The *axion* *vs* *the* **WIMP**

Sacha Davidson
IPN de Lyon/CNRS, France

1. an axion review
   - why the axion in particle physics?
   - put an (invisible) axion: astrophysical constraints
   - the axion in cosmology: $COLD$ Dark Matter
2. how to distinguish **the axion vs the WIMP?**
   - direct detection
   - non-linear structure formation : ingredients, scenarios ...and things to do?
3. (what we did) arXiv:1307.8024 with M Elmer

Why the axion:

gauge boson sector of QCD:

$$-\frac{1}{4}G^{A}_{\mu\nu}G^{\mu\nu A} \qquad\qquad A: 1..8$$

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$\vec{E}^2 + \vec{B}^2$ $\qquad\qquad$ $\vec{E}\cdot\vec{B}$

But... θ is CPV! neutron edm $\Rightarrow \theta \lesssim 10^{-10}$

Pich deRafael
Pospelov, Ritz

Why the axion: QCD is a model-builders nightmare

gauge boson sector of QCD: input g_s,

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and instantons dynamically generate $\theta \sim 1$ ... neutron edm $\Rightarrow \theta \lesssim 10^{-10}$

How to make θ unobservable? *Aha!* There are quarks and the axial anomaly: a chiral rotn through η contributes:

$$\delta\mathcal{L} \propto \eta\partial_\mu J_5^\mu = \eta\frac{g_s^2 N}{8\pi^2}G\widetilde{G} + \eta\sum_f m_f \overline{q}_f\gamma_5 q_f$$

(N $\Leftrightarrow$ coloured fermion reps)

a chiral phase rotn moves θ onto (coloured) fermion mass matrix...still CPV

Why the axion: QCD is a model-builders nightmare

Model builders nightmare: a theory that *dynamically generates the wrong couplings*

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$\Rightarrow$ **solution**: add fields, such that "generalised" chiral rotns ($\equiv$ PQ sym) are a sym of classical theory.

Peccei Quinn

To build an (Invisible) axion model

ShifmanVainshteinZakharov
Srednicki NPB85

1. aim to obtain a "Peccei-Quinn" symmetry = a global symmetry of the classical Lagrangian, broken by colour anomalies ($\simeq$ some generalisation of chiral rotns)

2. for instance (SVZ), add a gauge-singlet scalar with $Q_{PQ} = 2$ and SU(2) singlet quarks $\Psi_{L,R}$ with $Q_{PQ} = \pm 1$, so

$$\mathcal{L} = \mathcal{L}_{SM} + \partial_\mu \Phi^\dagger \partial^\mu \Phi + i\overline{\Psi} \not{D} \Psi + \{\lambda \Phi \overline{\Psi} \Psi + h.c.\} + V(\Phi)$$

3. arrange to break the PQ sym spontaneously, at high scale, such that all new particles are heavy except the goldstone = axion

4. so can rotate θ to the phase of Φ...which is a dynamical field...who will get a mass and want to sit at zero.

   ...so if CDM is an oscillating axion field, the nedm oscillates at $m_a \sim 10^{10}$ s^{-1}

The axion in particle physics (summary)

- strong CP problem of QCD: instantons choose θ — neutron edm $\Rightarrow \theta \lesssim 10^{-10}$
- solution : trade θ to a dynamical field a, with pot. min. at $\theta = 0$

phase of a complex SM-singlet scalar Φ, with big vev

Peccei Quinn
DineFischlerSrednicki,Zhitnitsky
Kim,ShifmanVainshteinZakharov

$$\Phi \to f_{PQ} e^{ia/f_{PQ}} \qquad f_{PQ} \sim 10^{11} \text{ GeV}$$

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$$\Phi \to f_{PQ} e^{ia/f_{PQ}} \qquad\qquad f_{PQ} \sim 10^{11}\ \mathrm{GeV}$$

$\Rightarrow$ only new particle at low-energy is the (pseudo-) goldstone a

$$\text{mixes to pion} \quad : \quad m_a \sim \frac{m_\pi f_\pi}{f_{PQ}} \simeq 6 \times 10^{-5} \frac{10^{11}\ \mathrm{GeV}}{f_{PQ}}\ \mathrm{eV}$$

...
Srednicki NPB85

$$\text{couplings to SM} \quad \propto \quad \frac{1}{f_{PQ}} \propto m_a$$

always to gluons $\Leftrightarrow$ nucleon

model $-$ dep to fermions (electrons) at tree

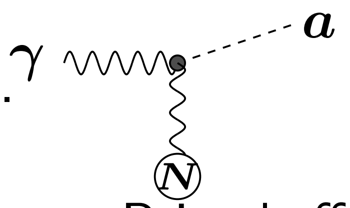
generically $\sim \dfrac{\alpha}{\pi f_{PQ}}$ to 2γ (triangle, and mixing with π)

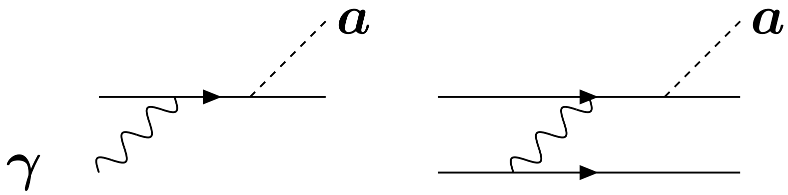
Astrophysical bounds

Raffelt...

axion light and (feebly) coupled to SM $\propto \frac{1}{f_{PQ}} \propto m_a$

$\Rightarrow$ produce in sun, He-burning stars(g_{ae}), supernovae(g_{aN})...





(axion couplings to e vs N vary across models by ~ 10)

upper bound on coupling to avoid rapid stellar energy loss:

$$m_a \lesssim 10^{-2} \text{ eV} \qquad\qquad (f_{PQ} \gtrsim 10^9 \text{ GeV})$$

the axion as Cold Dark Matter

1. non-thermal production (two populations) $\Rightarrow$ it redshifts like CDM

2. it grows density fluctuations like CDM

3. the *axion* vs the **WIMP**

Non-thermal axion production: the classical field is $Cold$ Dark Matter!

1. PQ phase transition : $\Phi \to f_{PQ} e^{ia/f_{PQ}}$
   a massless, random $-\pi f_{PQ} \leq a_0 \leq \pi f_{PQ}$ from one horizon to the next

2. QCD Phase Transition ($T \sim 200$ MeV): $m_a(t) : 0 \to f_\pi m_\pi / f_{PQ}$ (tilt mexican hat)
   $*$ ... at $H < m_a$, "misaligned" axion field starts oscillating around the minimum
   $*$ energy density $m_a^2 \langle a_0 \rangle^2 / R^3(t)$ $\Omega_a \lesssim 0.27 \Rightarrow m_a \langle a_0 \rangle = \ldots$

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Relate $\langle a_0 \rangle$ to f_{PQ}? When was inflation?

Scenario 1: PQPT before inflation
no: a_0 random ($10^{-7} f_{PQ}$ ok...so any $m_a \lesssim 10^{-2}$ eV ok...)
But, also $\delta a/a \sim H_I/(2\pi f_{PQ})$, gives isocurvature $\delta\rho/\rho$,
Planck $\Rightarrow H_I \lesssim 10^7 \sqrt{f/10^{12}}$ GeV, or non-canonical kin.terms for a...

WantzShellard
HanannHRW
FolkertsCristianoRedondo

Scenario 2: PQPT after inflation
yes: $\langle a_0^2 \rangle_{U\ today} \sim \pi^2 f_{PQ}^2 / 3$
density today higher for smaller mass $\Rightarrow$ correct Ω for $m_a \gtrsim 10^{-5}$eV)

Another contribution to CDM (if PQPT after inflation): cold axion particles

1. Suppose inflation before Peccei-Quinn Phase Trans.

   avoid CMB bounds on isocurvature fluctuations
   obtain varrying axion field across the U

2. then at PQPT $\Phi \to f_{PQ} e^{ia/f_{PQ}}$
   * a random in each horizon...
   * ...one string/horizon

3. QCD Phase Transition ($T \sim 200$ MeV):
   * strings go away (radiate cold axion particles, $\vec{p} \sim H \lesssim 10^{-6} m_a$)

   Hiramatsu etal 1012.5502

if PQPT after inflation ⇒ **CDM = oscillating axion field + cold particles**

Axion density fluctuations

1. has adiabatic density fluctuations inherited from surroundings at the QCDPT

2. on LSS scales, have same linear growth equations as in WIMPs

   Ratra, Hwang+Noh

3. (very) small scale differences....

   - there is pressure and Jeans length $\sim 1/\sqrt{H(t)m_a}$ (and funny c_s on smaller scales?)

   - if PQPT after inflation, a random from one horizon to next, so $\delta\rho_a/\rho_a \sim \mathcal{O}(1)$ on QCDPT horizon scale (5km then, 0.1 pc today)... axion "miniclusters"

     Hogan,Rees

4. the axion field does not turn into particles by parametric resonance

   Kolb,Singh,Srednicki

The *axion* vs the **WIMP**

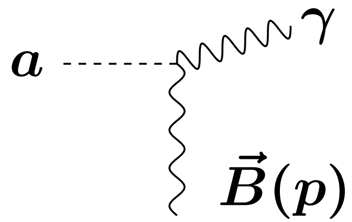
1. direct detection?

2. *might axions differ from WIMPs during non-linear structure formation?*

   (**Umm...** non-linear/N-body is hard!)

The *axion* vs the **WIMP**

1. direct detection? $\frac{a}{f_{PQ}} F\tilde{F} \sim a\vec{\gamma} \cdot \frac{\vec{B}\omega_\gamma}{f_{PQ}} \sim$



   not seen an axion (or a WIMP?) yet

2. *might axions differ from WIMPs during non-linear structure formation?*

   (**Umm...** non-linear/N-body is hard!)

The axion vs the WIMP — is non-linear structure formation different?

? non-perturbative dynamics...
$\Rightarrow$ write an axion DM code and compare to N-body?

The axion vs the WIMP — is non-linear structure formation different?

? non-perturbative dynamics...
⇒ write an axion DM code and compare to N-body?

...there is diverse literature...

Sikivie: Erken,Sikivie,Tam,Yang

1. at $T_\gamma \sim$ keV, "gravitational thermalisation" of axions drives them to a "Bose-Einstein Condensate"
2. BEC can support vortices, which allow caustics in the galactic DM distribution

⇔ **axion DM signature?**

The path integral should tell you everything...

The path integral (in Closed Time Path formalism) allows to compute:

1. the expectation value of the field ("classical field" of 1PI action)

2. the expectation value of the two pt function (propagator, number density)

... (higher point functions...)

hypothesis 1: the field and (particle) number density are the relevant variables.

Sacha's translation dictionary and assumptions

The path integral (in Closed Time Path formalism) allows to compute:

1. the expectation value of the field ("classical field" of 1PI action)
   = coherent state   condensed regime /**Bose Einstein condensate** /

2. the expectation value of the two pt function (propagator, number density)
   kinetic regime / cold particles / never a BE condensate

hypothesis 1 the field and (particle) number density are the relevant variables.

Parenthese: what is a Bose Einstein condensate? (I don't know. Please tell me if you do!)

Is a BE condensate just a (non-relativistic) charge-carrying classical field?
Or as well as being "coherent", should the modes be concentrated around a $\vec{p}$?

1. in equilibrium stat mech: bosons pile into the $\vec{p} = 0$ mode

2. in equilibrium Finite Temp FT: a phase transition $\leftrightarrow$ form a vev
   store a density of conserved charge in a homogeneous + isotropic classical field

3. for alkali gases in atomic traps: coherent collective behaviour (all the same $\vec{p}$; but not necc $\vec{p} = 0$)

? Axions are effectively non-interacting, so whether the classical field is a superposition of BE condensates, or a BE condensate...is the same?

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hypothesis 1 the field and (particle) number density are the relevant variables.

hypothesis 2 structure formation cares about $\boldsymbol{T^{\mu\nu}}$.

$\Rightarrow$ axions are $simple$: free scalar field, and/or non-rel. particles, coupled to gravity.
gravity is classical, cosmology is $\mathcal{O}(G_N)$

back to Sikivie's scenario (with the hypotheses)

Erken,Sikivie,Tam,Yang

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   Saikawa, Yamaguchi etal confirm in QFT that the gravitational intn rate, of classical axion field, exceeds H for $T \lesssim$ keV.

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   what we asked: is that interaction rate a $thermalisation$ rate? leading order unitary eqns??

   what we did: look for entropy production...find at $\mathcal{O}(G_N p^2/m_a^2)$...negligeable rate

   DavidsonElmer

2. BEC can support vortices, which allow caustics in the galactic DM distribution $\Leftrightarrow$ signature of rotating halo of axion DM?

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   DavidsonElmer

   post scriptum: NB all these calns done for the field

   (for PQPT after inflation, $k \sim H_{QCDPT} < 10^{-6} m_a$)

   but no need to thermalise field modes ; field is a BE condensate!

2. BEC can support vortices, which allow caustics in the galactic DM distribution $\Leftrightarrow$ axion DM signature?

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   ...sure, lab BECs have vortices, but they have self-interactions...

   Rindler-Daller+Shapiro study a rotating galactic halo formed of classical scalar field. They show that for sufficiently strong $\lambda\phi^4$, with sufficient angular momentum, its energetically favourable to form a vortex. $\lambda \sim m_a^2/f_{PQ}^2 \lesssim 10^{-40}$ is to small.

What we did

arXiv:1307.8024, with Martin Elmer

What does gravity do with the axion field?

consider early evolution of the Universe, until $\delta\rho \sim \rho$
Dynamics inside the horizon thermalisation is causal, so neglecting $H^2/m_a^2, \ldots$ for axion field in perturbed FRW (Newtonian gauge, ϕ = Newtonian potential comes from metric) can be obtained from

$$T^{\mu\nu}{}_{;\nu} = 0 \qquad , \qquad \nabla^2\phi = 4\pi G_N \delta\rho$$

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1. other people get Eqns of motion for the axion field a:

$$\ddot{a} + 3H\dot{a} + k^2 a + m_a^2 a \sim \frac{Gm^2}{q^2} aaa$$

... non-linear...can obtain time evolution of number of axions of momentum q:

$$i\frac{\partial n_a(q)}{\partial t} \simeq 4\pi G m_a \sum_{\vec{k}} \frac{\delta\rho(k)}{k^2}\{n_a\}$$

gives rate for axions to emit a graviton of any wavelength.

Interpretation of Sikivie : the rate to emit gravitons is a thermalisation rate.

What does gravity do with axions?

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$$T^{\mu\nu}{}_{;\nu} = 0 \qquad , \qquad \nabla^2\phi = 4\pi G_N \delta\rho$$

1. other people get Eqns of motion for the axion field (or its number density):
$$i\frac{\partial n_a(q)}{\partial t} \simeq 4\pi G m_a \sum_{\vec{k}} \frac{\delta\rho(k)}{k^2}\{n_a\}$$
non-linear... calculate rate for axions to emit a graviton of any wavelength.

2. Or (what we did), get Eqns of motion for a fluid with scalar perturbations
$$\ddot{\delta} + 2H\dot{\delta} - 4\pi\rho\delta + \frac{c_s^2}{R^2(t)}\nabla^2\delta = 0$$
can solve (in fourier space); gives evolution of axion density fluctuations.

Our Interpretation: 1. and 2. describe the same physics. The gravitational interaction rate of 1. includes the growth of inhomogeneities given by 2.

But "include" $\neq$ "is" : could *part* of the gravitational interactions be thermalising the axions? But this needs a bath/fluctuations to sum?? $\Rightarrow$ find some entropy?

Looking for dissipation in the gravitational interactions of axions

1. Assume no dissipation/thermalisation at leading order of classical equations of motion usual non-equilibrium field theory — must sum a bath of fluctuations to dissipate with t-reversal invariant eqns

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3. match axion in perturbed U onto imperfect fluid in FRW:

$$T^i{}_j(\vec{x},t) = -\frac{(1+2\phi)}{R^2(t)}\partial_i\phi\partial_j\phi \quad = \quad -\eta(t)(\partial_j U^i(\vec{x},t) + \partial^i U_j(\vec{x},t))$$

   $\eta =$ viscosity, $U = 4-$velocity. An imperfect fluid can grow density fluctuations, but contains dissipation...

4. estimate a dissipation scale:
$<$ the Jeans length $1/\sqrt{m_a H}$, distance below which fluctuations oscillate due to axion pressure

summary ...about our caln

Some doubts about gravitational thermalisation in Sikivie's scenario: it remains to be shown that the gravitational interaction rate of axions is a thermalisation rate:

leading order classical equations (no entropy generation?) expand the U and grow density fluctuations in axions

If gravity grows density fluctuations and thermalises, then there should be entropy production: we found some dissipative gravitational interactions, but suppressed by p_a^2/m_a^2...

Probably "gravitational thermalisation" of the axion field is a red herring anyway: not need to "thermalise" the field, its already a Bose Einstein condensate, or superposition thereof

Summary(review of well-known things)

The CPV θ parameter of QCD can be replaced by a light scalar field $\Leftrightarrow$ the $axion!$

The axion coupling to SM $\propto m_a$, so for $m_a \lesssim 10^{-2}$ eV, axion emission does not cool stars to fast.

Non-thermal production mechanisms in cosmology allow the axion to be a viable $Cold$ DM candidate:

$*$ redshifts like CDM

$*$ grow density fluctuations like CDM

Summary: to distinguish axions from WIMPs?

direct detection: find WIMPs or axions in terrestrial searches (CAST, ADMX...)

structure formation:

– axions redshift like WIMPs

– density fluctuations grow the same in linear regime

+ *are axions different from WIMPs during non-linear structure formation ?*

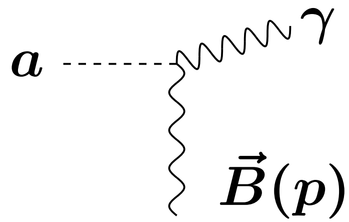
- difficult dynamics : ask a N-body friend to write an axion code

- simple theory :free scalar (field and/or particles) coupled to gravity
  many interesting analytic proposals...vortices, caustics (Sikivie)...lots to do :)

...can gravity move axions between field and particles?
(interesting because $T_{\mu\nu}$ is different I think)

Backup

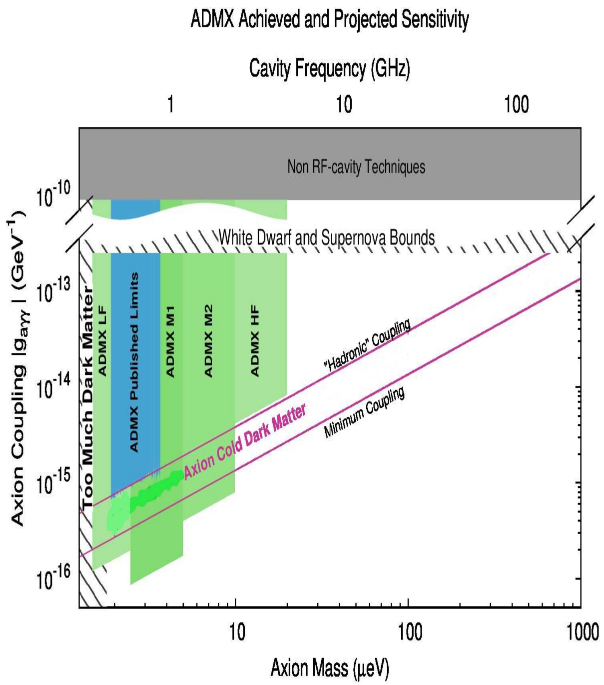
Direct detection (of axions)

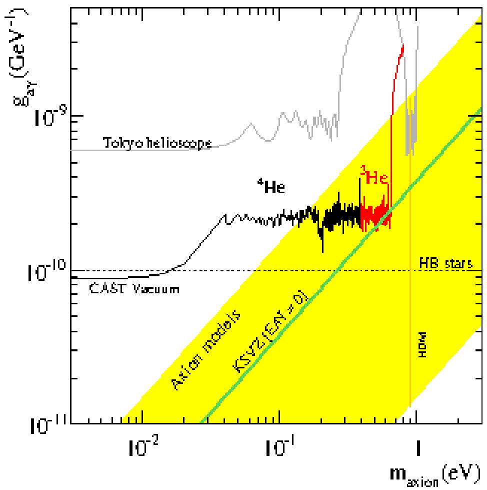


1. $a \to \gamma$ conversion in $\vec{B}$ field. (with gradient, to transfer correct $\vec{p}$...a diff $\vec{B}$ for each m_a)
   (a) CernAxionSolarTel: LHC magnet, points at sun, convert solar a to γs (also Sumico)
   (b) ADMX: dark matter axions ($E_\gamma \sim m_a \sim$ microwave)

2. spherical mirror in $\vec{B}$ field: a convert in $\vec{B}$ to γ focused by mirror= antennae

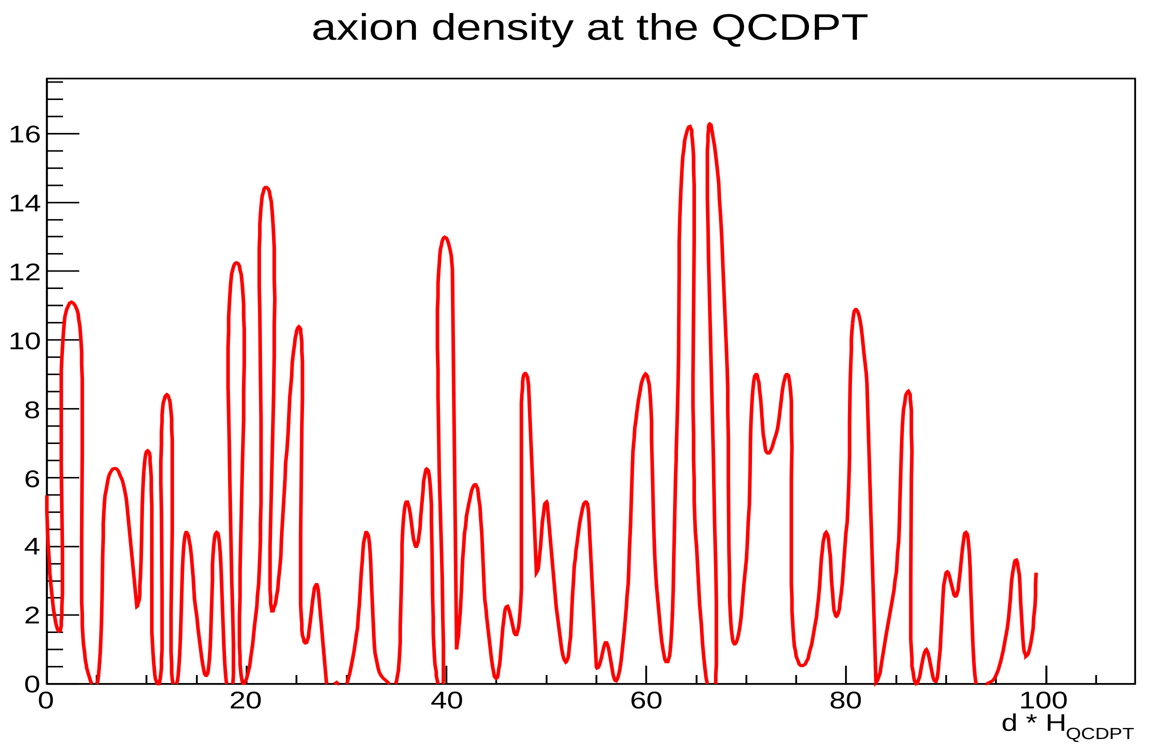
Horns etal 1212.2970





Inhomogeneities are $\mathcal{O}(1)$ on the QCD horizon scale

$a(\vec{x}, t)$ random from one horizon(~ 5km) to next; $\rho_a(\vec{x}, t) \simeq m_a^2 a^2(\vec{x}, t)$



$\Rightarrow$ its *not* a spatially homogeneous distribution of particles various momenta

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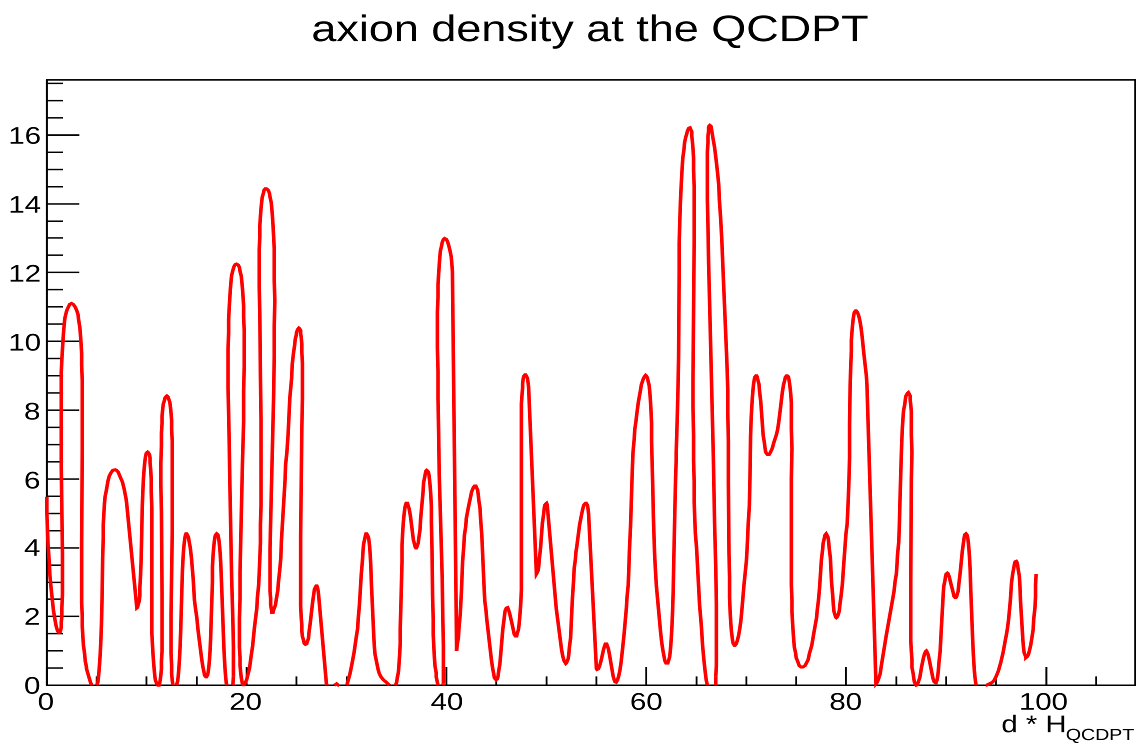

But how can axions form a *homogeneous-on-QCD-horizon-scale* bose-einstein condensate = zero mode of field? ??

$v = H_{QCDPT}/m_a \lesssim 10^{-6}c$...not "free-stream" QCD-horizon distance before t_{eq}:

$$d(t) = \int^t \frac{H_{QCDPT}}{m_a R(t')} dt' \sim \frac{H_{QCDPT}}{m_a} \frac{1}{H(t)R(t)} = \frac{R(t)}{m_a} \ll \frac{R(t)}{H_{QCDPT}}$$

(RD U, $R(t)$ =1@QCDPT)

The (beautiful) calculation of Saikawa and Yamaguchi

Suppose PQ PT after inflation. The classical axion field can be represented as a coherent state of axion particles (of momentum $\lesssim H_{QCDPT}$).
QFT rate for axions (momentum $\vec{k}$) to emit gravitons:

$$i\frac{\partial \hat{n}_k}{\partial t} = \left[\hat{H}_{int}, \hat{n}_k\right] \simeq \frac{G_E}{H(t)^2}\rho_a^2 \gg H(t)n_k$$

Saikawa+Yamaguchi

(evaluated in coherent state $\Leftrightarrow$ classical field caln.)

Sikivie interprets as gravitational thermalisation rate: hugely occupied low-$\vec{p}$ modes, equilibrium after $T_\gamma \lesssim$ keV, $\rightarrow$ BE condensate.

But are some of those gravitons expanding the U, and some growing fluctuations?

Why is that a thermalisation rate??

thermalisation in closed unitary systems?

$$\text{entropy} = \sum_{states\ s} P_s \ln P_s \quad \text{increases}$$

- unitary evolution creates no entropy $\Leftrightarrow$ NO entropy generation in closed systems ... BUT... can calculate "effective" thermalisation: a subset of observables evolve towards equilibrium expectations
$\Rightarrow$ the "rest" of the system is the bath??

- ex: couple two SHOs. Solve one, substitute into Eqns of second, and find dissipation.

- ...$K - \bar{K}$ evolution is non-unitatry, because not also follow 2π 3π states...

? $\Rightarrow$ divide axions+gravity into

1. U expansion + structure growth
2. other fluctuations which are the bath?

gravity and the second law

1. undergraduate memories say that gravitational collapse of a gas cloud to a star respects the second law...

2. story of $\Omega_{baryon} = 1$ U

   (a) quasi-homogeneous dust clouds collapse
   (b) ...generations of stars, supernovae, black holes...
   (c) ... ... proton decays...
   (d) venerable homogeneous and isotropic U full of photons and gravitons

3. so gravitational thermalisation of axions will happen.
   But does it happen before the U a year old?

Particles vs fields

Develop field operator

$$\hat{a}(t,\vec{x}) = \frac{1}{[R(t)L]^{3/2}} \int \frac{d^3k}{(2\pi)^3} \left\{ \hat{b}_{\vec{k}} \frac{\chi(t)}{\sqrt{2\omega}} e^{i\vec{k}\cdot\vec{x}} + \hat{b}^\dagger_{\vec{k}} \frac{\chi^*(t)}{\sqrt{2\omega}} e^{-i\vec{k}\cdot\vec{x}} \right\}$$

then write the coherent state:

$$|a(\vec{x},t)\rangle \quad \propto \quad \exp\left\{ \int \frac{d^3p}{(2\pi)^3} a(\vec{p},t) b^\dagger_{\vec{p}} \right\} |0\rangle$$

which satisfies $\hat{b}_{\vec{q}}|a(\vec{x},t)\rangle = a(\vec{q},t)|a(\vec{x},t)\rangle$ (can check $\hat{b}_{\vec{q}}\{1 + \int \frac{d^3p}{(2\pi)^3} a(\vec{p},t) b^\dagger_{\vec{p}}\}|0\rangle = a(\vec{q},t)|0\rangle$)
where the classical field is

$$a(t,\vec{x}) = \frac{1}{[R(t)L]^{3/2}} \int \frac{d^3k}{(2\pi)^3} \left\{ a(\vec{k},t) \frac{\chi(t)}{\sqrt{2\omega}} e^{i\vec{k}\cdot\vec{x}} + a^*(\vec{q},t) \frac{\chi^*(t)}{\sqrt{2\omega}} e^{-i\vec{k}\cdot\vec{x}} \right\}$$

Brodsky+Heurer,Donoghue etal
Olive+Montonen...I+Z,C-T...

What is quantum?

Classical = saddle-point configurations of the path integral
$\Rightarrow$ attribute dimensions to fields/parameters $\ni$ [action]= E*t, and no $\hbar$ in selected classical limit (this is *not* unique)

Summary: particles or fields can be obtained in a "classical" (= no $\hbar$) limit. However, $\hbar$ is differently distributed in the Lagrangian in the two limits, so to get from one to another requires $\hbar$...

in particular, to define a number of quanta, in the field picture, requires $\hbar$.

ex 1: massive scalar electrodynamics

$$\mathcal{L} = (D_\mu \phi)^\dagger D^\mu \phi - \tilde{m}^2 \phi^\dagger \phi - \frac{1}{4} FF \qquad , \quad D_\mu = \partial_\mu - i\tilde{e} A_\mu$$

Classical field limit: $[\phi, A] = \sqrt{E/L}$, $[m] = 1/L$, $[\tilde{e}] = 1/\sqrt{EL}$.

No $\hbar$ in classical EoM. OK that $[m^2] = 1/L^2$ because gravity couples is the stress-energy tensor, function of the fields.

If in Maxwells Eqns, want $j^0 = i\tilde{e}(\dot{\phi}^\dagger \phi - \phi^\dagger \dot{\phi})$ to be eN/V, then need number of charge-carrying quanta $\Rightarrow e = \tilde{e}\hbar$.

De même, if classically m a particle mass, need $m = \tilde{m}\hbar$.

ex 2: the SHO Hamiltonian is (no $\hbar$)

$$H = \frac{1}{2m} P^2 + \frac{m\nu^2}{2} X^2$$

where ν is the oscillator frequency.

But to $quantise$, = introduce creation and annihilation ops, requires $\hbar$.
To write the total energy as $\omega(N + 1/2)$, requires $\hbar$ to convert frequency to energy $\omega = \hbar\nu$, and downstairs in the defn of N, because its the number of $quanta$.

Erken,Sikivie,Tam,Yang

Sikivie's scenario—some questions and guesses

1. at $T_\gamma \sim$ keV, "gravitational thermalisation" of axions drives them to a Bose-Einstein Condensate

Saikawa Yamaguchi

- which axions (field or particles) ?
  ESTY, S+Y consider the axion field
- what is grav thermalisation?
  ESTY,S+Y: grav interactions of axions
  SD+ME: entropy-producing grav interactions
- what is a BEC? /when does a classical field support vortices?
  DRS:classical field supports vortices for ranges of mass, coupling, not including axion.

2. BEC can support vortices, which allow caustics in the galactic DM distribution $\Leftrightarrow$ axion DM signature?

BEC galactic halos:
Rindler-Daller+Shapiro

- how do galaxies form in a "BEC" (?does it stay a "BEC"?)?
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