

NLO merging with PYTHIA 8

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(DESY)

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Outline

- Comments on leading-order ME+PS merging.
- NLO multi-jet merging in PYTHIA 8.
- NNLO outlook.
- Summary.

The ME+PS merging problem

Problem: We want to describe soft/collinear and hard jets in one sample, but we do not know the boundary between “soft” and “hard”. Parton showers are good for describing soft/collinear jets. Fixed-order matrix elements are good for well-separated jets.

⇒ Should we just “add” several PS and ME samples?

- Just adding ME and PS gives massive double counting.
→ Instead, use ME above a cut t_{MS} , and PS below t_{MS} .
- This still has problems: ME overlaps and cut dependence.
→ Apply the same weights above and below the cut.

→ MEPS merging

CKKW(-L)¹ merging

The CKKW-L prescription is:

- ◇ Calculate the tree-level MEs.

$$\langle \mathcal{O} \rangle = B_0 \int \mathcal{O}(S_{+0j}) B_1 \Theta(t(S_{+1}) - t_{MS}) \mathcal{O}(S_{+1j})$$

¹ JHEP 0111 (2001) 063 (Catani, Krauss, Kuhn, Webber), JHEP 0205 (2002) 046 (Lönnblad) ... 4 / 30

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- ◇ Start the PS on the reweighted ME configuration.
Veto the event if the PS emission if it gives a state in the ME region.
- ◇ Combine by adding all accepted events.

$$\langle \mathcal{O} \rangle = B_0 \times \Pi_{S_{+0}}(\rho_0, t_{MS}) \mathcal{O}(S_{+0j}) \\ + \int B_1 \Theta(t(S_{+1}) - t_{MS}) w_f^0 w_{\alpha_s}^0 \Pi_{S_{+0}}(\rho_0, \rho_1) \mathcal{O}(S_{+1j})$$

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Illustration

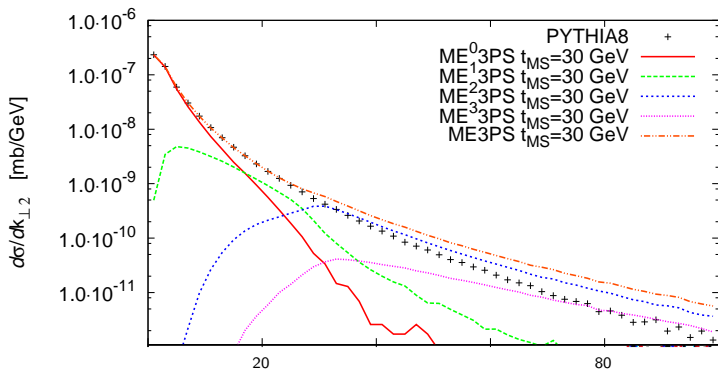


Figure: Separation between the first and second jet for W +jets, when clustering to exactly two jets. The coloured lines show the different reweighted multi-parton MEs.

CKKW-L results

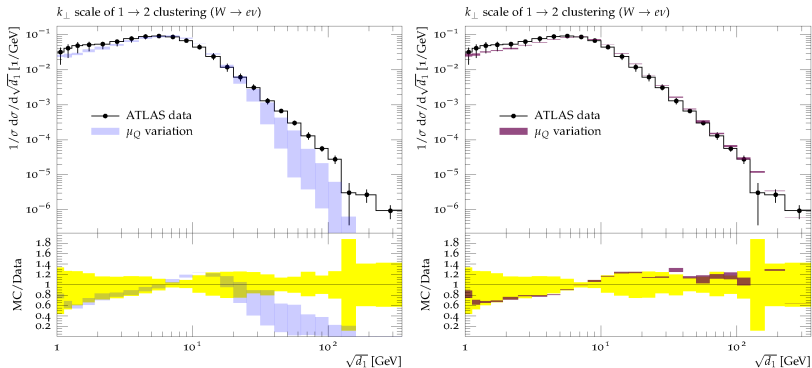


Figure: k_{\perp} -separation between the first and second jet for W +jets, when clustering to exactly two jets. The bands are obtained by varying the PS starting scale in $\mu_Q \in [\frac{1}{2}M_W, 2M_W]$

... however

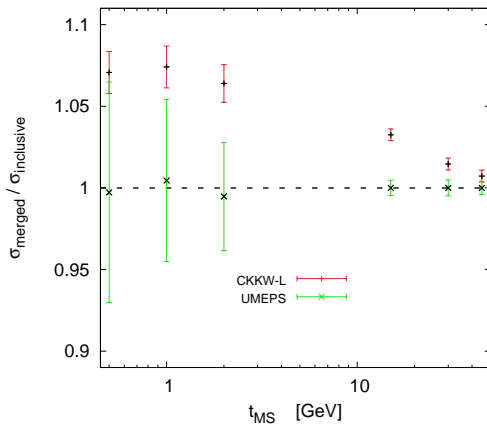


Figure: Ratio of the inclusive cross section after merging, compared to the tree-level inclusive cross section.

The problem with CKKW-L

The ME includes terms that are not compensated by the PS approximate virtual corrections (i.e. Sudakov factors).

These are the improvements that we need to describe multiple hard jets!
But if we simply add samples, the “improvements” will degrade the inclusive (0-jet) cross section by $\ln^m(t_{MS}/Q)$ -terms!

Traditional approach: Don't use a too small merging scale.

→ Uncancelled terms numerically not important.

Unitary approach¹:

Use a (PS) unitarity inspired approach exactly cancel the dependence of the inclusive cross section on ρ_{MS} , thus preserving the inclusive cross section.

CKKW(-L)¹ merging

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- ◇ Combine by adding all accepted events.

$$\langle \mathcal{O} \rangle = B_0 \times \Pi_{S_{+0}}(\rho_0, t_{MS}) \mathcal{O}(S_{+0j}) \\ + \int B_1 \Theta(t(S_{+1}) - t_{MS}) w_f^0 w_{\alpha_s}^0 \Pi_{S_{+0}}(\rho_0, \rho_1) \mathcal{O}(S_{+1j})$$

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CKKW(-L)¹ merging

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- ◇ Calculate the tree-level MEs.
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- ◇ Start the PS on the reweighted ME configuration.
Veto the event if the PS emission if it gives a state in the ME region.
- ◇ Combine by adding all accepted events.

$$\langle \mathcal{O} \rangle = B_0 \left(1 - \int d\rho w_f^0 w_{\alpha_s}^0 P_0(\rho) \Pi_{S_{+0}}(\rho_0, \rho) \Theta(t(S_{+1}) - t_{MS}) \right) \mathcal{O}(S_{+0j}) \\ + \int B_1 \Theta(t(S_{+1}) - t_{MS}) w_f^0 w_{\alpha_s}^0 \Pi_{S_{+0}}(\rho_0, \rho_1) \mathcal{O}(S_{+1j})$$

¹ JHEP 0111 (2001) 063 (Catani, Krauss, Kuhn, Webber), JHEP 0205 (2002) 046 (Lönnblad) ...

UMEPS merging

The UMEPS prescription is:

- ◇ Calculate the tree-level MEs.
- ◇ Reweight with Sudakovs, α_s - and PDF-ratios.
- ◇ Process 1j-ME again: Reweight, then project onto 0-parton state.
- ◇ Start the PS on the reweighted ME configuration.
Veto the PS emission if it gives a state in the ME region.
- ◇ Combine by adding reweighted events, and subtracting reweighted-projected events.

$$\langle \mathcal{O} \rangle = B_0 \left(1 - \int d\rho w_f^0 w_{\alpha_s}^0 \frac{B_1}{B_0} \Pi_{S_{+0}}(\rho_0, \rho) \Theta(t(S_{+1}) - t_{MS}) \right) \mathcal{O}(S_{+0j}) \\ + \int B_1 \Theta(t(S_{+1}) - t_{MS}) w_f^0 w_{\alpha_s}^0 \Pi_{S_{+0}}(\rho_0, \rho_1) \mathcal{O}(S_{+1j})$$

Comments on UMEPS

This sketch directly extends to the case of (α_s - or Sudakov-) weighted $+n$ -jet states (\widehat{B}_n), e.g. two-jet UMEPS merging:

$$\begin{aligned} \langle \mathcal{O} \rangle = & \int d\phi_0 \left\{ \mathcal{O}(S_{+0j}) \left[B_0 - \int_s \widehat{B}_{1 \rightarrow 0} - \int_s \widehat{B}_{2 \rightarrow 0} \right] \right. \\ & + \int \mathcal{O}(S_{+1j}) \left[\widehat{B}_1 - \int_s \widehat{B}_{2 \rightarrow 1} \right] \\ & \left. + \int \cdots \int \mathcal{O}(S_{+2j}) \widehat{B}_2 \right\} \end{aligned}$$

Integrated version of the real-emission matrix elements are available anyway in MEPS since we need them to perform the Sudakov weighting.

The "subtract what you add" prescription means that this will produce *counter-events* with negative weight \rightarrow Statistics might be problematic, if the integrations are done separately.

Unitarisation should be helpful for inclusive observables, heavy flavour (fix t_{MS} by flavour thresholds, e.g. always fill full $g \rightarrow b\bar{b}$ phase space with ME, independent of the process), MC tuning with merging, . . .

UMEPS results

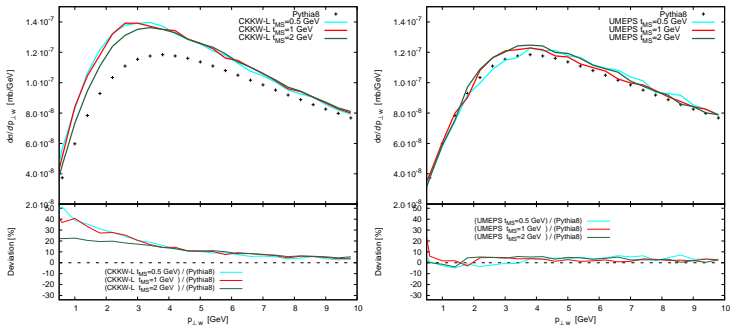


Figure: p_{\perp} of the W-boson in the Sudakov region (for 2-jet merging, $E_{CM} = 7$ TeV)

⇒ CKKW-L overshoots for (very) low merging scales.

⇒ UMEPS describes the Sudakov peak nicely.

(For jet observables (high- p_{\perp} tails etc.) UMEPS does as nicely as CKKW-L.)

Is leading-order enough?

UMEPS is a leading-order method, i.e. it contains only approximate virtual corrections.

We want to use the full NLO results whenever possible.

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We want to use the full NLO results whenever possible.

Basic idea: Do NLO multi-jet merging for UMEPS:

- ◇ Subtract approximate UMEPS $\mathcal{O}(\alpha_s)$ -terms, add back full NLO.
 - ◇ To preserve the inclusive (NLO) cross section, add approximate NNLO.
- ⇒ UNLOPS¹.

For UNLOPS merging, we need exclusive NLO inputs:

$$\tilde{B}_n = B_n + V_n + I_{n+1|n} + \int d\Phi_{\text{rad}} (B_{n+1|n} \Theta(\rho_{\text{MS}} - t(S_{+n+1}, \rho)) - D_{n+1|n})$$

¹ JHEP1303(2013)166 (Leif Lönnblad, SP), Similar scheme in JHEP1308(2013)114 (Simon Plätzer) 12 / 30

Technicalities: How do we get \tilde{B}_n ?

Like everyone else, we need exclusive NLO cross sections as input. Note that

$$\begin{aligned}\tilde{B}_n &= B_n + V_n + I_{n+1|n} + \int (B_{n+1|n} - D_{n+1|n}) - \int B_{n+1|n} \Theta(t(S_{n+1}, \rho) - \rho_{MS}) \\ &= B_{n,inc}^{NLO} - \int_{\rho_{ms}} B_{n+1|n}\end{aligned}$$

and remember that, after the first emission, matched calculations give

$$\begin{aligned}\text{Born-like events} & \qquad \qquad \qquad + \text{radiative events} \\ &= \bar{B}_n \Delta(p_{\perp c}) \mathcal{O}(S_{+0}) \qquad \qquad \qquad + \int \left[\bar{B}_n \frac{B_{n+1}^s}{B_n} \Delta(p_{\perp}) + (B_{n+1} - B_{n+1}^s) \right] \mathcal{O}(S_{+1}) \\ &= \bar{B}_n \left[1 - \int_{p_{\perp c}} \frac{B_{n+1}^s}{B_n} \Delta(p_{\perp}) \right] \mathcal{O}(S_{+0}) \qquad + \int \left[\bar{B}_n \frac{B_{n+1}^s}{B_n} \Delta(p_{\perp}) + (B_{n+1} - B_{n+1}^s) \right] \mathcal{O}(S_{+1})\end{aligned}$$

with $B_{n+1}^s = B_{n+1} \cdot F(p_{\perp})$ in POWHEG and $B_{n+1}^s = D_{n+1|n}^{PS}$ in MC@NLO, and

$$\bar{B}_n = B_n + V_n + I_{n+1|n} + \int (B_{n+1|n}^s - D_{n+1|n}) \ , \quad \Delta(p_{\perp}) = \exp \left(- \int_{p_{\perp}} \frac{B_{n+1}^s}{B_n} \right)$$

Thus, if we project onto an underlying Born for radiative events, we get $\bar{B}_n + (B_{n+1|n} - B_{n+1|n}^s)$. By having a subtraction sample $\int_{\rho_{ms}} B_{n+1|n}$, we get \tilde{B}_n .

The UNLOPS method

Start with UMEPS:

$$\langle \mathcal{O} \rangle = \int d\phi_0 \left\{ \mathcal{O}(S_{+0j}) \left(B_{0+} - \int_s \hat{B}_{1 \rightarrow 0} - \int_s \hat{B}_{2 \rightarrow 0} \right) + \int \mathcal{O}(S_{+1j}) \left(\hat{B}_1 - \int_s \hat{B}_{2 \rightarrow 1} \right) + \iint \mathcal{O}(S_{+2j}) \hat{B}_2 \right\}$$

The UNLOPS method

Remove all unwanted $\mathcal{O}(\alpha_s^n)$ - and $\mathcal{O}(\alpha_s^{n+1})$ -terms:

$$\langle \mathcal{O} \rangle = \int d\phi_0 \left\{ \mathcal{O}(S_{+0j}) \left(- \left[\int_s \widehat{\mathbf{B}}_{1 \rightarrow 0} \right]_{-1,2} - \int_s \widehat{\mathbf{B}}_{2 \rightarrow 0} \right) + \int \mathcal{O}(S_{+1j}) \left(\left[\widehat{\mathbf{B}}_1 \right]_{-1,2} - \left[\int_s \widehat{\mathbf{B}}_{2 \rightarrow 1} \right]_{-2} \right) + \iint \mathcal{O}(S_{+2j}) \widehat{\mathbf{B}}_2 \right\}$$

The UNLOPS method

Add full NLO results:

$$\langle \mathcal{O} \rangle = \int d\phi_0 \left\{ \mathcal{O}(S_{+0j}) \left(\tilde{\mathbf{B}}_0 - \left[\int_s \hat{\mathbf{B}}_{1 \rightarrow 0} \right]_{-1,2} - \int_s \hat{\mathbf{B}}_{2 \rightarrow 0} \right) \right. \\ \left. + \int \mathcal{O}(S_{+1j}) \left(\tilde{\mathbf{B}}_1 + \left[\hat{\mathbf{B}}_1 \right]_{-1,2} - \left[\int_s \hat{\mathbf{B}}_{2 \rightarrow 1} \right]_{-2} \right) + \iint \mathcal{O}(S_{+2j}) \hat{\mathbf{B}}_2 \right\}$$

The UNLOPS method

Unitarise:

$$\langle \mathcal{O} \rangle = \int d\phi_0 \left\{ \mathcal{O}(S_{+0j}) \left(\tilde{B}_0 - \int_s \tilde{B}_{1 \rightarrow 0} + \int_s B_{1 \rightarrow 0} - \left[\int_s \hat{B}_{1 \rightarrow 0} \right]_{-1,2} - \int_s B_{2 \rightarrow 0}^\uparrow - \int_s \hat{B}_{2 \rightarrow 0} \right) \right. \\ \left. + \int \mathcal{O}(S_{+1j}) \left(\tilde{B}_1 + \left[\hat{B}_1 \right]_{-1,2} - \left[\int_s \hat{B}_{2 \rightarrow 1} \right]_{-2} \right) + \iint \mathcal{O}(S_{+2j}) \hat{B}_2 \right\}$$

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UNLOPS merging of zero and one parton at NLO:

$$\langle \mathcal{O} \rangle = \int d\phi_0 \left\{ \mathcal{O}(S_{+0j}) \left(\tilde{B}_0 - \int_s \tilde{B}_{1 \rightarrow 0} + \int_s B_{1 \rightarrow 0} - \left[\int_s \hat{B}_{1 \rightarrow 0} \right]_{-1,2} - \int_s B_{2 \rightarrow 0}^\uparrow - \int_s \hat{B}_{2 \rightarrow 0} \right) \right. \\ \left. + \int \mathcal{O}(S_{+1j}) \left(\tilde{B}_1 + \left[\hat{B}_1 \right]_{-1,2} - \left[\int_s \hat{B}_{2 \rightarrow 1} \right]_{-2} \right) + \iint \mathcal{O}(S_{+2j}) \hat{B}_2 \right\}$$

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Iterate for the case of M different NLO calculations, and N tree-level calculations:

$$\langle \mathcal{O} \rangle = \sum_{m=0}^{M-1} \int d\phi_0 \int \cdots \int \mathcal{O}(S_{+mj}) \left\{ \tilde{\mathbf{B}}_m + \left[\hat{\mathbf{B}}_m \right]_{-m,m+1} + \int_s \mathbf{B}_{m+1 \rightarrow m} \right. \\ \left. - \sum_{i=m+1}^M \int_s \tilde{\mathbf{B}}_{i \rightarrow m} - \sum_{i=m+1}^M \left[\int_s \hat{\mathbf{B}}_{i \rightarrow m} \right]_{-i,i+1} - \sum_{i=m+1}^M \int_s \mathbf{B}_{i+1 \rightarrow m}^\dagger - \sum_{i=M+1}^N \int_s \hat{\mathbf{B}}_{i \rightarrow m} \right\} \\ + \int d\phi_0 \int \cdots \int \mathcal{O}(S_{+Mj}) \left\{ \tilde{\mathbf{B}}_M + \left[\hat{\mathbf{B}}_M \right]_{-M,M+1} - \left[\int_s \hat{\mathbf{B}}_{M+1 \rightarrow M} \right]_{-M} - \sum_{i=M+1}^N \int_s \hat{\mathbf{B}}_{i+1 \rightarrow M} \right\} \\ + \sum_{n=M+1}^N \int d\phi_0 \int \cdots \int \mathcal{O}(S_{+nj}) \left\{ \hat{\mathbf{B}}_n - \sum_{i=n+1}^N \int_s \hat{\mathbf{B}}_{i \rightarrow n} \right\}$$

The UNLOPS method

UNLOPS merging of zero and one parton at NLO:

$$\langle \mathcal{O} \rangle = \int d\phi_0 \left\{ \mathcal{O}(S_{+0j}) \left(\tilde{B}_0 - \int_s \tilde{B}_{1 \rightarrow 0} + \int_s B_{1 \rightarrow 0} - \left[\int_s \hat{B}_{1 \rightarrow 0} \right]_{-1,2} - \int_s B_{2 \rightarrow 0}^\dagger - \int_s \hat{B}_{2 \rightarrow 0} \right) \right. \\ \left. + \int \mathcal{O}(S_{+1j}) \left(\tilde{B}_1 + \left[\hat{B}_1 \right]_{-1,2} - \left[\int_s \hat{B}_{2 \rightarrow 1} \right]_{-2} \right) + \iint \mathcal{O}(S_{+2j}) \hat{B}_2 \right\}$$

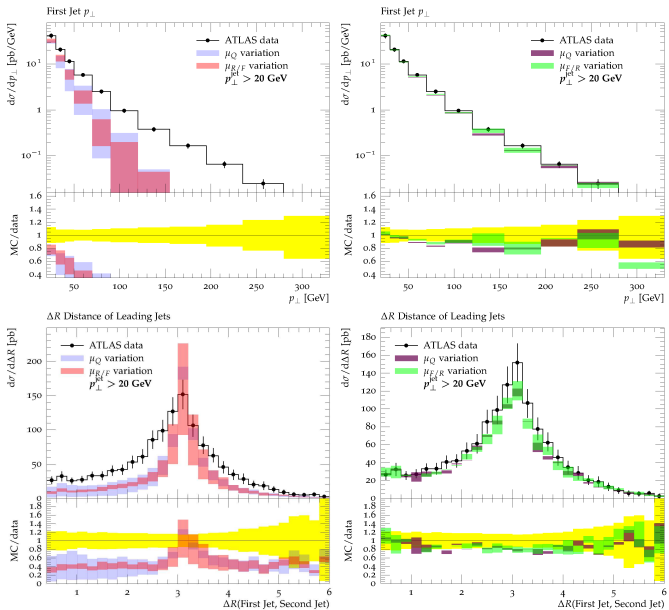
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Inputs (B_n, \tilde{B}_n) taken from external tools.

Merging done internally in PYTHIA 8.

UNLOPS results (W+jets)



Inclusive sample containing (W + no resolved)@NLO, (W + one resolved)@NLO and (W + two resolved)@LO.

NLO merged results (H+jets, the real reason why we unitarise)

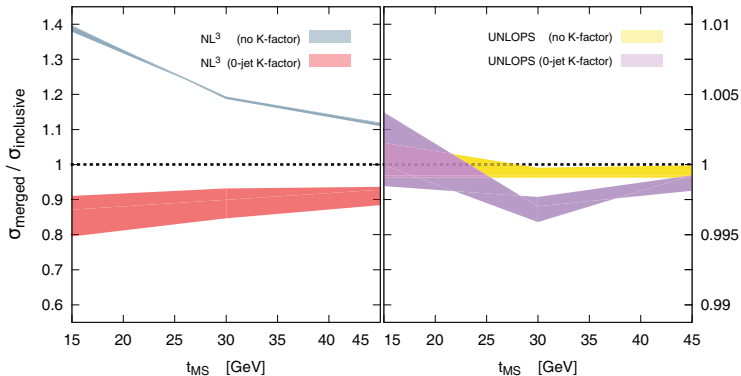


Figure: Ratio of the inclusive cross section for $gg \rightarrow H$ after merging (H+0)@NLO, (H+1)@NLO and (H+2)@LO, compared to the NLO inclusive cross section.

\Rightarrow NL³ (=CKKW-L@NLO) has problems for processes with large, loop-driven NLO corrections.

NLO merged results (squarks+jets)

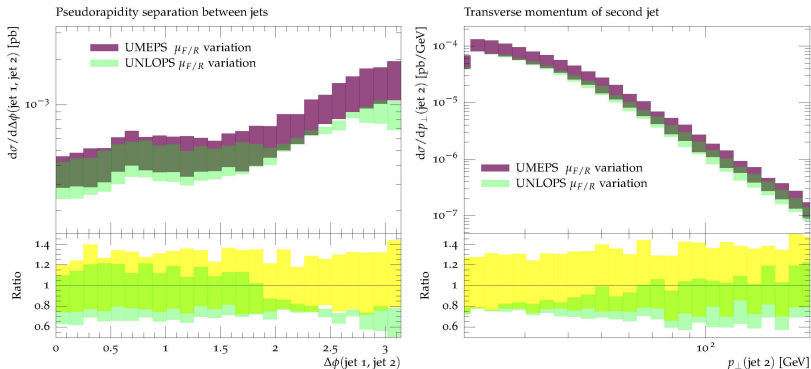


Figure: $\Delta\phi_{12}$ and $p_{\perp 2}$ for u-squark pair production ($m_{\tilde{u}} = 500$ GeV, $m_{\chi_0} = 500$ GeV, $\text{BR}(\tilde{u} \rightarrow u\chi_0) \approx 1$) after merging (squarks+0)@NLO¹, (squarks+1)@LO and (squarks+2)@LO.

¹ arXiv:1305.4061 (Gavin, Hangst, Krämer, Mühlleitner, Pellen, Popenda, Spira)

UNLOPS with PYTHIA8 ← SHERPA – polygamy is fun

If MC@NLO provides event output after the first emission, then this can be used to construct \tilde{B}_n .

SHERPA is fast, has lots of OLP interfaces, and does now provide such LHEF output. We can use this as input for PYTHIA8!

A simple, yet helpful feature is: We can set + transfer μ_f, μ_r on an event-by-event basis (not there for other LHEF-generators yet). This allows to UNLOPS-merge processes with no intrinsic hard scale (e.g. pure QCD jets).

No plots yet, work in progress. Thanks for the support!

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Before continuing, some UNLOPS downsides:

1. Any old, bad Sudakov will do.
2. NLO correction not smeared over resolved-real phase space.
3. Statistics becomes an issue for low ρ_{MS} .

Points 1. and 2. could also be sold as positive.

Comparison to other schemes

Other codes don't go through the trouble of unitarisation. Why?

MEPS@NLO¹: Improved, colour- & spin-correct Sudakov of MC@NLO for the first emission.

→ No uncanceled logs for the soft, real emission parton, only beneficial power-corrections from [R-D]-events

Improved resummation in process-independent way.

How about logs from soft-ish non-softest jets (e.g. hardest jet in 2j ME)?

MinLO²: Applies analytical NNLL information, which cancels the necessary terms when merging two multiplicities.

Can be (and was) moulded into an NNLO matching³.

Depends on the process class through necessary NNLL coefficients.

FxFx⁴: Restricts the range of t_{MS} . Violation numerically small.

Few counter events. MLM-style veto instead of Sudakovs.

¹ JHEP1301(2013)144, JHEP1304(2013)027 (Gehrmann + Höche, Krauss, Schönherr, Siegert)

² JHEP1305(2013)082 (Hamilton, Nason, Oleari, Zanderighi)

³ arXiv:1309.0017 (Hamilton, Nason, Re, Zanderighi)

⁴ JHEP1212(2012)061 (Frederix, Frixione)

So what's next? NNLO?

Note that in UNLOPS, the lowest-multiplicity input cross section is *not* reweighted. To go to NNLO, we have to identify terms of $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(\alpha_s^2)$...

$$\begin{aligned} \langle \mathcal{O} \rangle = & \int d\phi_0 \left\{ \mathcal{O}(S_{+0j}) \left(\tilde{\mathbf{B}}_0 - \int_s \tilde{\mathbf{B}}_{1 \rightarrow 0} + \int_s \mathbf{B}_{1 \rightarrow 0} - \left[\int_s \hat{\mathbf{B}}_{1 \rightarrow 0} \right]_{-1,2} \right. \right. \\ & \left. \left. - \int_s \mathbf{B}_{2 \rightarrow 0}^\dagger - \int_s \hat{\mathbf{B}}_{2 \rightarrow 0} \right) \right. \\ & + \int \mathcal{O}(S_{+1j}) \left(\tilde{\mathbf{B}}_1 + [\hat{\mathbf{B}}_1]_{-1,2} - \left[\int_s \hat{\mathbf{B}}_{2 \rightarrow 1} \right]_{-2} \right) \\ & \left. + \iint \mathcal{O}(S_{+2j}) \hat{\mathbf{B}}_2 \right. \end{aligned}$$

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 &\quad \left. \left. - \int_s \mathbf{B}_{2 \rightarrow 0}^\dagger - \int_s \hat{\mathbf{B}}_{2 \rightarrow 0} \right) \right. \\
 \mathcal{O}(\alpha_s) &\quad \left. + \int \mathcal{O}(S_{+1j}) \left(\tilde{\mathbf{B}}_1 + [\hat{\mathbf{B}}_1]_{-1,2} - \left[\int_s \hat{\mathbf{B}}_{2 \rightarrow 1} \right]_{-2} \right) \right. \\
 \mathcal{O}(\alpha_s^2) &\quad \left. + \iint \mathcal{O}(S_{+2j}) \hat{\mathbf{B}}_2 \right.
 \end{aligned}$$

Terms entering due to PS weights are $\mathcal{O}(\alpha_s^2)_{\text{PS}} \times \mathcal{O}(\alpha_s^1)_{\text{ME}}$ and $\mathcal{O}(\alpha_s^0)_{\text{PS}} \times \mathcal{O}(\alpha_s^2)_{\text{ME}}$

So what's next? NNLO?

Note that in UNLOPS, the lowest-multiplicity input cross section is *not* reweighted. To go to NNLO, we have to identify terms of $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(\alpha_s^2)$...

$$\begin{aligned} \langle \mathcal{O} \rangle = & \int d\phi_0 \left\{ \mathcal{O}(S_{+0j}) \left(\left[\int_s \widehat{\mathbf{B}}_{1 \rightarrow 0} \right]_{-1,2} \right. \right. \\ & \left. \left. - \left[\int_s \widehat{\mathbf{B}}_{2 \rightarrow 0} \right]_{-2} \right) \right. \\ & + \int \mathcal{O}(S_{+1j}) \left(\widetilde{\mathbf{B}}_1 + \left[\widehat{\mathbf{B}}_1 \right]_{-1,2} - \left[\int_s \widehat{\mathbf{B}}_{2 \rightarrow 1} \right]_{-2} \right) \\ & \left. + \iint \mathcal{O}(S_{+2j}) \widehat{\mathbf{B}}_2 \right. \end{aligned}$$

... now remove these terms

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... now remove these terms ... and instead use the exclusive NNLO result.

NNLO with UNLOPS

Alternatively, we can use the inclusive cross section for NNLO matching:

$$\begin{aligned}
 \langle \mathcal{O} \rangle = & \int d\phi_0 \left\{ \mathcal{O}(S_{+0j}) \left(\overline{\overline{B}}_0 - \int_s \overline{B}_{1 \rightarrow 0} - \left[\int_s \widehat{B}_{1 \rightarrow 0} \right]_{-1,2} - \left[\int_s \widehat{B}_{2 \rightarrow 0} \right]_{-2} \right) \right. \\
 & + \int \mathcal{O}(S_{+1j}) \left(\overline{B}_1 + \left[\widehat{B}_1 \right]_{-1,2} - \int_s \widehat{B}_{2 \rightarrow 1} \right) \\
 & \left. + \iint \mathcal{O}(S_{+2j}) \widehat{B}_2 \right\}
 \end{aligned}$$

- Relatively clean only because the NLO cross section was not reweighted. Process-independent.
- ρ_{MS} -definition must be infrared safe at NNLO¹.
- Needs NNLO generator to produce $\widetilde{\overline{B}}_0$ or $\overline{\overline{B}}_0$. For $\widetilde{\overline{B}}_0$, the ρ_{MS} -definition has to be known within the NNLO calculation. Otherwise, $\overline{\overline{B}}_0$ should best be fully differential, so that the PS can project onto 0-parton states.

(Note to self: stop japping, do it. But NNLO “events” unobtainable so far...)

¹ see e.g. discussion in arXiv:1311.0286 (Alioli, Bauer, Berggren, Tackmann, Walsh, Zuberi) 20 / 30

Question: Precalculated samples for merging?

In unitarised merging, the parton shower on the full $\mathcal{O}(S_{+nj})$ -expression will **never** veto an event.

It would thus be highly desirable to, *once and for all*, integrate all contributions to $\mathcal{O}(S_{+nj})$, e.g.

$$\mathcal{O}(S_{+0j}) \left[\bar{B}_0 - \int_s \hat{B}_{1 \rightarrow 0} - \int_s \hat{B}_{2 \rightarrow 0} \right] \quad \text{UMEPS}$$

$$\mathcal{O}(S_{+0j}) \left[\bar{\bar{B}}_0 - \int_s \bar{B}_{1 \rightarrow 0} - \left[\int_s \hat{B}_{1 \rightarrow 0} \right]_{-1,2} - \int_s \hat{B}_{2 \rightarrow 0} \right] \quad \text{UNLOPS}$$

$$\mathcal{O}(S_{+0j}) \left[\bar{\bar{B}}_0 - \int_s \bar{B}_{1 \rightarrow 0} - \left[\int_s \hat{B}_{1 \rightarrow 0} \right]_{-1,2} - \left[\int_s \hat{B}_{2 \rightarrow 0} \right]_{-2} \right] \quad \text{UNLOPS@NNLO}$$

before showers and detector simulation.

This would be significantly more efficient than what I do right now!

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before showers and detector simulation.

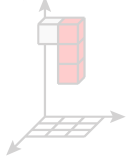
This would be significantly more efficient than what I do right now!

SHERPA contains almost all ingredients for the first two, and has an improved Sudakov. Should we do a unitarisation, and then an NNLO matching in SHERPA? Anyone interested?

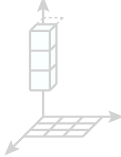
Summary

- To describe data, we need to infuse parton showers with matrix elements.
- CKKW-L tree-level merging is included in PYTHIA 8.
But it does not work for very small merging scales.
- UMEPS tree-level merging is included in PYTHIA 8.
UMEPS almost cancels the merging scale dependence.
But it's not NLO.
- Two NLO merging schemes are implemented in PYTHIA 8:
NL³ and UNLOPS. The latter is our preferred choice.
- All merging schemes in PYTHIA 8 run on LHEF input, e.g. from Sherpa :)

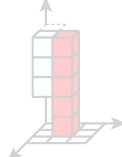
Zero-jet NLO input:



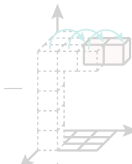
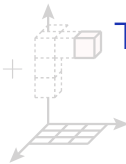
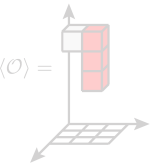
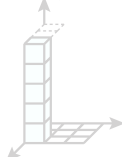
One-jet tree-level input:



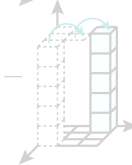
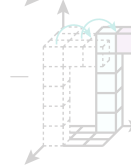
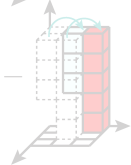
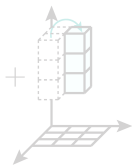
One-jet NLO input:



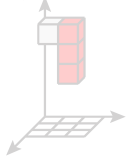
Two-jet tree-level input:



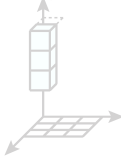
Thank you for your time.



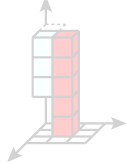
Zero-jet NLO input:



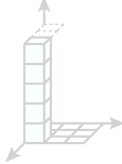
One-jet tree-level input:



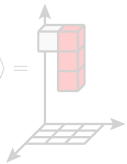
One-jet NLO input:



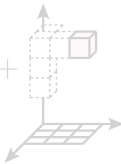
Two-jet tree-level input:



$\langle O \rangle =$



+

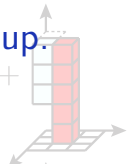


-

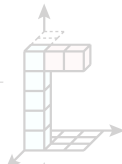


Back-up.

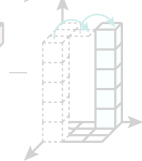
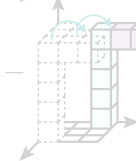
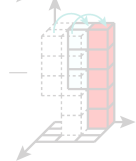
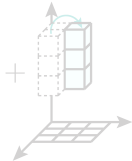
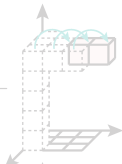
+



+



-



UMEPS definitions

$$w_n = \frac{x_n^+ f_n^+(x_n^+, \rho_n)}{x_n^+ f_n^+(x_n^+, \mu_F)} \frac{x_n^- f_n^-(x_n^-, \rho_n)}{x_n^- f_n^-(x_n^-, \mu_F)}$$

$$\times \prod_{i=1}^n \left[\frac{\alpha_s(\rho_i)}{\alpha_s(\mu_R)} \frac{x_{i-1}^+ f_{i-1}^+(x_{i-1}^+, \rho_{i-1})}{x_{i-1}^+ f_{i-1}^+(x_{i-1}^+, \rho_i)} \frac{x_{i-1}^- f_{i-1}^-(x_{i-1}^-, \rho_{i-1})}{x_{i-1}^- f_{i-1}^-(x_{i-1}^-, \rho_i)} \Pi_{S_{+i-1}}(x_{i-1}, \rho_{i-1}, \rho_i) \right]$$

$$\widehat{B}_n = B_n w_n$$

$$\int_s \widehat{B}_{n \rightarrow m} = \left[\prod_{a=m+1}^{n-1} \int d\rho_a dz_a d\varphi_a \Theta(\rho_{MS} - \rho_a) \right] \int d\rho_n dz_n d\varphi_n B_n w_n$$

$$\langle \mathcal{O} \rangle = \sum_{n=0}^N \int d\phi_0 \int \dots \int \mathcal{O}(S_{+nj}) \left\{ \widehat{B}_n - \sum_{i=n+1}^N \int_s \widehat{B}_{i \rightarrow n} \right\} .$$

In CKKW-L, w_n contains an additional factor $\Pi_{S_{+n}}(x_n, \rho_n, \rho_{MS})$.
 UMEPS induces this through $\int_s \widehat{B}_{n+1 \rightarrow n}$ instead.

UMEPS'

Here, we illustrate how to include the full kinematical information also below the merging scale.

$$d\sigma_{+0}^u \equiv \widehat{B}_{+0} - \int_s \widehat{B}_{0+1 \rightarrow 0} \quad d\sigma_{+1}^u \equiv \widehat{B}_{+1}$$

So far, we have

$$\begin{aligned} \langle \mathcal{O} \rangle = & d\sigma_{+0}^u \times [\Delta(\rho_0, \rho_c) \mathcal{O}_0(S_{+0}) + \Delta(\rho_{MS}, \rho_1) d\rho_1 dz_1 P(\rho_1, z_1) \times (\Delta(\rho_1, \rho_c) \mathcal{O}_1(S_{+1}) + \dots)] \\ & + d\sigma_{+1}^u \times [\Delta(\rho_1, \rho_c) \mathcal{O}_1(S_{+1}) + \Delta(\rho_1, \rho_2) d\rho_2 dz_2 P(\rho_2, z_2) \times (\Delta(\rho_2, \rho_c) \mathcal{O}_2(S_{+2}) + \dots)] \\ & \times \Theta(\rho_1 - \rho_{MS}) \end{aligned}$$

But we could write

$$\begin{aligned} \langle \mathcal{O} \rangle = & d\sigma_{+0}^u \times [\Delta(\rho_0, \rho_c) \mathcal{O}_0(S_{+0}) + \Delta(\rho_{MS}, \rho_1) d\rho_1 dz_1 P(\rho_1, z_1) \times (\Delta(\rho_1, \rho_c) \mathcal{O}_1(S_{+1}) \quad)] \\ & + d\sigma_{+1}^u \times [\Delta(\rho_1, \rho_c) \mathcal{O}_1(S_{+1}) + \Delta(\rho_1, \rho_2) d\rho_2 dz_2 P(\rho_2, z_2) \times (\Delta(\rho_2, \rho_c) \mathcal{O}_2(S_{+2}) + \dots)] \\ & \times \Theta(\rho_1 - \rho_{MS}) \\ & + d\sigma_{+1}^d \times [\Delta(\rho_1, \rho_c) \mathcal{O}_1(S_{+1}) + \Delta(\rho_1, \rho_2) d\rho_2 dz_2 P(\rho_2, z_2) \times (\Delta(\rho_2, \rho_c) \mathcal{O}_2(S_{+2}) + \dots)] \\ & \times \Theta(\rho_{MS} - \rho_1) \\ & - d\sigma_{+0}^u \times \Delta(\rho_{MS}, \rho_1) d\rho_1 dz_1 P(\rho_1, z_1) \times \Delta(\rho_1, \rho_c) \mathcal{O}_1(S_{+1}) \\ & - \int \left\{ d\sigma_{+1}^d \times [\Delta(\rho_1, \rho_c) + \Delta(\rho_1, \rho_2) d\rho_2 dz_2 P(\rho_2, z_2) \times (\Delta(\rho_2, \rho_c) + \dots)] \times \Theta(\rho_{MS} - \rho_1) \right. \\ & \left. - d\sigma_{+0}^u \times \Delta(\rho_{MS}, \rho_1) d\rho_1 dz_1 P(\rho_1, z_1) \times \Delta(\rho_1, \rho_c) \right\} \mathcal{O}_0(S_{+0}) \end{aligned}$$

UMEPS' = UMEPS

This is just

$$\begin{aligned}
 \langle \mathcal{O} \rangle &= d\sigma_{+0}^u \mathcal{O}_0(S_{+0}) - \int d\sigma_{+1}^d \times \Theta(\rho_{MS} - \rho_1) \mathcal{O}_0(S_{+0}) \\
 &+ d\sigma_{+1}^u \times [\Delta(\rho_1, \rho_c) \mathcal{O}_1(S_{+1}) + \Delta(\rho_1, \rho_2) d\rho_2 dz_2 P(\rho_2, z_2) \times (\Delta(\rho_2, \rho_c) \mathcal{O}_2(S_{+2}) + \dots)] \\
 &\quad \times \Theta(\rho_1 - \rho_{MS}) \\
 &+ d\sigma_{+1}^d \times [\Delta(\rho_1, \rho_c) \mathcal{O}_1(S_{+1}) + \Delta(\rho_1, \rho_2) d\rho_2 dz_2 P(\rho_2, z_2) \times (\Delta(\rho_2, \rho_c) \mathcal{O}_2(S_{+2}) + \dots)] \\
 &\quad \times \Theta(\rho_{MS} - \rho_1)
 \end{aligned}$$

which for $d\sigma_{+1}^d = d\sigma_{+1}^u$ becomes

$$\begin{aligned}
 \langle \mathcal{O} \rangle &= d\sigma_{+0}^u \mathcal{O}_0(S_{+0}) \\
 &+ d\sigma_{+1}^u \times [\Delta(\rho_1, \rho_c) \mathcal{O}_1(S_{+1}) + \Delta(\rho_1, \rho_2) d\rho_2 dz_2 P(\rho_2, z_2) \times (\Delta(\rho_2, \rho_c) \mathcal{O}_2(S_{+2}) + \dots)] \\
 &\quad \times \Theta(\rho_1 - \rho_c)
 \end{aligned}$$

i.e. simply the UMEPS result for $\rho_{MS} = \rho_c$.

Note that this does not depend on the form of $d\sigma_{+0}^u$ or $d\sigma_{+1}^u$, and can thus also be applied for UNLOPS to add real-emission kinematics with $\rho(S_{+n+1}) < \rho_{MS}$.

Extreme merging scale values

In unitarised merging, t_{MS} can be varied between the PS cut-off ρ_c and ∞ . The t_{MS} dependence is almost exactly cancelled (**caveat: jet definition**)

For $t_{\text{MS}} \rightarrow \infty$, the spectrum of the first add. emission is given by the PS. The below- t_{MS} behaviour can be fixed¹, leading e.g. to the following form:

$$\langle \mathcal{O} \rangle = \left[d\sigma_{+0}^u - \int d\sigma_{+1}^d \times \Theta(\rho_{\text{MS}} - \rho_1) \right] \mathcal{O}_0(S_{+0}) + \text{PS} \left[d\sigma_{+1}^u \Theta(\rho_1 - \rho_{\text{MS}}) \right] + \text{PS} \left[d\sigma_{+1}^d \Theta(\rho_{\text{MS}} - \rho_1) \right]$$

which for $d\sigma_{+1}^d = d\sigma_{+1}^u$ becomes

$$\langle \mathcal{O} \rangle = d\sigma_{+0}^u \mathcal{O}_0(S_{+0}) + \text{PS} \left[d\sigma_{+1}^u \Theta(\rho_1 - \rho_c) \right]$$

i.e. simply the UMEPS result for $\rho_{\text{MS}} = \rho_c$.

The merging scale is a *technical* parameter, much in the same way that the shower cut-off (or `ptminsq/hfact` in the POWHEG-BOX) is.

¹See also arXiv:1211.5467 (Simon Plätzer)

What is $\widetilde{\overline{B}}_0$?

For UNLOPS@NNLO, $\widetilde{\overline{B}}_0$ is given by

$$\widetilde{\overline{B}}_0 \mathcal{O}_{+0} = \left[\overline{B}_0 - \overline{B}_1 \Theta(\rho(S_{+1}, \rho) - \rho_{MS}) \right] \mathcal{O}_{+0}$$

where we need to project

$$\begin{aligned} \overline{B}_0 &= \left[B_0 + V_0 + \int D_{1|0} \right] \mathcal{O}_{+0} + \int [B_1 \mathcal{O}_{+1} - D_{1|0} \mathcal{O}_{+0}] \\ &+ \left[W_0 + \iint G_{2|0} + \iint (1-h) H_{2|0} \right] \mathcal{O}_{+0} \\ &+ \int \left[V_1 \mathcal{O}_{+1} + \int D_{2|1} \mathcal{O}_{+1} - \int E_{2|0} \mathcal{O}_{+0} + \int F_{2|0} \mathcal{O}_{+0} + \int h H_{2|0} \mathcal{O}_{+0} \right] \\ &+ \iint \left[B_2 \mathcal{O}_{+2} - D_{2|1} \mathcal{O}_{+1} + E_{2|0} \mathcal{O}_{+0} - F_{2|0} \mathcal{O}_{+0} - G_{2|0} \mathcal{O}_{+0} - H_{2|0} \mathcal{O}_{+0} \right] \end{aligned}$$

onto 0-parton states.

$E_{2|0}$ cancels spurious div's from integrating $D_{2|1}$ over the full 2-parton PS;
 $F_{2|0}$ cancels double-unresolved div's in $\iint B_2$ that would otherwise cancel against $\int V_1$;

$G_{2|0}$ cancels double-unresolved div's in $\iint B_2$ that would cancel against W_0 ;
 $H_{2|0}$ cancels div's that are genuinely shared between $\iint B_2$, $\int V_1$ and W_0 (if present), or other spurious div's (if existing).

What is $\tilde{\tilde{B}}_0$?

With $\Theta_n(>) = \Theta(t(S_{+n}, \rho) - \rho_{MS})$ and $\Theta_n(<) = \Theta(\rho_{MS} - t(S_{+n}, \rho))$, we find

$$\begin{aligned}
 \tilde{\tilde{B}}_0 = & \left[B_0 + V_0 + \int D_{1|0} \right] \mathcal{O}_{+0} \\
 + & \int \left[B_1 \Theta_1(<) \mathcal{O}_{+1} - D_{1|0} \mathcal{O}_{+0} \right] + \left[W_0 + \iint G_{2|0} + \iint (1-h) H_{2|0} \right] \mathcal{O}_{+0} \\
 + & \int \left[V_1 \Theta_1(<) \mathcal{O}_{+1} + \int D_{2|1} \Theta_1(<) \mathcal{O}_{+1} - \int E_{2|0} \mathcal{O}_{+0} + \int F_{2|0} \mathcal{O}_{+0} + \int h H_{2|0} \mathcal{O}_{+0} \right. \\
 + & \left. \iint \left[B_2 \Theta_1(<) \Theta_2(>) \mathcal{O}_{+2} + B_2 \Theta_1(<) \Theta_2(<) \mathcal{O}_{+2} - D_{2|1} \Theta_1(<) \mathcal{O}_{+1} \right. \right. \\
 & \left. \left. + E_{2|0} \mathcal{O}_{+0} - F_{2|0} \mathcal{O}_{+0} - G_{2|0} \mathcal{O}_{+0} - H_{2|0} \mathcal{O}_{+0} \right] \right]
 \end{aligned}$$

Note the term $B_2 \Theta_1(<) \Theta_2(>) \mathcal{O}_{+2}$, which contains a state S_{+2} with two resolved partons, but no state with only one resolved parton. Alternatively, we could have defined $\tilde{\tilde{B}}_0$ without this term. Then, it would be, in an NNLO-matched calculation, be included e.g. through $\int_s \widehat{B}_{2 \rightarrow 0}$. This is how we defined $\tilde{\tilde{B}}_0$ in the main text. Above, we also defined the sum $\overline{\overline{B}}_0 - \overline{\overline{B}}_1$ to not contain such terms.