Status of predictions for the total $t\bar{t}$ cross section and measurement of the pole mass

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(See also "High precision fundamental constants at the TeV scale", arXiv:1405.4781 [hep-ph])

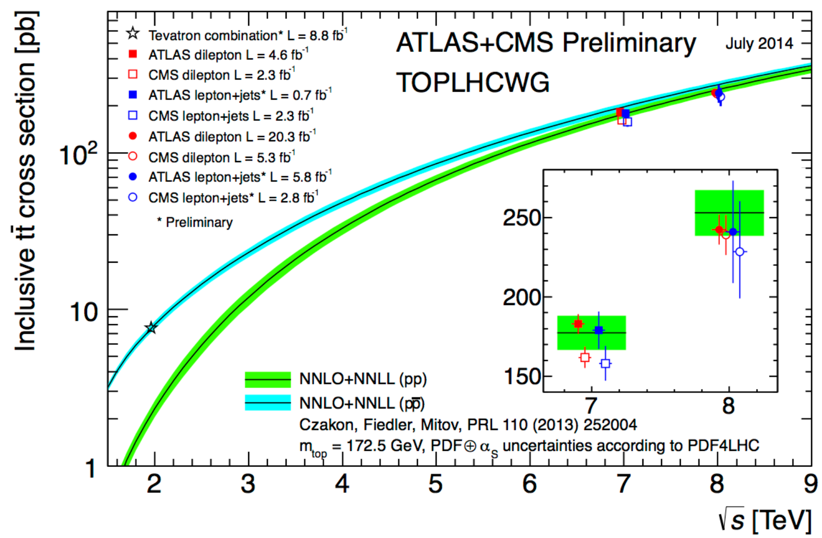
Total $t\bar{t}$ cross section

measurements (in pb)

$$\sigma_{t\bar{t}}^{\text{Tevatron}} = 7.60^{+0.41}_{-0.41}(\text{D0+CDF})$$

$$\sigma_{t\bar{t}}^{\text{LHC @7 TeV}} = \begin{cases} 162^{+7}_{-7} & (\text{CMS}) \\ 177^{+11}_{-10} & (\text{ATLAS}) \end{cases}$$

$$\sigma_{t\bar{t}}^{\text{LHC @8 TeV}} = \begin{cases} 237^{+13}_{-13} & (\text{CMS}) \\ 242^{+10}_{-10} & (\text{ATLAS}) \end{cases}$$



Top mass from kinematic measurements

$$m_t = \begin{cases} 173.20 \pm 0.87\text{GeV} & (\text{Tevatron comb. } 8.7 \text{ fb}^{-1}) \\ 173.29 \pm 0.95\text{GeV} & (\text{LHC comb. } 4.9 \text{ fb}^{-1}) \end{cases}$$

Relation to theoretical mass definition?

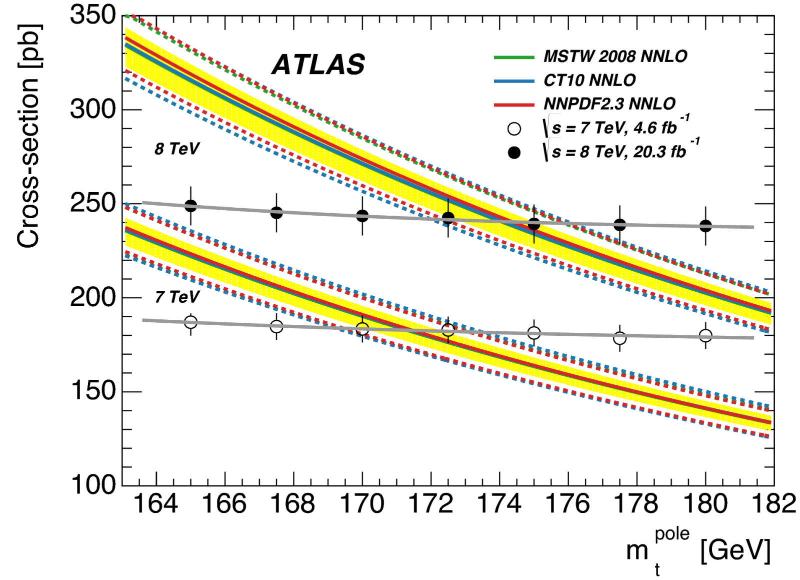
Difference $\sim 1\text{GeV}$ to well-defined mass definition expected

Theory prediction for $\sigma_{t\bar{t}}$ in QCD:
function of α_s, m_t, PDFs

Proposal: determine m_t in
well-defined scheme (pole, $\overline{\text{MS}}$,...)
from $\sigma_{t\bar{t}}$ measurement

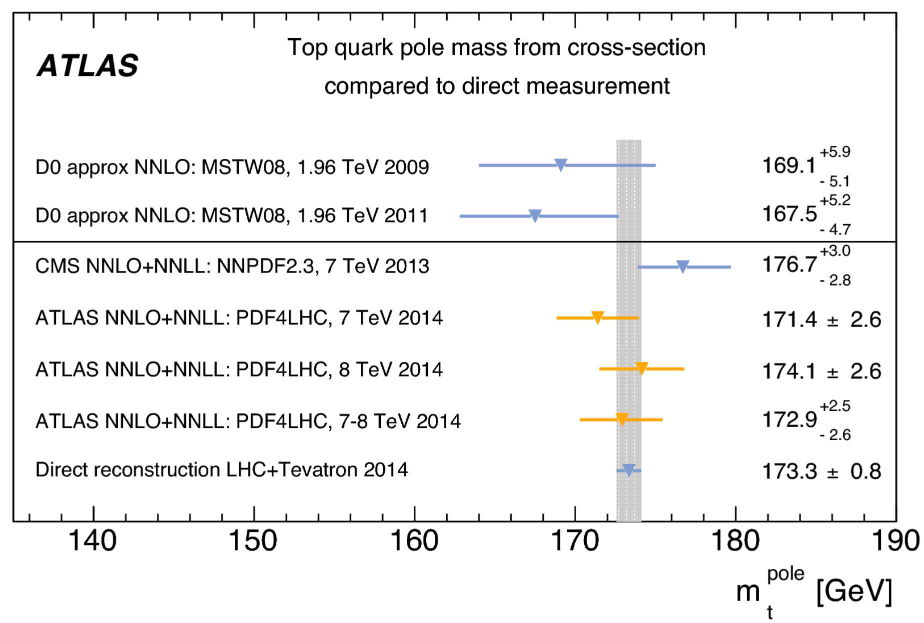
(Langenfeld/Moch/Uwer 09)

Experimental measurement
depends on m_t^{MC}



Latest experimental results:

- CMS:
  $m_t^{\text{pole}} = 176.7^{+3.8}_{-3.4}\,\text{GeV}$
  (using NNPDF2.3)
- ATLAS:
  $m_t^{\text{pole}} = 172.9^{+2.5}_{-2.6}\,\text{GeV}$
  (using PDF4LHC)



Full NNLO calculation (Bärnreuther/Czakon/Fiedler/Mitov 12–13)

NNLL resummation

Soft threshold logarithms $\alpha_s \log \beta$ (Czakon/Mitov/Sterman 09)

Threshold logs and Coulomb corrections α_s/β (Beneke/Falgari/CS 09)

Resummation for distributions (Kidonakis, Ahrens et al. ⇒ Adrian's talk)

Programs including exact NNLO result

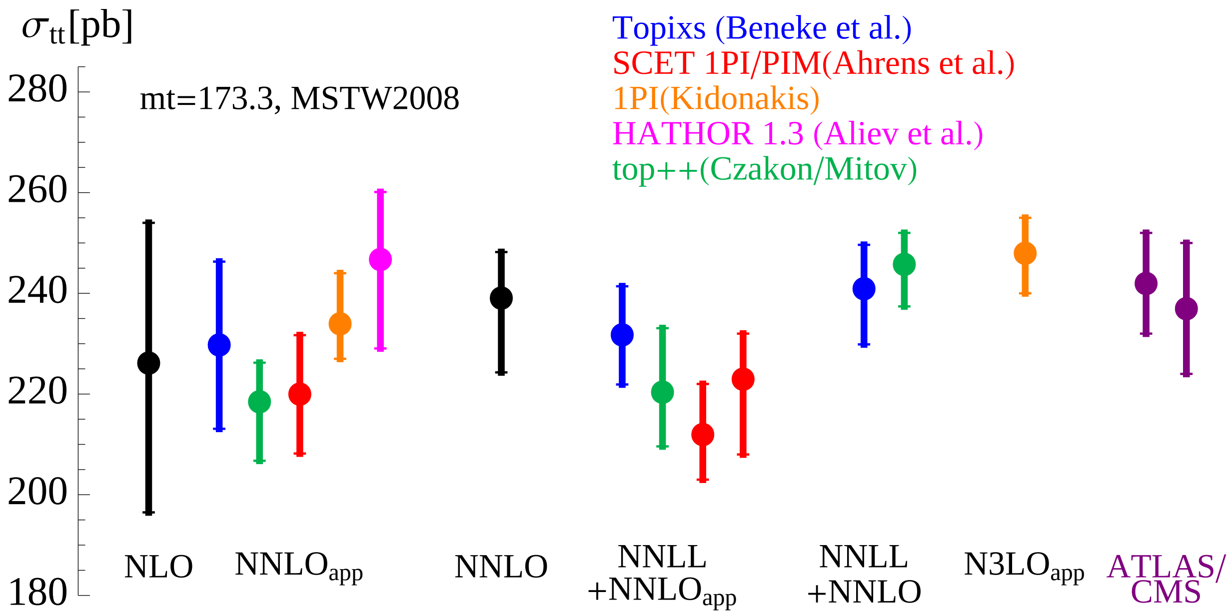
- TOP++ V2.0: NNLO+NNLL (soft) (Czakon/Mitov)
- HATHOR V1.5: NNLO (Aliev et al.)
- TOPIXS V2.0 NNLO+NNLL (soft+Coulomb) (Beneke et al.)

EW corrections $\sim 2\%$ (Bernreuther/Fücker/Si; Kühn/Scharf/Uwer, 05/06)

QED (e.g. $q\gamma$ induced) $\sim 1\%$ (Hollik/Kollar 07)

Comparison of different approximations (excluding PDF$+\alpha_s$ uncertainties)

- $\pm 5\%$ scale uncertainty at NNLO; ± 3–4% at NNLL

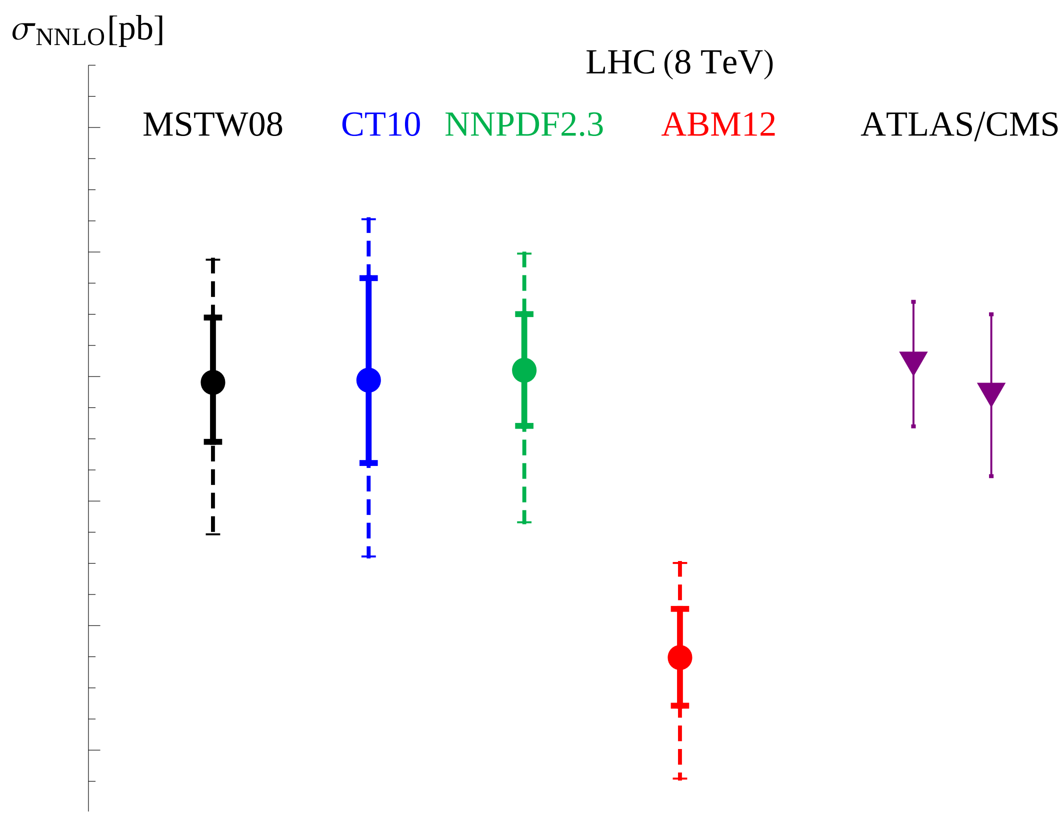


Total top-pair production cross-section

Comparison of different approximations (excluding PDF+α_s uncertainties)

- $\pm 5\%$ scale uncertainty at NNLO; ± 3–4% at NNLL

PDF+α_s uncertainties now comparable to scale uncertainty



Reduction of scale uncertainty from threshold resummation

$$\text{NNLO}: \quad 239.18^{+\ 9.29(3.9\%)}_{-14.85(6.2\%)}\text{pb} \Rightarrow \begin{cases} \text{NNLL(top++)}: & 245.89^{+6.24(2.5\%)}_{-8.41(3.4\%)}\text{pb} \\ \text{NNLL(topixs)}: & 241.04^{+\ 8.65(3.6\%)}_{-11.09(4.3\%)}\text{pb} \end{cases}$$

top++: Mellin space resummation (Sterman 87; Catani/Trentadue 89)

- Includes 2-loop constant term H_2 in threshold expansion

$$\sigma_{t\bar{t}}^{\text{NLLL}}|_{H_2=0} = 242.74 \text{ pb}$$

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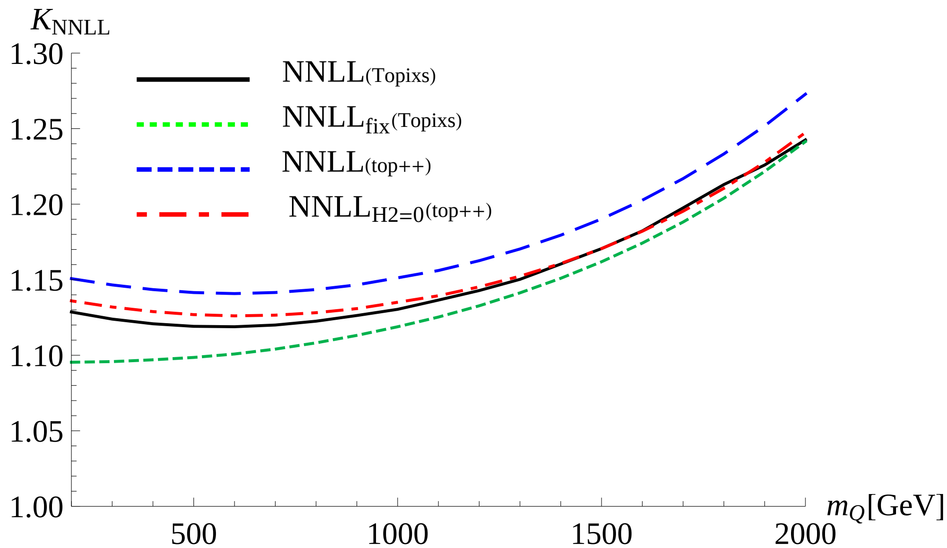
`topixs:` combined soft/Coulomb resummation

- RGE for momentum-space resummation (Becher/Neubert 06)
- dependence on scales μ_f, $\mu_h \sim 2M$: $\Delta_{\text{scale}}\sigma_{t\bar{t}}^{\text{NNLL}} = {}^{+5.64}_{-6.56}$ pb
- resummation uncertainty: choice of $\mu_s \sim M\beta^2$, kinematic ambiguities, higher-order terms: $\Delta_{\text{res}}\sigma_{t\bar{t}}^{\text{NNLL}} = {}^{+6.56}_{-4.01}$ pb

Heavy Quarks as test case for resummation methods



$(K_{\text{NNLL}} = \sigma^{\text{NNLL}}/\sigma^{\text{NLO}}$,

LHC $\sqrt{s} = 8$ TeV)

NNLL: momentum-space, running $\mu_s = 2m_Q\beta^2$ (`Topixs` default)

NNLL$_{\text{fix}}$: momentum-space, fixed μ_s (`Topixs`)

NNLL (top++): Mellin-space (Cacciari et al. 11; Czakon/Mitov 11-13)

NNLL$_{H_2=0}$ (top++): Mellin-space, two-loop constant term set to zero

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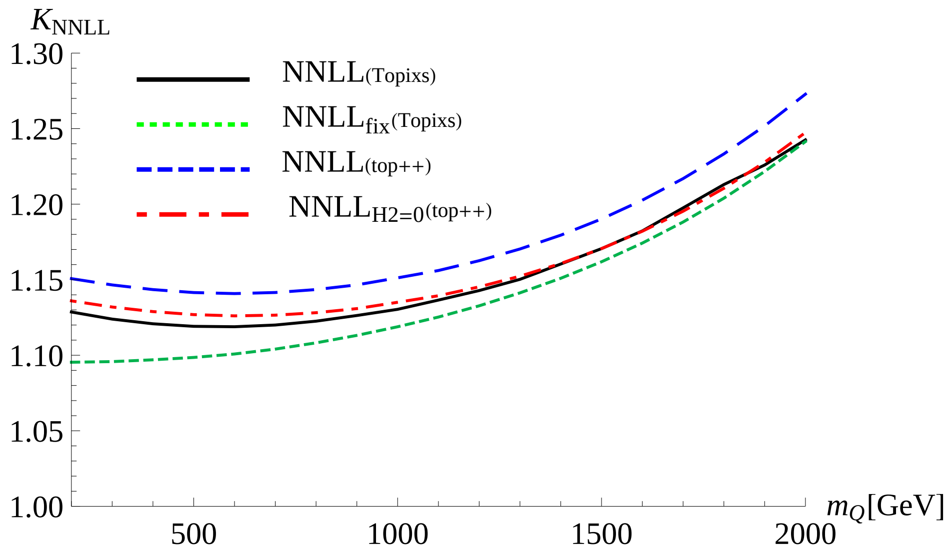


$(K_{\text{NNLL}} = \sigma^{\text{NNLL}}/\sigma^{\text{NLO}}$,

LHC $\sqrt{s} = 8$ TeV)

$\Rightarrow$ resummation methods agree well for larger masses

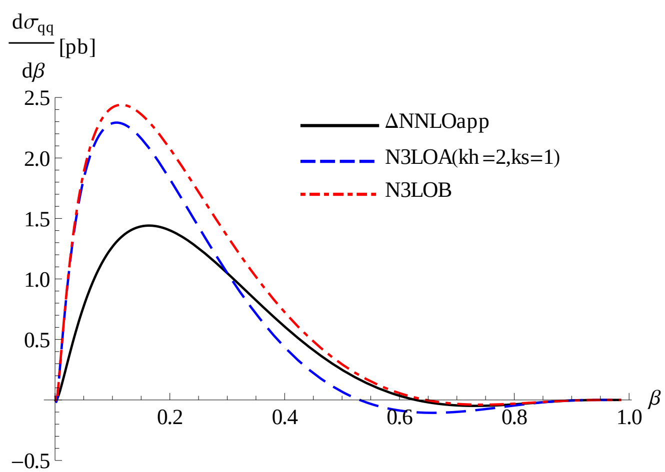
- differences at m_t: estimate of resummation ambiguities and higher-order effects
- main difference: treatment of $H_2 \Rightarrow \alpha_s^3 \log \beta^2$ terms (NNLL')

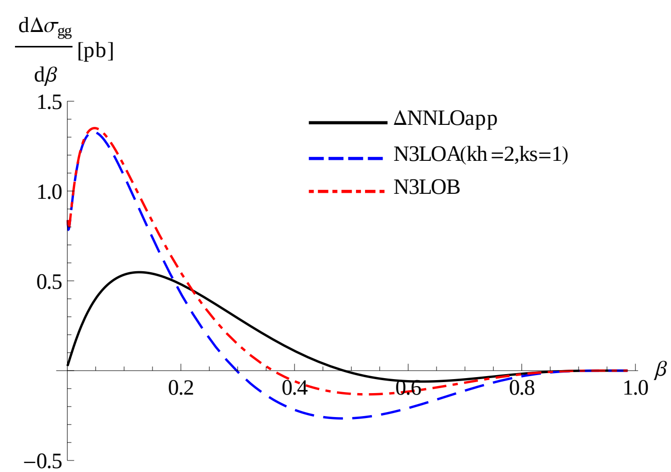
Expand NNLL to $\mathcal{O}(\alpha_s^3)$, e.g. (Beneke/Falgari/Klein/CS 13)

$$\Delta\sigma^{(3)}_{q\bar q,\text{NNLL}} = 12945.4\log^6\beta - 37369.1\log^5\beta + 27721.4\log^4\beta + 41839.4\log^3\beta + \frac{1}{\beta}\left(-6278.5\log\beta + 3862.5\log^2\beta + 2804.7\log^3\beta - 2994.5\log^4\beta\right) + \frac{153.9\log^2\beta + 122.9\log\beta - 145}{\beta^2} + \underbrace{\left\{\log\beta^{1,2}, 1/\beta, C^{(3)}\right\}}_{\text{not known exactly}} + \text{scale dep.}$$

N^{3}LO$_A$: keep all terms, including μ_s, μ_h-dependence and constants

N^{3}LO$_B$: only keep terms known exactly





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Approx. N3LO from one-particle inclusive kinematics (Kidonakis 14)

$$\text{NNLO}: \quad 239.18^{+\ 9.29(3.9\%)}_{-14.85(6.2\%)}\text{pb} \Rightarrow \begin{cases} \text{N3LO}_A: & 244.87^{+3.5(1.5\%)}_{-6.7(2.8\%)}\text{pb} \\ \text{N3LO}_B: & 245.90^{+6.7(2.7\%)}_{-5.0(2.0\%)}\text{pb} \\ \text{N3LO}_{\text{1PI}}: & 248^{+7(2.8\%)}_{-8(3.2\%)}\text{pb} \end{cases}$$

But: strong dependence of incompletely known terms on soft scale:

$$\Delta_{\mu_s}\sigma^{\text{N3LO}_A}_{t\bar{t}} = {}^{+3.8}_{-12.1}\ \text{pb}$$

$\Rightarrow$ need input beyond NNLL, use only for uncertainty estimate.

Follow method from (ATLAS-CONF-2011-54)

Fit m_t-dependence of theoretical cross-section:

$$\sigma_{t\bar{t}}^{\text{th}}(m_t) = \left(\frac{172.5}{m_t}\right)^4 \left(c_0 + c_1(m_t - 172.5) + c_2(m_t - 172.5)^2 + c_3(m_t - 172.5)^3\right) \text{ pb},$$

$$c_0 = 166.5,\ c_1 = -1.15,\ c_2 = 5.1 \times 10^{-3},\ c_3 = 8.5 \times 10^{-5}$$

Use fit of dependence of experimental result on m_t^{MC}

maximize joint likelihood assuming $m_t = m_t^{\text{MC}}$

$$f(m_t) = \int f_{\text{th}}(\sigma|m_t) \cdot f_{\text{exp}}(\sigma|m_t) d\sigma\,,$$

with normalized Gaussians

$$f_{\text{th}} = \frac{1}{\sqrt{2\pi}\Delta\sigma_{t\bar{t}}^{\text{th}}(m_t)} \exp\left[-\frac{\left(\sigma - \sigma_{t\bar{t}}^{\text{th}}(m_t)\right)^2}{2(\Delta\sigma_{t\bar{t}}^{\text{th}}(m_t))^2}\right]$$

Determine uncertainty from 68% area in upper/lower region;
estimate uncertainty from assumption $m_t = m_t^{\text{MC}}$.

Experimental input with available parameterisation $\sigma_{t\bar{t}}(m_t)$

(Example results for NNLO, MSTW08)

Ref.	$\sqrt{s}$/TeV	$\sigma_{t\bar{t}}(172.5)$/pb	$\frac{d\sigma_{t\bar{t}}}{dm_t}(172.5)$	m_t/GeV
arXiv:1105.5384 (D0)	1.96	$7.56^{+0.63}_{-0.56}$	$-1.1\%\,\text{GeV}^{-1}$	$170.7^{+5.9}_{-6.8}$
arXiv:1406.5375 (ATLAS)	7	$182.9^{+7.1}_{-7.1}$	$-0.28\%\,\text{GeV}^{-1}$	$170.6^{+3.8}_{-4.3}$
arXiv:1208.2671 (CMS)	7	$161.9^{+6.7}_{-6.7}$	$-0.80\%\,\text{GeV}^{-1}$	$175.9^{+6.5}_{-5.5}$
arXiv:1406.5375 (ATLAS)	8	$242.4^{+10.3}_{-10.3}$	$-0.28\%\,\text{GeV}^{-1}$	$173.3^{+4.0}_{-4.5}$
arXiv:1312.7582 (CMS)	8	$239^{+13.1}_{-13.1}$	$-0.90\%\,\text{GeV}^{-1}$	$174.76^{+7.1}_{-5.7}$

Notes

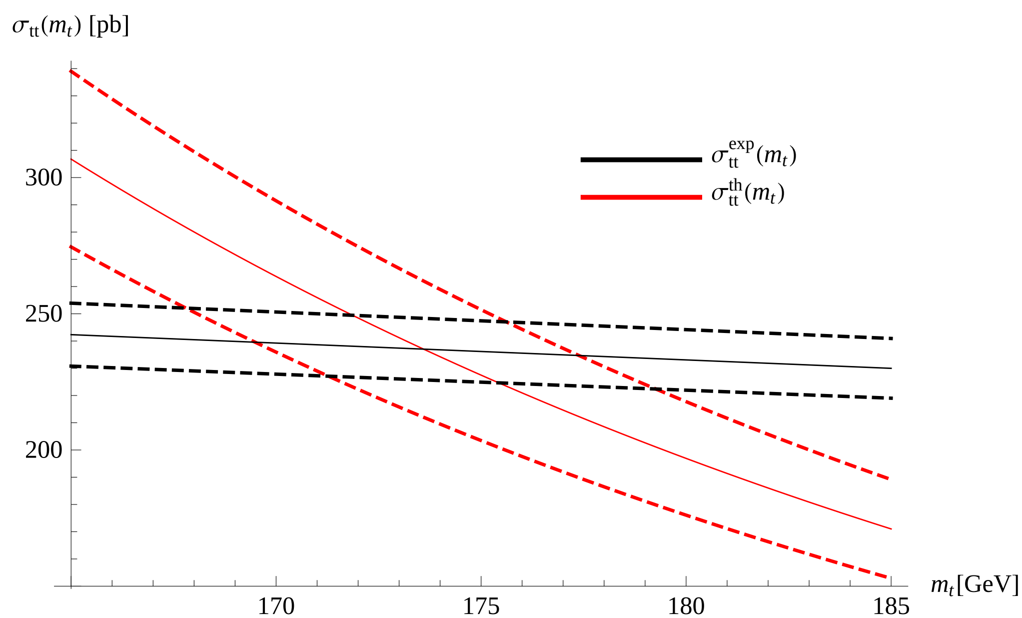
- scale and PDF uncertainty added linearly
- use constant relative error for experimental cross sections
- use parameterisations $\sigma_{t\bar{t}}(m_t)$ outside domain of validity in normalization of likelihood function

Further potential example measurement (ATLAS arXiv:1406.5375)

$$\sigma_{t\bar{t}}(8\mathrm{TeV}) = 242.4^{+10.3}_{-10.3}\mathrm{pb} \qquad \frac{d\sigma_{t\bar{t}}}{dm_t} = -0.28\%\,\mathrm{GeV}^{-1}$$

Results for NNLO, default PDF value for α_s

	MSTW08	CT10	NNPDF2.3	ABM11
m_t	$173.3^{+4.0}_{-4.5}$	$173.6^{+4.8}_{-5.3}$	$174.1^{+4.0}_{-4.4}$	$165.7^{+3.7}_{-4.0}$



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m_t	$173.3^{+4.0}_{-4.5}$	$173.6^{+4.8}_{-5.3}$	$174.1^{+4.0}_{-4.4}$	$165.7^{+3.7}_{-4.0}$

- Effect of NNLL prediction: $173.3^{+4.0}_{-4.5} \rightarrow 173.5^{+3.5}_{-3.9}$
- Effect of $m_t = m_t^{\text{MC}} \pm 1\,\text{GeV}$: $\Delta m_t = \pm 0.1\,\text{GeV}$
- 50% reduction of exp. uncertainty: $173.3^{+4.0}_{-4.5} \rightarrow 173.5^{+3.2}_{-3.7}$
- 50% reduction of th. uncertainty: $173.3^{+4.0}_{-4.5} \rightarrow 173.5^{+2.3}_{-2.3}$
- 50% reduction of both uncertainties: $173.3^{+4.0}_{-4.5} \rightarrow 173.2^{+1.8}_{-1.9}$
- CMS study with similar assumptions: $\Delta m_t \sim 1\text{GeV}$

Summary and conclusions

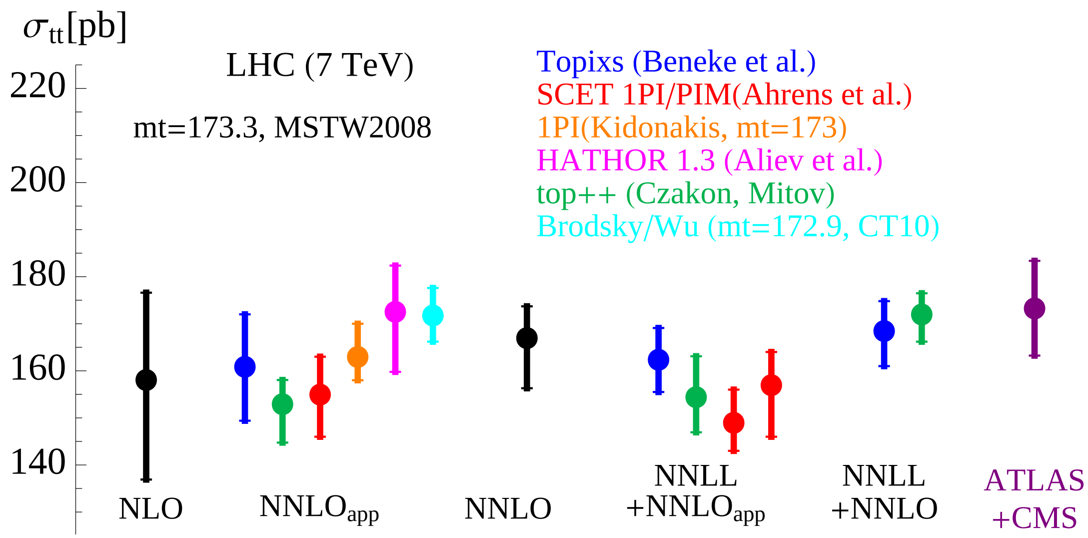
Theory predictions

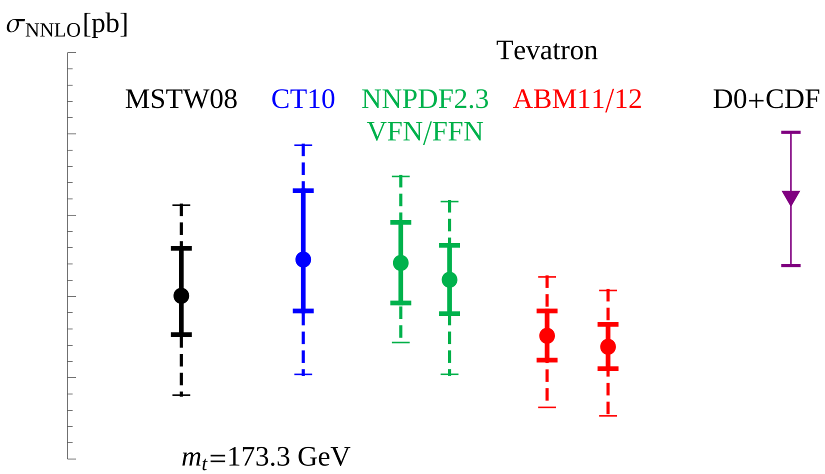
- full NNLO now available
- two programs implementing NNLO+NNLL resummation (top++, topixs)
- accuracy of NNLO+NNLL $\pm 3 + 4\%$
- similar PDF+α_s uncertainties
- N^3LO currently uses same input as NNLL

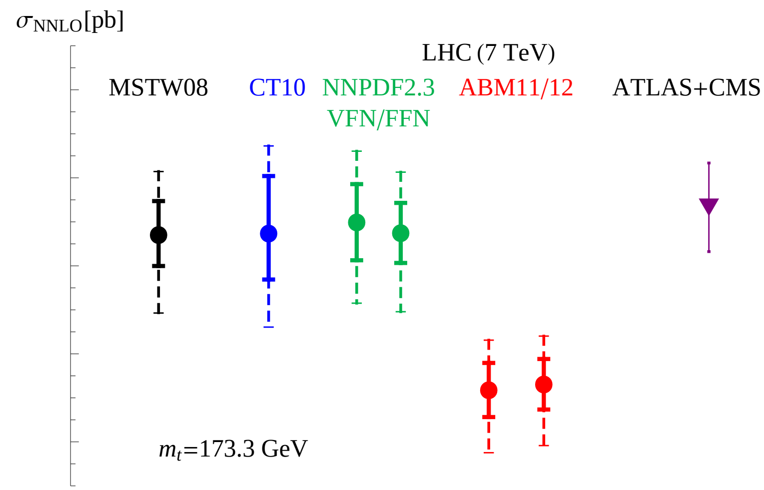
Top pole mass determination from total cross section

- in agreement with kinematical measurements
- currently limited to $\pm 2 - 3\%$ accuracy
- significant improvement requires further reduction of theory+PDF uncertainties

Comparison of different approximations





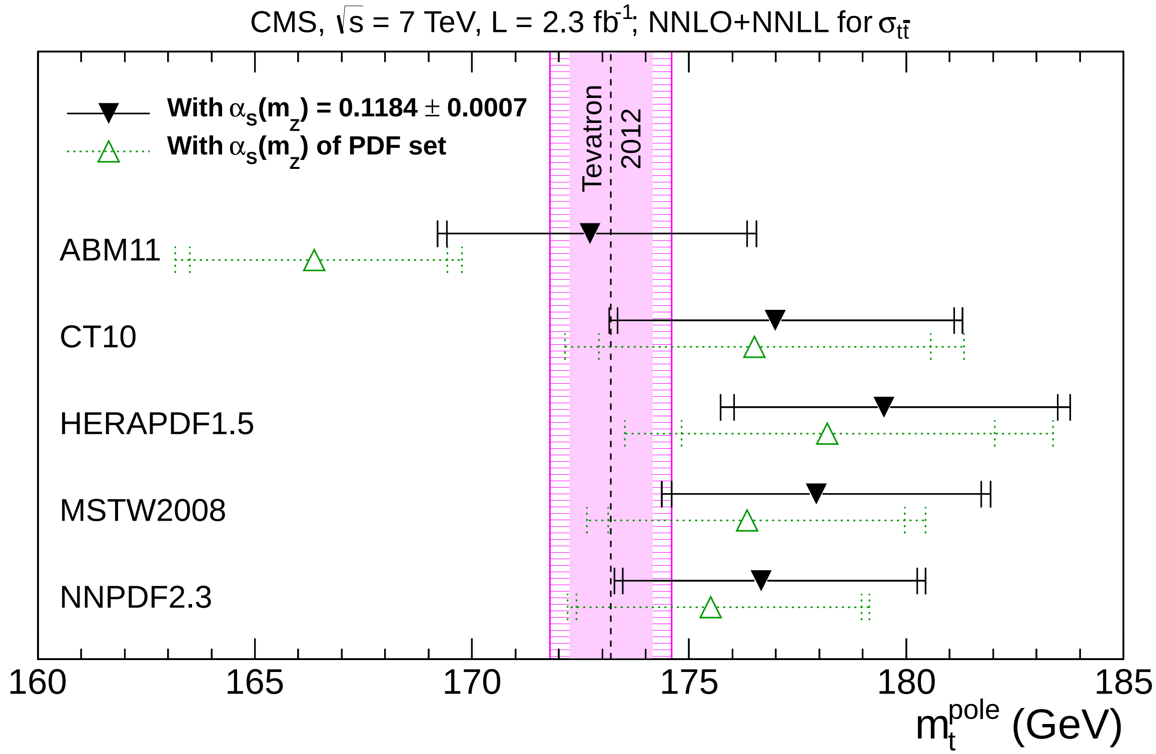


Latest experimental analysis (CMS arXiv:1307.1907) using most precise single measurement of cross section at $7\,\mathrm{TeV}$

$$\sigma_{t\bar{t}} = 161.9^{+6.7}_{-6.7}\mathrm{pb}$$

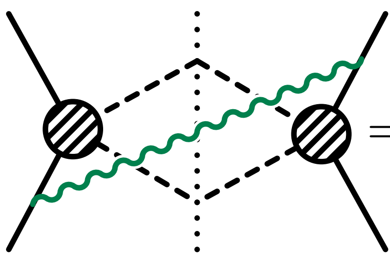
Results for different PDFs using NNLO+NNLL

(Bärnreuther/Czakon/Fiedler/Mitov 12–13)

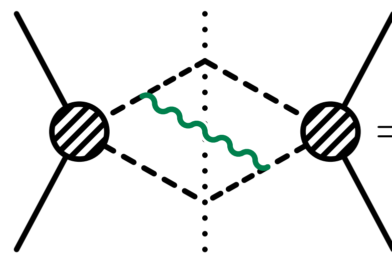


Enhanced QCD corrections in threshold limit $\beta = \sqrt{1 - 4m_t^2/\hat{s}} \to 0$

Soft corrections: (Resummation in Mellin space: Sterman 87; Catani, Trentadue 89, Kidonakis,Sterman 97, Bonciani et al. 98, ...)

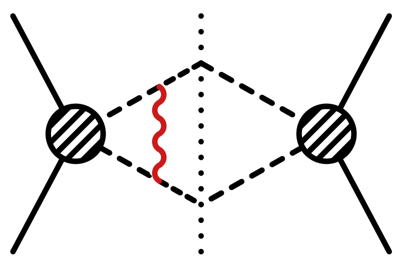


$\Rightarrow \alpha_s \; \log^2(8\beta^2)$



$\Rightarrow \alpha_s \; \log(8\beta^2)$

Coulomb gluon corrections (Fadin, Khoze 87; Peskin, Strassler 90, NRQCD,...)



$\Rightarrow \alpha_s \; \frac{1}{\beta}$

Resummed cross section

$$\hat{\sigma}_{pp'} \quad \propto \quad \sigma^{(0)} \exp\left[\ln\beta\, g_{\text{LL}}(\alpha_s \ln\beta) + g_{\text{NLL}}(\alpha_s \ln\beta) + \alpha_s g_{\text{NNLL}}(\alpha_s \ln\beta) + \ldots\right]$$
$$\times \sum_{k=0} \left(\frac{\alpha_s}{\beta}\right)^k \times \{1\,(\text{LL,NLL}); \alpha_s, \beta\,(\text{NNLL}); \ldots\} :$$

Total partonic cross section (Bonciani et al. 98, Moch/Uwer/Langenfeld)

$$\hat{\sigma}(t\bar{t})(\hat{s}) \quad \Rightarrow \log^n \beta \, , \frac{1}{\beta^m} \, , \; \beta = \sqrt{1 - \frac{4m_t^2}{\hat{s}}}$$

Pair invariant mass cross sections (Kidonakis,Sterman 97, Ahrens et al. 10)

$$\frac{d\hat{\sigma}(t\bar{t})}{dM_{t\bar{t}}} \quad \Rightarrow \left[\frac{\log^n(1-z)}{1-z}\right]_+ \, , \; z = \frac{M_{t\bar{t}}^2}{\hat{s}} \, , \qquad \text{PIM}_{\text{SCET}} : \log\left(\frac{1-z}{\sqrt{z}}\right)$$

One particle inclusive cross sections: (Laenen et al. 98, Ahrens et al. 11)

$$\frac{d\hat{\sigma}(t+X)}{ds_4} \quad \Rightarrow \left[\frac{\log^n\,(s_4/m^2)}{s_4}\right]_+ \; ; \; s_4 = p_X^2 - m_t^2 \, , \quad \text{1PI}_{\text{SCET}} : \log\left(s_4/\sqrt{m^2 + s_4}\right)$$