Perigee Parametrization in TrackFitResult

Status Report

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Perigee Parameters

- Signed distance from perigee to origin in the x-y plane. Positive (negative) when the angle between $\vec{d_0}$ and $\vec{p_t}$ is $+\pi/2$ $(-\pi/2)$.
  $|d_0| = \sqrt{x^2 + y^2}$
- Angle of $\vec{p_t}$ in the x-y plane.
  $\phi_0 = \arctan\left(\frac{p_y}{p_x}\right) \in [-\pi, \pi]$
- Signed curvature q/R.
  $\omega = \frac{Q}{\alpha \cdot p_t}$
- Z coordinate of the perigee in the r-z plane.
  $z_0 = z$
- Inverse slope of the track in the r-z plane.
  $\cot\Theta = \frac{p_z}{p_t}$

with $\alpha = 1/(1.5 \cdot 0.00299792458)$, obtained from c and the magnetic field strength of $1.5\,\mathrm{T}$ and the transverse momentum $p_t = \sqrt{p_x^2 + p_y^2}$

Cartesian parameters - Transformation

Cartesian parameters can be obtained from the perigee parameters:

$$\vec{x} = \begin{pmatrix} d_0 \sin\phi_0 \\ -d_0 \cos\phi_0 \\ z_0 \end{pmatrix}$$

$$\vec{p} = \frac{1}{|\omega \cdot \alpha|} \begin{pmatrix} \cos\phi_0 \\ \sin\phi_0 \\ \cot\theta \end{pmatrix}$$

Error Propagation

The perigee and cartesian coordinates are given by

$$\vec{\tau} = \left(d_0, \phi_0, \omega, z_0, \cot\theta\right),$$
$$\vec{x} = \left(x, y, z, p_x, p_y, p_z\right).$$

To propagate the error matrix we calculate the Jacobians

$$(A)_{ij} = \frac{\partial \tau_i}{\partial x_j},$$
$$(B)_{ij} = \frac{\partial x_i}{\partial \tau_j}.$$

with $\dim(A) = 5 \times 6$ and $\dim(B) = 6 \times 5$. Covariance matrix can be propagated between the two parametrizations by

$$C^{5\times 5}_{\mathrm{perigee}} = A^{5\times 6} \cdot C^{6\times 6}_{\mathrm{cartesian}} \cdot A^{T,6\times 5},$$
$$C^{6\times 6}_{\mathrm{cartesian}} = B^{6\times 5} \cdot C^{5\times 5}_{\mathrm{perigee}} \cdot B^{T,5\times 6}.$$

Test of Implementation

- Implementation works for random generated (5×5) covariance matrices.
- Transformation $(5 \times 5) \Rightarrow (6 \times 6) \Rightarrow (5 \times 5)$ only yields small numerical errors.
- Transformation does not work for the Genfit Output.

Failure with Genfit Output

$$\begin{bmatrix} 2.137\text{E-}05 & 7.677\text{E-}06 & -2.628\text{E-}07 & -5.951\text{E-}06 & -2.068\text{E-}06 & 2.312\text{E-}08 \\ 7.677\text{E-}06 & 2.758\text{E-}06 & -9.441\text{E-}08 & -2.138\text{E-}06 & -7.431\text{E-}07 & 8.307\text{E-}09 \\ -2.628\text{E-}07 & -9.441\text{E-}08 & 6\text{E-}05 & -2.777\text{E-}06 & 7.583\text{E-}06 & -7.438\text{E-}06 \\ -5.951\text{E-}06 & -2.138\text{E-}06 & -2.777\text{E-}06 & 2.02\text{E-}06 & 1.515\text{E-}07 & 2.599\text{E-}07 \\ -2.068\text{E-}06 & -7.431\text{E-}07 & 7.583\text{E-}06 & 1.515\text{E-}07 & 1.56\text{E-}06 & -7.144\text{E-}07 \\ 2.312\text{E-}08 & 8.307\text{E-}09 & -7.438\text{E-}06 & 2.599\text{E-}07 & -7.144\text{E-}07 & 1.391\text{E-}06 \end{bmatrix}$$

Transformation into perigee covariance matrix and back to cartesian yields

$$\begin{bmatrix} 1.661\text{E-}06 & -6.34\text{E-}08 & 7.731\text{E-}06 & -5.564\text{E-}07 & 1.362\text{E-}06 & -7.299\text{E-}07 \\ -6.34\text{E-}08 & 1.494\text{E-}05 & -3.98\text{E-}06 & -4.76\text{E-}06 & -2.41\text{E-}06 & 3.744\text{E-}07 \\ 7.731\text{E-}06 & -3.98\text{E-}06 & 6\text{E-}05 & -2.777\text{E-}06 & 7.583\text{E-}06 & -7.438\text{E-}06 \\ -5.564\text{E-}07 & -4.76\text{E-}06 & -2.777\text{E-}06 & 2.02\text{E-}06 & 1.515\text{E-}07 & 2.599\text{E-}07 \\ 1.362\text{E-}06 & -2.41\text{E-}06 & 7.583\text{E-}06 & 1.515\text{E-}07 & 1.56\text{E-}06 & -7.144\text{E-}07 \\ -7.299\text{E-}07 & 3.744\text{E-}07 & -7.438\text{E-}06 & 2.599\text{E-}07 & -7.144\text{E-}07 & 1.391\text{E-}06 \end{bmatrix}$$

Open Issues

- Use magnetic field of $1.5\,\mathrm{T}$ or get the magnetic field for each poca vector individually?
- Transformation of the Genfit output covariance matrix.