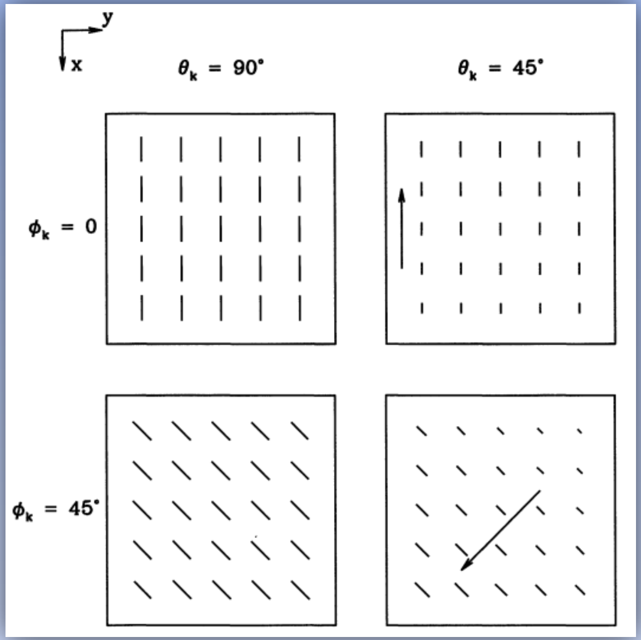
Linear polarization from single plane wave



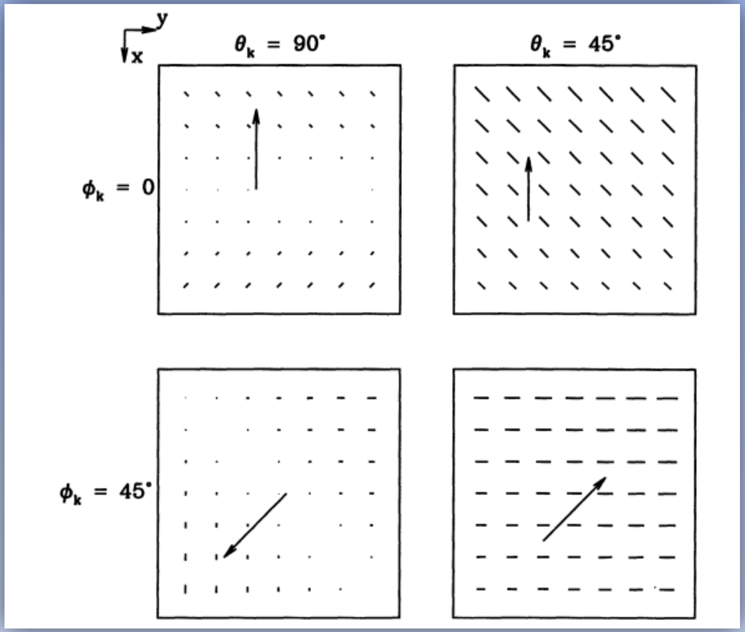
No net polarization from isotropy radiation

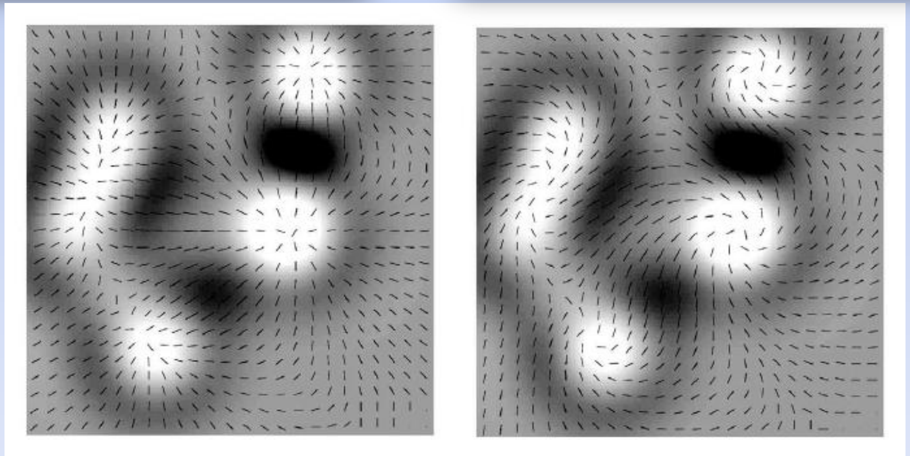
E-mode and B-mode

E-mode



B-mode





Giuseppe Iacobellis, 11/11/13

GUNN-PETERSON EFFECT

It's a property of *quasar* spectra due to the presence of **neutral hydrogen** in the interstellar medium.

Universe expansion produces *redshift* of *quasar emissions*, very far from observation telescopes

Wavelengths higher than *Lyman-alpha* limit are **"stretched"** till to reach the value in which the transition occurs

Even a small fraction of neutral hydrogen generates the **suppression of emission**, with the simultaneous **increase of the optical depth**

The detection of a *Gunn-Peterson* effect **suggests the neutrality of the Universe** at the *quasar redshift*

Solution of Boltzmann equation

Motivations for:

$$\vec{f}_1(x,\mu,t) = \theta_a(\eta)\left(\mu^2 - \frac{1}{3}\right)\begin{pmatrix}1\\1\end{pmatrix} + \theta_p(\eta)(1-\mu^2)\begin{pmatrix}1\\-1\end{pmatrix}$$

Frequency rate (Basko & Polnarev, 1980)

$$\frac{d\nu}{d\eta} = -\nu\left[H + \Delta H\left(\mu^2 - \frac{1}{3}\right) + \overline{\Delta H}(1-\mu^2)\right]$$

where: $\Delta H = H_z - (H_x + H_y)/2$ **anisotropy along *z***

$\overline{\Delta H} = H_x - H_y$ **polarization**

knowing that: $$\frac{1}{\nu}\frac{d\nu}{d\eta} = \frac{1}{2}\dot{h}_{ij}\hat{p}^i\hat{p}^j \qquad \frac{\Delta T}{\langle T\rangle} = \int_{t_{dec}}^{t_0} dt \frac{1}{2}\frac{\partial h_{ij}}{\partial t}\hat{n}^i\hat{n}^j$$

Numerical values and Observations

(subscript zero means "current time")

magnetic density $\rho_{B,0} = B_0^2/8\pi$ with $B_0 \simeq 4.6 \cdot 10^{-9} Gauss$

eccentricity $e_{dec} \simeq 0.64 \cdot 10^{-2}$ with $e_0 = 0$

Hubble radius $R_{H_0} = c/H_0 \sim 3 \times 10^3 Mpc \sim 10^{28} cm$

h parameter $h = 0.72 \pm 0.03$

Thomson cross section

$$\sigma_T = 8\pi\alpha^2\hbar^2/3m_e^2c^2 = 0.665245854 \times 10^{-24} cm^2$$

Numerical values and Observations (2)

Hubble parameter

$$H_0 = H(t_0) = h \times 10^2 km \cdot s^{-1} \cdot Mpc^{-1} = 3.2h \times 10^{-18} s^{-1}$$

decoupling epoch

$$\eta_{dec} \approx \frac{1}{H_0}$$

electronic density

$$n_{e,0} = \frac{n_{Eddington}}{V_{Universo}} \approx \frac{3 \cdot 10^{78}}{4\pi R_H^3}$$

N.B. In all final expressions we restored speed of light terms with appropriate powers.