

# NLO QCD calculations of Higgs production at LHC with GoSam

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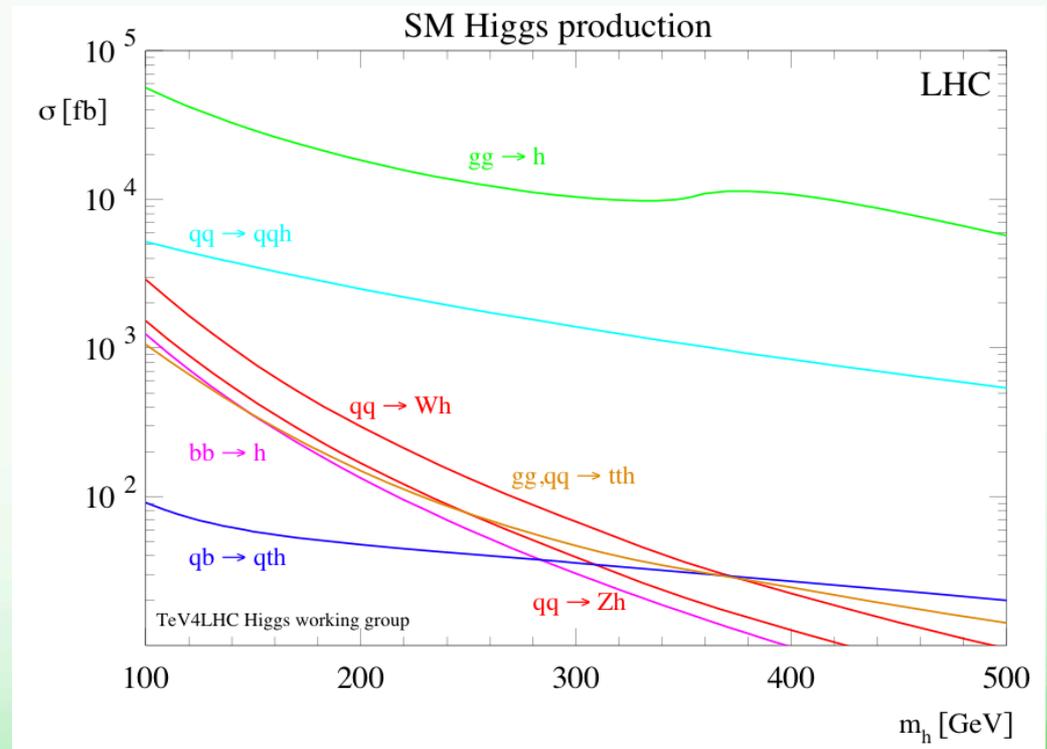
**Alexander von Humboldt**  
Stiftung / Foundation

# Outline

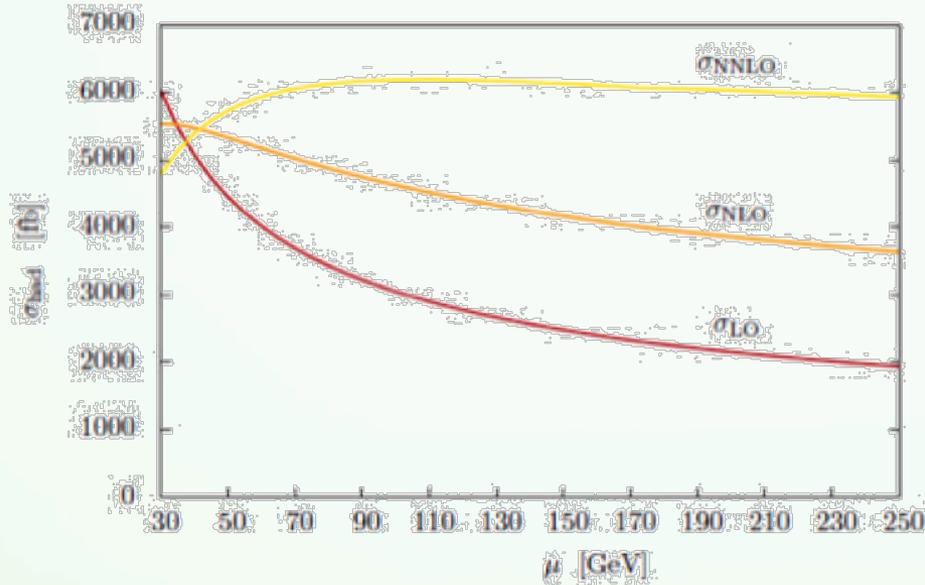
- Motivation, introduction GoSam
- Integrand decomposition
- Extension to higher rank
  - Computational strategy
  - Applications:  $H+2j$ ,  $H+3j$  in GF
- Ninja
  - Improved algorithm
  - Applications:  $H+2j$

# Motivation

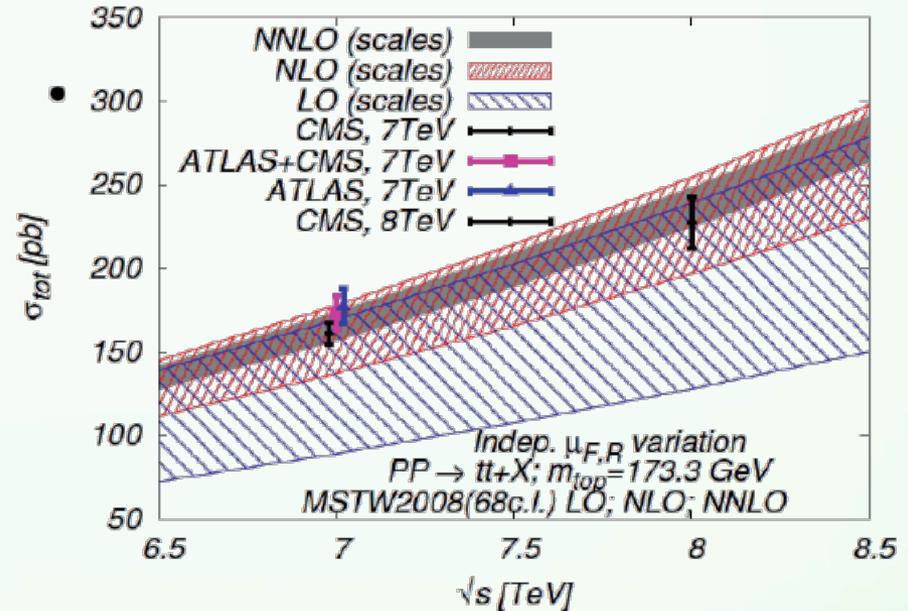
- Higgs discovery by Atlas and CMS
- Need to determine properties:
  - spin
  - CP properties
  - couplings



# Motivation for NLO



Higgs+jet at NNLO [Boughezal et al. (2013)]



Top pair production at NNLO [Czakon, Fiedler, Mitov (2013)]

- Reduce theoretical error
- Strong dependence on renormalization and factorization scale

# NLO calculations

$$\sigma^{NLO} = \int_m \left[ d^{(4)}\sigma^B + \int_{loop} d^{(d)}\sigma^V + \int_1 d^{(d)}\sigma^S \right] + \int_{m+1} \left[ d^{(4)}\sigma^R - d^{(4)}\sigma^S \right]$$

- NLO calculation consists of:
  - LO: Born diagram
  - Virtual corrections: loop diagrams ← GoSam
  - Real corrections: additional radiation
  - Subtraction terms to regulate infinities

# GoSam

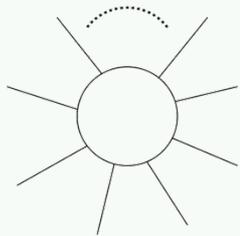
## Collaboration

Cullen, HvD, Greiner, Heinrich, Luisoni, Mastrolia, Mirabella, Ossola, Peraro, Reichel, Schlenk, von Soden-Fraunhofen, Tramontano

## Reduction algorithms

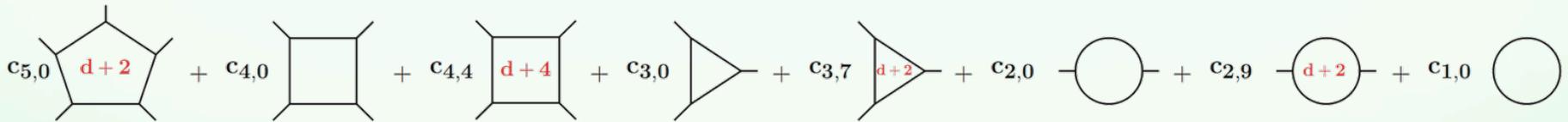
- *Samurai, Xsamurai*  
d-dimensional integrand-level reduction  
current default  
[Mastrolia, Ossola, Reiter, Tramontano]; [HvD (2013)]
- *Golem95, Golem95 higherrank extension*  
Tensorial reduction  
Numerically stable → rescue system  
[Binoth, Guillet, Heinrich, Pilon, Reiter]; [Guillet, Heinrich, von Soden-Fraunhofen]
- *Ninja*  
Integrand-level+Laurent expansion  
Stable and fast  
[Mastrolia, Mirabella, Peraro]; [HvD, Luisoni, Mastrolia, Mirabella, Ossola, Peraro]

# Amplitudes at one loop



$$\mathcal{M}_n \equiv \int \mathcal{A}_n(\bar{q}) d\bar{q} \equiv \int d^{-2\epsilon} \mu \int d^4 q \frac{N(q, \mu^2)}{\bar{D}_0 \dots \bar{D}_{n-1}}$$

Decompose:



$$\begin{aligned} \int d^{-2\epsilon} \mu^2 d^4 q \mathcal{A}_n(q) &= \int d\bar{q} \frac{c_{5,0} \mu^2}{D_0 D_1 D_2 D_3 D_4} + \int d\bar{q} \frac{c_{4,0} + c_{4,4} \mu^4}{D_0 D_1 D_2 D_3} \\ &+ \int d\bar{q} \frac{c_{3,0} + c_{3,7} \mu^2}{D_0 D_1 D_2} + \int d\bar{q} \frac{c_{2,0} + c_{2,9} \mu^2}{D_0 D_1} + \int d\bar{q} \frac{c_{1,0}}{D_0} \end{aligned}$$

# Amplitudes at one loop

$$\int d^{-2\epsilon} \mu^2 d^4 q \mathcal{A}_n(q) = \int d\bar{q} \frac{c_{5,0} \mu^2}{D_0 D_1 D_2 D_3 D_4} + \int d\bar{q} \frac{c_{4,0} + c_{4,4} \mu^4}{D_0 D_1 D_2 D_3} \\ + \int d\bar{q} \frac{c_{3,0} + c_{3,7} \mu^2}{D_0 D_1 D_2} + \int d\bar{q} \frac{c_{2,0} + c_{2,9} \mu^2}{D_0 D_1} + \int d\bar{q} \frac{c_{1,0}}{D_0}$$

Integrand level:

$$\mathcal{A}_n(q) = \frac{c_{5,0} \mu^2 + f_{01234}(q, \mu^2)}{D_0 D_1 D_2 D_3 D_4} + \frac{c_{4,0} + c_{4,4} \mu^4 + f_{0123}(q, \mu^2)}{D_0 D_1 D_2 D_3} \\ + \frac{c_{3,0} + c_{3,7} \mu^2 + f_{012}(q, \mu^2)}{D_0 D_1 D_2} + \frac{c_{2,0} + c_{2,9} \mu^2 + f_{01}(q, \mu^2)}{D_0 D_1} + \frac{c_{1,0} + f_0(q, \mu^2)}{D_0}$$

$$\int d^{-2\epsilon} \mu^2 \int d^4 q \frac{f_{ij\dots}(q, \mu^2)}{D_i D_j \dots} = 0$$

# Parametric form integrand

$$\begin{aligned} \mathcal{A}_n = & \sum_{ijklm} \frac{\Delta_{ijklm}(q, \mu^2)}{D_i D_j D_k D_l D_m} + \sum_{ijkl} \frac{\Delta_{ijkl}(q, \mu^2)}{D_i D_j D_k D_l} + \\ & + \sum_{ijk} \frac{\Delta_{ijk}(q, \mu^2)}{D_i D_j D_k} + \sum_{ij} \frac{\Delta_{ij}(q, \mu^2)}{D_i D_j} + \sum_i \frac{\Delta_i(q, \mu^2)}{D_i} \end{aligned}$$

- Residues multivariate polynomials
- Need rank to determine generic form
- Renormalizability requires rank  $\leq$  propagators

# Parametric form integrand

$$\begin{aligned} \mathcal{A}_n = & \sum_{ijklm} \frac{\Delta_{ijklm}(q, \mu^2)}{D_i D_j D_k D_l D_m} + \sum_{ijkl} \frac{\Delta_{ijkl}(q, \mu^2)}{D_i D_j D_k D_l} + \\ & + \sum_{ijk} \frac{\Delta_{ijk}(q, \mu^2)}{D_i D_j D_k} + \sum_{ij} \frac{\Delta_{ij}(q, \mu^2)}{D_i D_j} + \sum_i \frac{\Delta_i(q, \mu^2)}{D_i} \end{aligned}$$

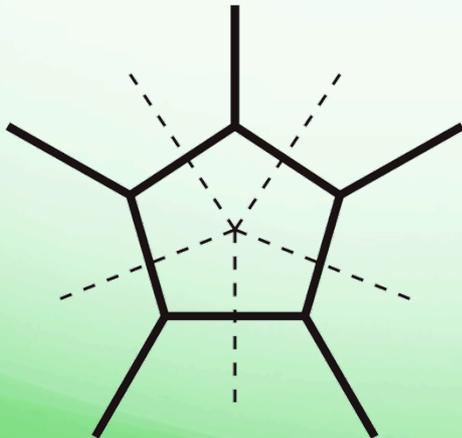
- Form residues process independent
- Values of coefficients process dependent

# Integrand decomposition

$$\mathcal{A}_n = \sum_{ijklm} \frac{\Delta_{ijklm}(q, \mu^2)}{D_i D_j D_k D_l D_m} + \sum_{ijkl} \frac{\Delta_{ijkl}(q, \mu^2)}{D_i D_j D_k D_l} +$$

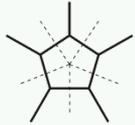
$$+ \sum_{ijk} \frac{\Delta_{ijk}(q, \mu^2)}{D_i D_j D_k} + \sum_{ij} \frac{\Delta_{ij}(q, \mu^2)}{D_i D_j} + \sum_i \frac{\Delta_i(q, \mu^2)}{D_i}$$

Quintuple cut



$$\Delta_{ijklm}(\bar{q}) = \text{Res}_{ijklm} \left\{ \frac{N(\bar{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}} \right\}$$

# Integrand decomposition

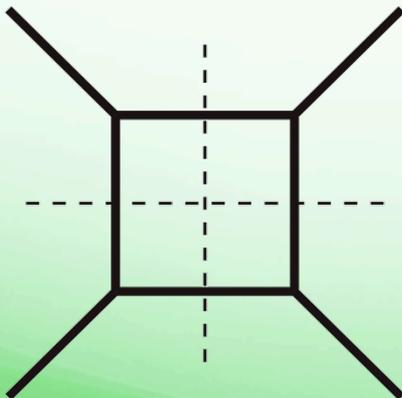


$$\Delta_{ijklm}(\bar{q}) = \text{Res}_{ijklm} \left\{ \frac{N(\bar{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}} \right\}$$

1 coefficient

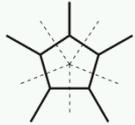
$$\begin{aligned} \mathcal{A}_n = & \sum_{ijklm} \frac{\Delta_{ijklm}(q, \mu^2)}{D_i D_j D_k D_l D_m} + \sum_{ijkl} \frac{\Delta_{ijkl}(q, \mu^2)}{D_i D_j D_k D_l} + \\ & + \sum_{ijk} \frac{\Delta_{ijk}(q, \mu^2)}{D_i D_j D_k} + \sum_{ij} \frac{\Delta_{ij}(q, \mu^2)}{D_i D_j} + \sum_i \frac{\Delta_i(q, \mu^2)}{D_i} \end{aligned}$$

Quadruple cut



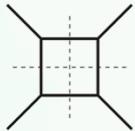
$$\Delta_{ijkl}(\bar{q}) = \text{Res}_{ijkl} \left\{ \frac{N(\bar{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}} - \sum_{i \ll m}^{n-1} \frac{\Delta_{ijklm}(\bar{q})}{\bar{D}_i \bar{D}_j \bar{D}_k \bar{D}_l \bar{D}_m} \right\}$$

# Integrand decomposition



$$\Delta_{ijklm}(\bar{q}) = \text{Res}_{ijklm} \left\{ \frac{N(\bar{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}} \right\}$$

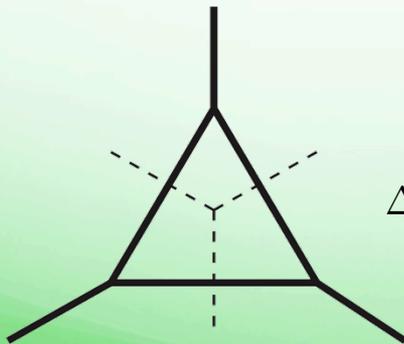
1 coefficient



$$\Delta_{ijkl}(\bar{q}) = \text{Res}_{ijkl} \left\{ \frac{N(\bar{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}} - \sum_{i << m}^{n-1} \frac{\Delta_{ijklm}(\bar{q})}{\bar{D}_i \bar{D}_j \bar{D}_k \bar{D}_\ell \bar{D}_m} \right\}$$

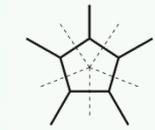
5 coefficients

## Triple cut



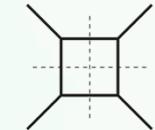
$$\Delta_{ijk}(\bar{q}) = \text{Res}_{ijk} \left\{ \frac{N(\bar{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}} - \sum_{i << m}^{n-1} \frac{\Delta_{ijklm}(\bar{q})}{\bar{D}_i \bar{D}_j \bar{D}_k \bar{D}_\ell \bar{D}_m} - \sum_{i << \ell}^{n-1} \frac{\Delta_{ijkl}(\bar{q})}{\bar{D}_i \bar{D}_j \bar{D}_k \bar{D}_\ell} \right\}$$

# Integrand decomposition



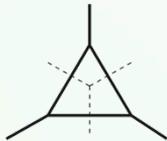
$$\Delta_{ijklm}(\bar{q}) = \text{Res}_{ijklm} \left\{ \frac{N(\bar{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}} \right\}$$

1 coefficient



$$\Delta_{ijkl}(\bar{q}) = \text{Res}_{ijkl} \left\{ \frac{N(\bar{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}} - \sum_{i \ll m}^{n-1} \frac{\Delta_{ijklm}(\bar{q})}{\bar{D}_i \bar{D}_j \bar{D}_k \bar{D}_\ell \bar{D}_m} \right\}$$

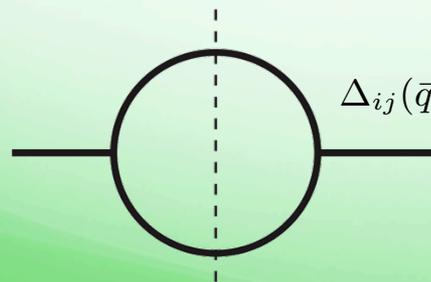
5 coefficients



$$\Delta_{ijk}(\bar{q}) = \text{Res}_{ijk} \left\{ \frac{N(\bar{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}} - \sum_{i \ll m}^{n-1} \frac{\Delta_{ijklm}(\bar{q})}{\bar{D}_i \bar{D}_j \bar{D}_k \bar{D}_\ell \bar{D}_m} - \sum_{i \ll \ell}^{n-1} \frac{\Delta_{ijkl}(\bar{q})}{\bar{D}_i \bar{D}_j \bar{D}_k \bar{D}_\ell} \right\}$$

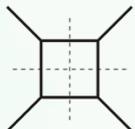
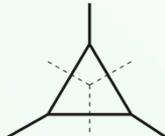
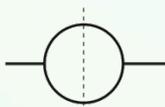
10 coefficients

## Double cut

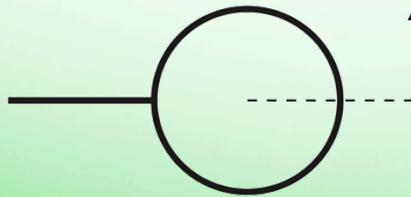


$$\Delta_{ij}(\bar{q}) = \text{Res}_{ij} \left\{ \frac{N(\bar{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}} - \sum_{i \ll m}^{n-1} \frac{\Delta_{ijklm}(\bar{q})}{\bar{D}_i \bar{D}_j \bar{D}_k \bar{D}_\ell \bar{D}_m} - \sum_{i \ll \ell}^{n-1} \frac{\Delta_{ijkl}(\bar{q})}{\bar{D}_i \bar{D}_j \bar{D}_k \bar{D}_\ell} - \sum_{i \ll k}^{n-1} \frac{\Delta_{ijk}(\bar{q})}{\bar{D}_i \bar{D}_j \bar{D}_k} \right\}$$

# Integrand decomposition

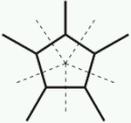
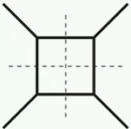
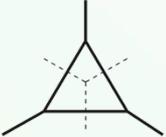
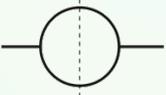
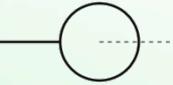
|   |  |                 |
|---|--|-----------------|
|  | $\Delta_{ijklm}(\bar{q}) = \text{Res}_{ijklm} \left\{ \frac{N(\bar{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}} \right\}$  | 1 coefficient   |
|  | $\Delta_{ijkl}(\bar{q}) = \text{Res}_{ijkl} \left\{ \frac{N(\bar{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}} - \sum_{i < j < k < m}^{n-1} \frac{\Delta_{ijklm}(\bar{q})}{\bar{D}_i \bar{D}_j \bar{D}_k \bar{D}_\ell \bar{D}_m} \right\}$  | 5 coefficients  |
|  | $\Delta_{ijk}(\bar{q}) = \text{Res}_{ijk} \left\{ \frac{N(\bar{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}} - \sum_{i < j < k < m}^{n-1} \frac{\Delta_{ijklm}(\bar{q})}{\bar{D}_i \bar{D}_j \bar{D}_k \bar{D}_\ell \bar{D}_m} - \sum_{i < j < k < \ell}^{n-1} \frac{\Delta_{ijkl}(\bar{q})}{\bar{D}_i \bar{D}_j \bar{D}_k \bar{D}_\ell} \right\}$  | 10 coefficients |
|  | $\Delta_{ij}(\bar{q}) = \text{Res}_{ij} \left\{ \frac{N(\bar{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}} - \sum_{i < j < k < m}^{n-1} \frac{\Delta_{ijklm}(\bar{q})}{\bar{D}_i \bar{D}_j \bar{D}_k \bar{D}_\ell \bar{D}_m} - \sum_{i < j < k < \ell}^{n-1} \frac{\Delta_{ijkl}(\bar{q})}{\bar{D}_i \bar{D}_j \bar{D}_k \bar{D}_\ell} - \sum_{i < j < k}^{n-1} \frac{\Delta_{ijk}(\bar{q})}{\bar{D}_i \bar{D}_j \bar{D}_k} \right\}$ | 10 coefficients |

## Single cut



$$\Delta_i(\bar{q}) = \text{Res}_i \left\{ \frac{N(\bar{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}} - \sum_{i < j < k < m}^{n-1} \frac{\Delta_{ijklm}(\bar{q})}{\bar{D}_i \bar{D}_j \bar{D}_k \bar{D}_\ell \bar{D}_m} - \sum_{i < j < k < \ell}^{n-1} \frac{\Delta_{ijkl}(\bar{q})}{\bar{D}_i \bar{D}_j \bar{D}_k \bar{D}_\ell} + \right. \\ \left. - \sum_{i < j < k}^{n-1} \frac{\Delta_{ijk}(\bar{q})}{\bar{D}_i \bar{D}_j \bar{D}_k} - \sum_{i < j}^{n-1} \frac{\Delta_{ij}(\bar{q})}{\bar{D}_i \bar{D}_j} \right\}$$

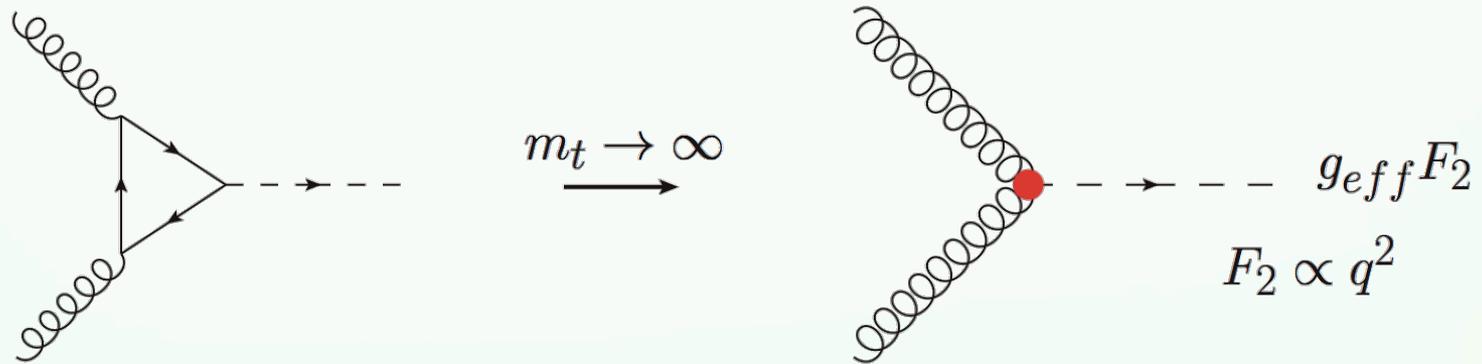
# Integrand decomposition

|   |   |                 |
|---|---|-----------------|
|  | $\Delta_{ijklm}(\bar{q}) = \text{Res}_{ijklm} \left\{ \frac{N(\bar{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}} \right\}$   | 1 coefficient   |
|  | $\Delta_{ijkl}(\bar{q}) = \text{Res}_{ijkl} \left\{ \frac{N(\bar{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}} - \sum_{i < j < k < m}^{n-1} \frac{\Delta_{ijklm}(\bar{q})}{\bar{D}_i \bar{D}_j \bar{D}_k \bar{D}_\ell \bar{D}_m} \right\}$   | 5 coefficients  |
|  | $\Delta_{ijk}(\bar{q}) = \text{Res}_{ijk} \left\{ \frac{N(\bar{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}} - \sum_{i < j < k}^{n-1} \frac{\Delta_{ijklm}(\bar{q})}{\bar{D}_i \bar{D}_j \bar{D}_k \bar{D}_\ell \bar{D}_m} - \sum_{i < j < k}^{n-1} \frac{\Delta_{ijkl}(\bar{q})}{\bar{D}_i \bar{D}_j \bar{D}_k \bar{D}_\ell} \right\}$  | 10 coefficients |
|  | $\Delta_{ij}(\bar{q}) = \text{Res}_{ij} \left\{ \frac{N(\bar{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}} - \sum_{i < j < k < m}^{n-1} \frac{\Delta_{ijklm}(\bar{q})}{\bar{D}_i \bar{D}_j \bar{D}_k \bar{D}_\ell \bar{D}_m} - \sum_{i < j < k}^{n-1} \frac{\Delta_{ijkl}(\bar{q})}{\bar{D}_i \bar{D}_j \bar{D}_k \bar{D}_\ell} - \sum_{i < j < k}^{n-1} \frac{\Delta_{ijk}(\bar{q})}{\bar{D}_i \bar{D}_j \bar{D}_k} \right\}$   | 10 coefficients |
|  | $\Delta_i(\bar{q}) = \text{Res}_i \left\{ \frac{N(\bar{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}} - \sum_{i < j < k < m}^{n-1} \frac{\Delta_{ijklm}(\bar{q})}{\bar{D}_i \bar{D}_j \bar{D}_k \bar{D}_\ell \bar{D}_m} - \sum_{i < j < k}^{n-1} \frac{\Delta_{ijkl}(\bar{q})}{\bar{D}_i \bar{D}_j \bar{D}_k \bar{D}_\ell} + \right. \\ \left. - \sum_{i < j < k}^{n-1} \frac{\Delta_{ijk}(\bar{q})}{\bar{D}_i \bar{D}_j \bar{D}_k} - \sum_{i < j}^{n-1} \frac{\Delta_{ij}(\bar{q})}{\bar{D}_i \bar{D}_j} \right\}$ | 5 coefficients  |

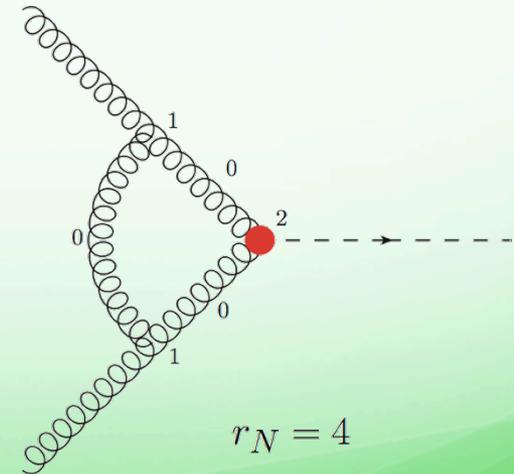
Hexagon:

$$\binom{6}{5} \cdot 1 + \binom{6}{4} \cdot 5 + \binom{6}{3} \cdot 10 + \binom{6}{2} \cdot 10 + \binom{6}{1} \cdot 5 = 461 \text{ coefficients}$$

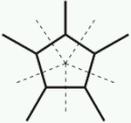
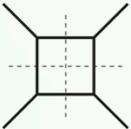
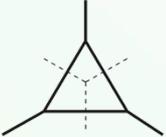
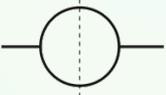
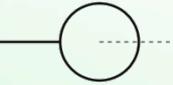
# Effective vertices



- Effective vertex in loop adds two powers of  $q$
- New rule: rank  $\leq$  propagators + 1



# Integrand decomposition

|   |   |                      |
|---|---|----------------------|
|  | $\Delta_{ijklm}(\bar{q}) = \text{Res}_{ijklm} \left\{ \frac{N(\bar{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}} \right\}$   | 1 → 1 coefficient    |
|  | $\Delta_{ijkl}(\bar{q}) = \text{Res}_{ijkl} \left\{ \frac{N(\bar{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}} - \sum_{i < j < k < m}^{n-1} \frac{\Delta_{ijklm}(\bar{q})}{\bar{D}_i \bar{D}_j \bar{D}_k \bar{D}_l \bar{D}_m} \right\}$  | 5 → 6 coefficients   |
|  | $\Delta_{ijk}(\bar{q}) = \text{Res}_{ijk} \left\{ \frac{N(\bar{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}} - \sum_{i < j < k}^{n-1} \frac{\Delta_{ijklm}(\bar{q})}{\bar{D}_i \bar{D}_j \bar{D}_k \bar{D}_l \bar{D}_m} - \sum_{i < j < k < l}^{n-1} \frac{\Delta_{ijkl}(\bar{q})}{\bar{D}_i \bar{D}_j \bar{D}_k \bar{D}_l} \right\}$  | 10 → 15 coefficients |
|  | $\Delta_{ij}(\bar{q}) = \text{Res}_{ij} \left\{ \frac{N(\bar{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}} - \sum_{i < j < k < l < m}^{n-1} \frac{\Delta_{ijklm}(\bar{q})}{\bar{D}_i \bar{D}_j \bar{D}_k \bar{D}_l \bar{D}_m} - \sum_{i < j < k < l}^{n-1} \frac{\Delta_{ijkl}(\bar{q})}{\bar{D}_i \bar{D}_j \bar{D}_k \bar{D}_l} - \sum_{i < j < k}^{n-1} \frac{\Delta_{ijk}(\bar{q})}{\bar{D}_i \bar{D}_j \bar{D}_k} \right\}$   | 10 → 20 coefficients |
|  | $\Delta_i(\bar{q}) = \text{Res}_i \left\{ \frac{N(\bar{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}} - \sum_{i < j < k < l < m}^{n-1} \frac{\Delta_{ijklm}(\bar{q})}{\bar{D}_i \bar{D}_j \bar{D}_k \bar{D}_l \bar{D}_m} - \sum_{i < j < k < l}^{n-1} \frac{\Delta_{ijkl}(\bar{q})}{\bar{D}_i \bar{D}_j \bar{D}_k \bar{D}_l} + \sum_{i < j < k}^{n-1} \frac{\Delta_{ijk}(\bar{q})}{\bar{D}_i \bar{D}_j \bar{D}_k} - \sum_{i < j}^{n-1} \frac{\Delta_{ij}(\bar{q})}{\bar{D}_i \bar{D}_j} \right\}$ | 5 → 15 coefficients  |

Hexagon:

[Mastrolia, Mirabella, Peraro (2012)]; [HvD (2013)]

$$\binom{6}{5} \cdot 1 + \binom{6}{4} \cdot 6 + \binom{6}{3} \cdot 15 + \binom{6}{2} \cdot 20 + \binom{6}{1} \cdot 15 = (461 \rightarrow) 786 \text{ coefficients}$$

# Discrete Fourier Transformation

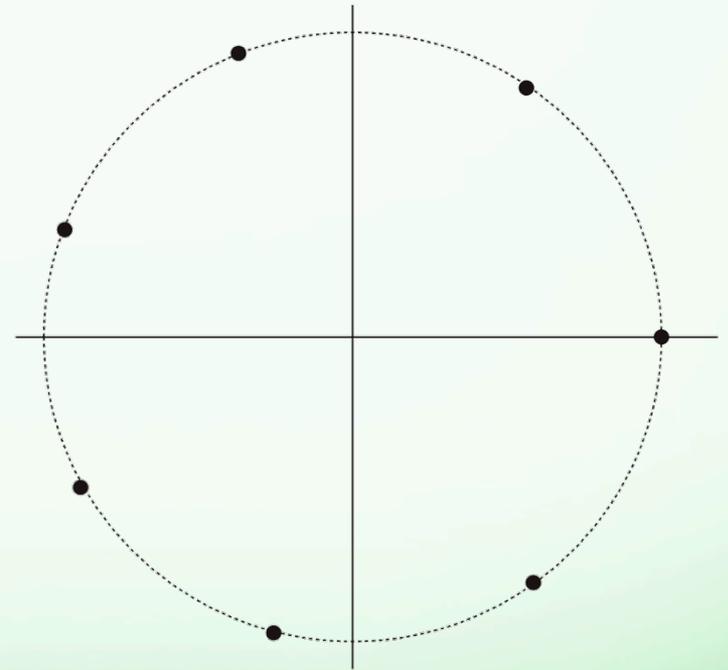
$$P(x) = c_0 + c_1x + c_2x^2 + \dots + c_nx^n$$

$$x_k = \rho \exp \left[ -2\pi i \frac{k}{n+1} \right]$$

$$P_k = P(x_k) = \sum_{l=0}^n c_l \rho^l \exp \left[ -2\pi i \frac{k}{(n+1)} l \right]$$

$$\sum_{n=0}^{N-1} \exp \left[ 2\pi i \frac{k}{N} n \right] \exp \left[ -2\pi i \frac{k'}{N} n \right] = N \delta_{kk'}$$

$$c_l = \frac{\rho^{-l}}{n+1} \sum_{k=0}^n P_k \exp \left[ 2\pi i \frac{k}{n+1} l \right]$$



[Mastrolia, Ossola, Papadopoulos, Pittau (2008)]

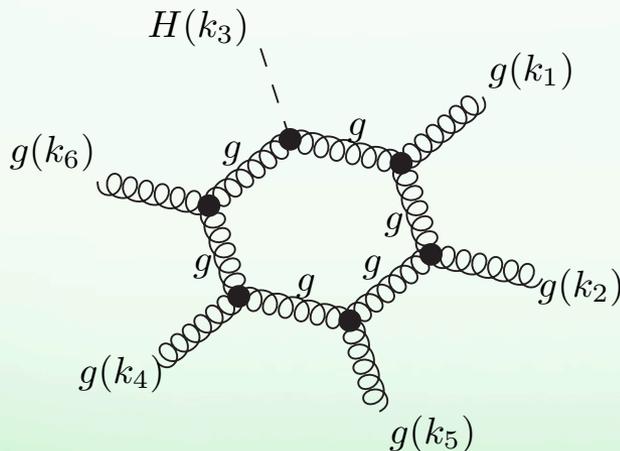
# Sampling strategy

- Cuts impose conditions on the relation between variables
- These can lead to infinities when using the DFT naively
- Solution: Branch for each situation
- Especially in higherrank numerous instances
- Xsamurai: Systematic implementation

[HvD, Acta Phys. Polon. B44 (2013) 11, 2223-2230]

# Higgs plus jets in GF@NLO

- Computational challenges
  - Over 10,000 diagrams
  - Higher-rank terms
  - 60 rank-7 hexagons



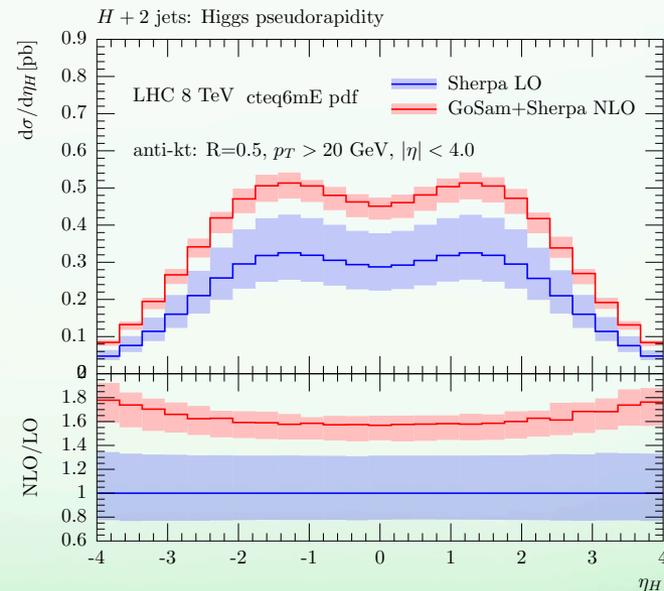
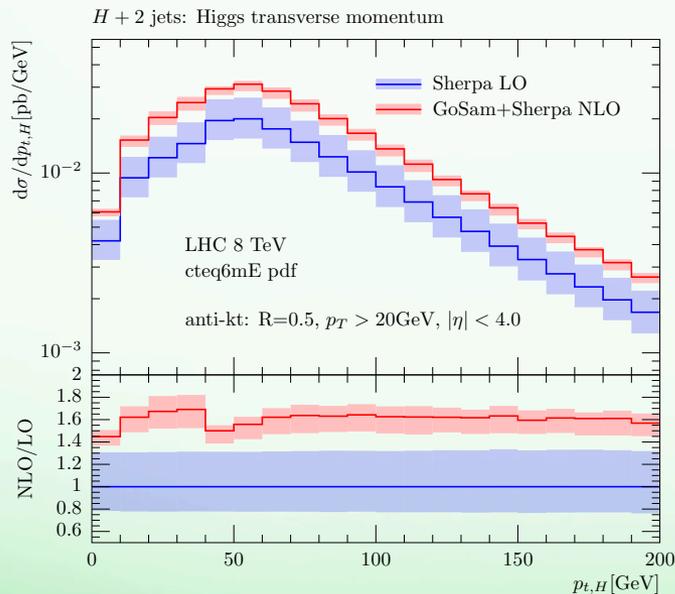
|                         |                  |
|-------------------------|------------------|
| <b>H+0j</b>             | <b>1 NLO</b>     |
| $gg \rightarrow H$      | 1 NLO            |
| <b>H+1j</b>             | <b>62 NLO</b>    |
| $qq \rightarrow Hg$     | 14 NLO           |
| $gg \rightarrow Hg$     | 48 NLO           |
| <b>H+2j</b>             | <b>926 NLO</b>   |
| $qq' \rightarrow Hqq'$  | 32 NLO           |
| $qq \rightarrow Hqq$    | 64 NLO           |
| $qg \rightarrow Hqg$    | 179 NLO          |
| $gg \rightarrow Hgg$    | 651 NLO          |
| <b>H+3j</b>             | <b>13179 NLO</b> |
| $qq' \rightarrow Hqq'g$ | 467 NLO          |
| $qq \rightarrow Hqqg$   | 868 NLO          |
| $qg \rightarrow Hqgg$   | 2519 NLO         |
| $gg \rightarrow Hggg$   | 9325 NLO         |

Complex calculations → GoSam enhanced

grouping, optimization through Form4.0, numerical polarization vectors, parallelization

# Higgs + 2 jets in GF@NLO

- Results obtained with GoSam+Sherpa
- Agreement with MCFM (v6.4) [Campbell, Ellis, Williams]



[HvD, Greiner, Luisoni, Mastrolia, Mirabella, Ossola, Peraro, von Soden-Fraunhofen, Tramontano (2013)]

# Higgs + 3 jets GF@NLO: cross section

[Cullen, HvD, Greiner, Luisoni, Mastrolia, Mirabella, Ossola, Peraro, Tramontano (2013)]

- Cross section obtained with hybrid setup:
  - GoSam + Sherpa for Born and virtual contributions
  - MadGraph+MadDipole+MadEvent for real contributions, subtraction terms, integrated dipoles

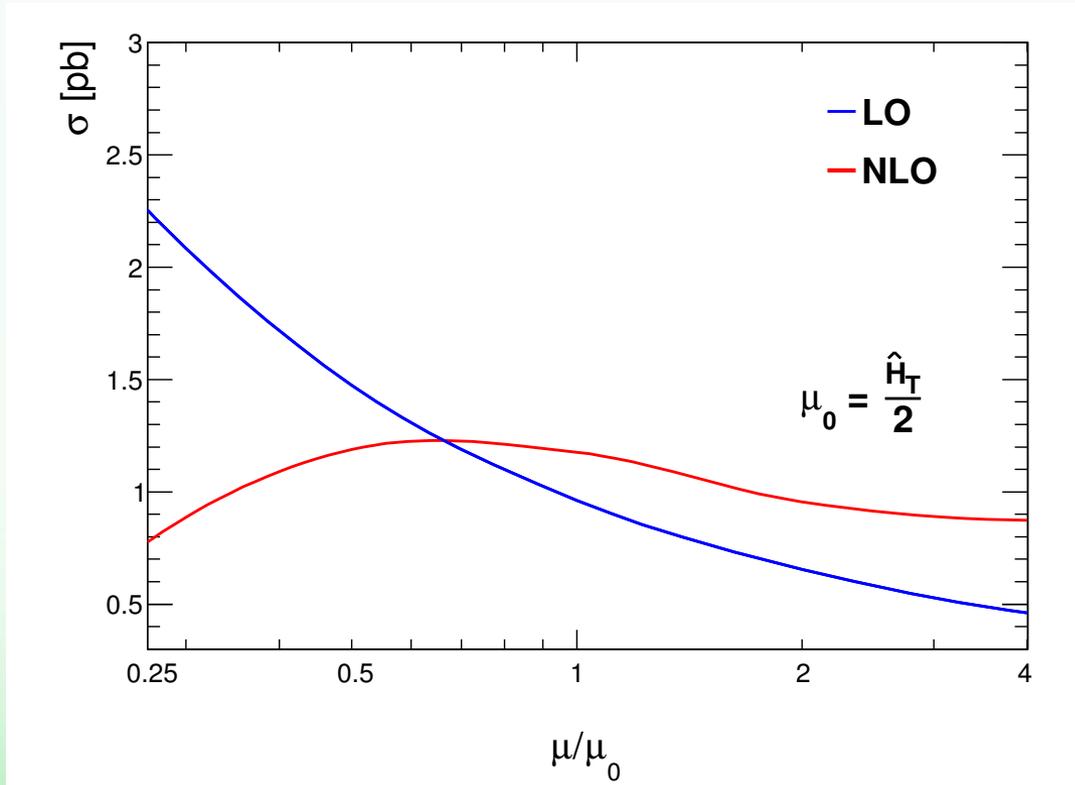
# Higgs + 3 jets GF@NLO: cross section

[Cullen, HvD, Greiner, Luisoni, Mastrolia, Mirabella, Ossola, Peraro, Tramontano (2013)]

- Tests performed on the cross section
  - NLO H+2j: Agreement between hybrid setup and GoSam+Sherpa
  - LO H+3j: Agreement MadGraph and Sherpa
  - NLO H+3j: Independence from  $\alpha$ -parameter (subtr.+int.dipoles)

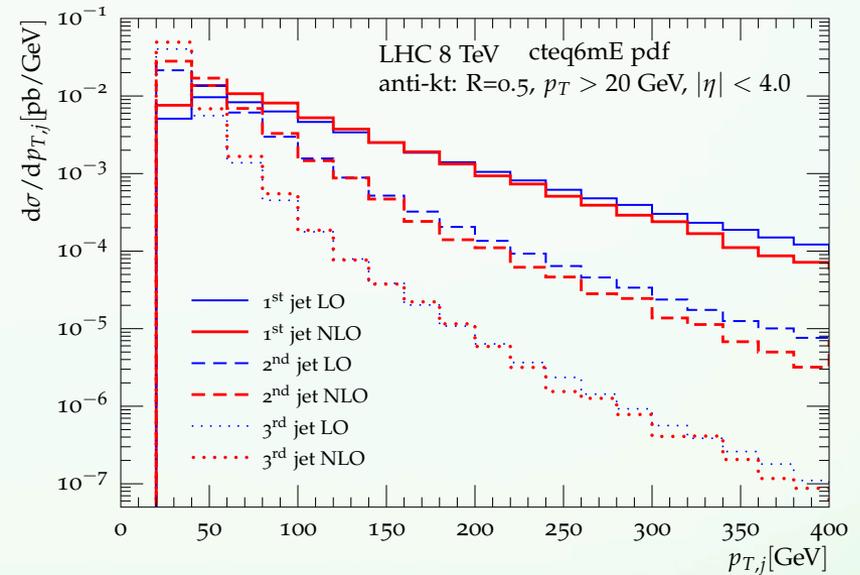
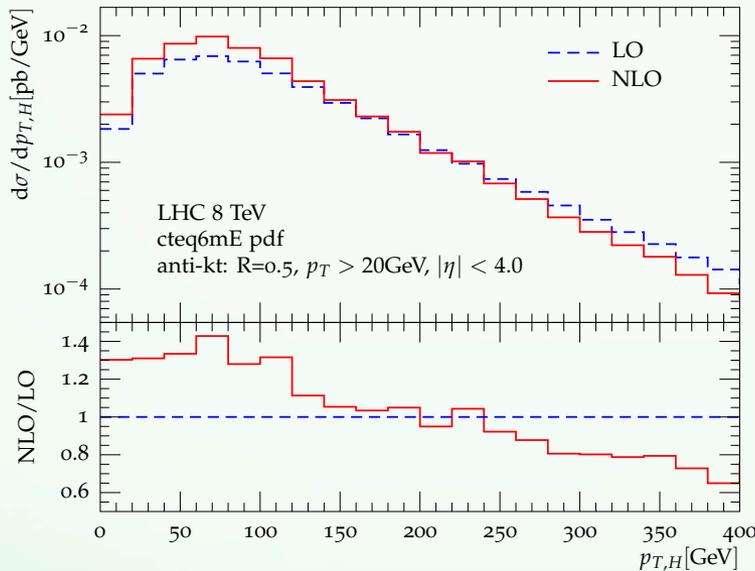
# Higgs + 3 jets GF@NLO: results

[Cullen, HvD, Greiner, Luisoni, Mastrolia, Mirabella, Ossola, Peraro, Tramontano (2013)]



# Higgs + 3 jets GF@NLO: distributions

[Cullen, HvD, Greiner, Luisoni, Mastrolia, Mirabella, Ossola, Peraro, Tramontano (2013)]



- $pp \rightarrow Hjjj$  can be paired with available MC programs for further phenomenological analyses

# Ninja

- New reduction algorithm based on Laurent expansion [Mastrolia, Mirabella, Peraro (2012)]
- Improved in all directions
  - Faster (timings per PSP)
  - More stable (less bad points)
  - More precise (in correct digits)
- Higher rank included

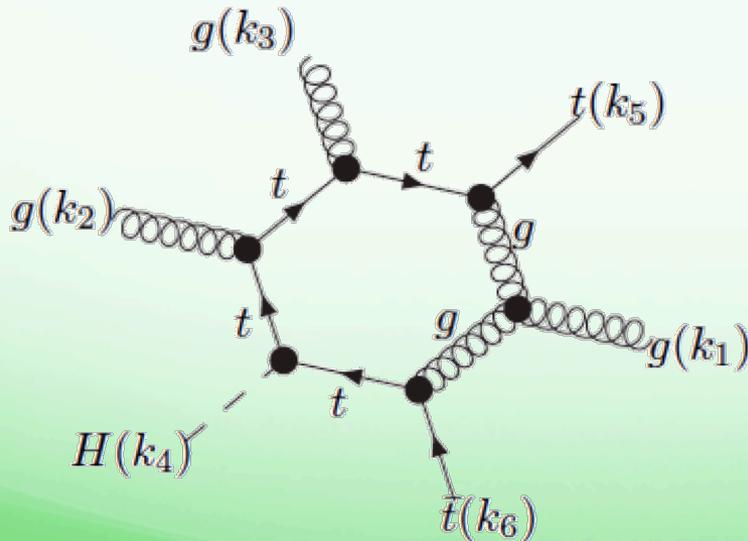
[Peraro (2014)]

[HvD, Luisoni, Mastrolia, Mirabella, Ossola, Peraro (2013)]

# $pp \rightarrow Ht\bar{t} + 1 \text{ jet @ NLO}$

[HvD, Luisoni, Mastrolia, Mirabella, Ossola, Peraro (2013)]

- First application of GoSam+Ninja
- Two different mass scales: Higgs and top
- 51 hexagons in the gluon-gluon channel



| $t\bar{t}H + 1j$            | <b>1895 NLO</b> |
|-----------------------------|-----------------|
| $qq \rightarrow Ht\bar{t}g$ | 320 NLO         |
| $gg \rightarrow Ht\bar{t}g$ | 1575 NLO        |

# pp → Htt̄ + 1 jet results

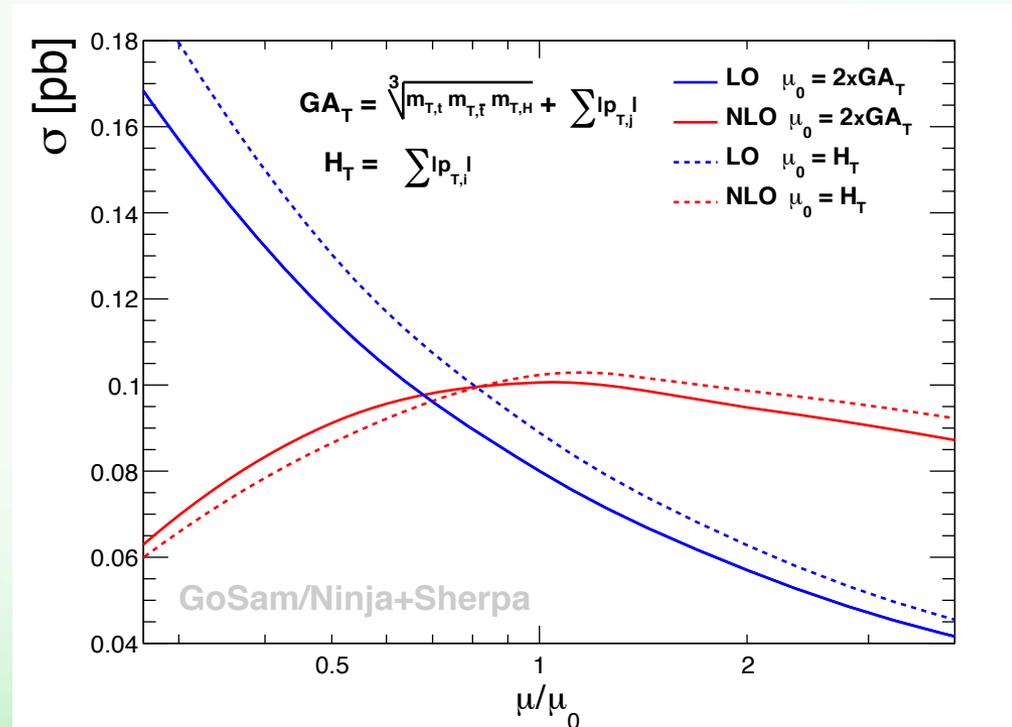
[HvD, Luisoni, Mastrolia, Mirabella, Ossola, Peraro (2013)]

$$\mu_R = \mu_F = \mu_0 \quad H_T = \sum_i |p_T^i|$$

$$GA_T = (m_T^H m_T^t m_T^{\bar{t}})^{1/3} + \sum_{jets} |p_T^j|$$

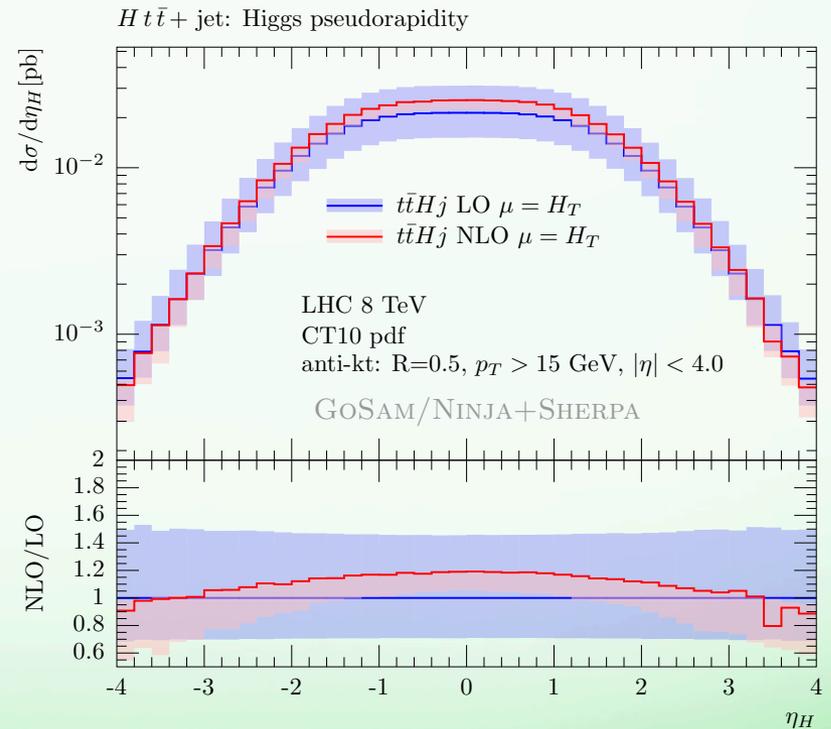
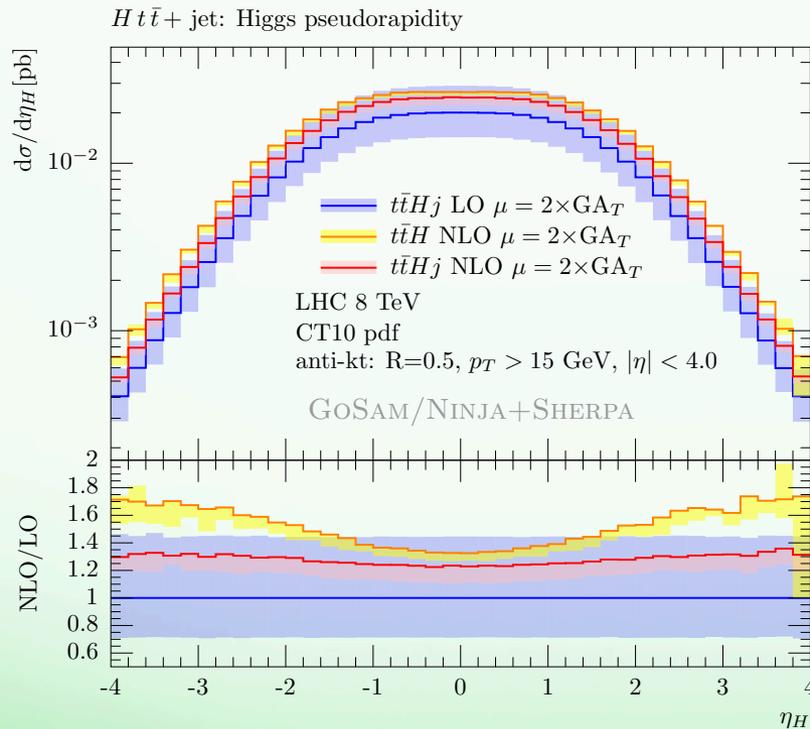
$$\mu_0 = 2GA_T \quad \mu_0 = H_T$$

| Central Scale | $\sigma_{LO}$ [fb]        | $\sigma_{NLO}$ [fb]      |
|---------------|---------------------------|--------------------------|
| $2GA_T$       | $80.03^{+35.64}_{-23.02}$ | $100.6^{+0.00}_{-9.43}$  |
| $H_T$         | $88.93^{+41.41}_{-26.13}$ | $102.3^{+0.00}_{-15.82}$ |



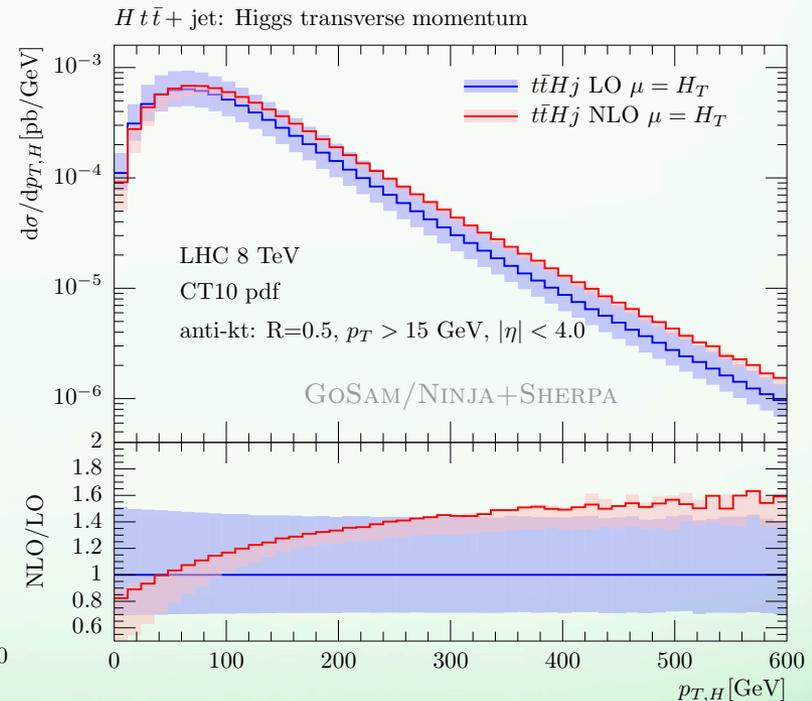
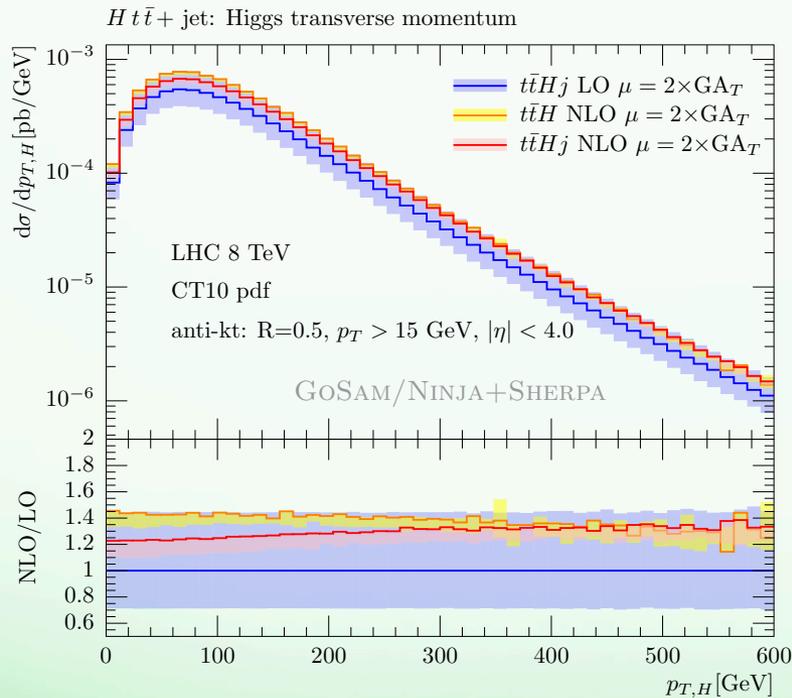
# pp $\rightarrow H t \bar{t} + 1$ jet results

[HvD, Luisoni, Mastrolia, Mirabella, Ossola, Peraro (2013)]



# $pp \rightarrow H t \bar{t} + 1 \text{ jet results}$

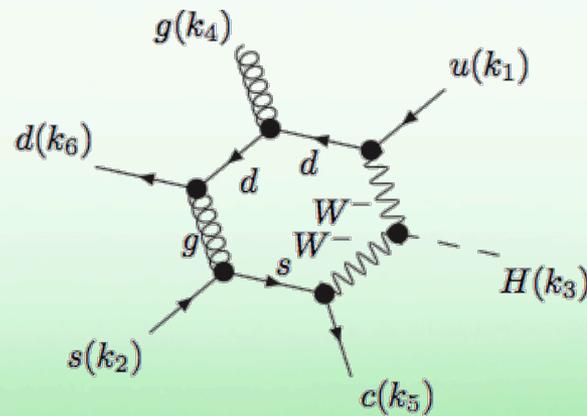
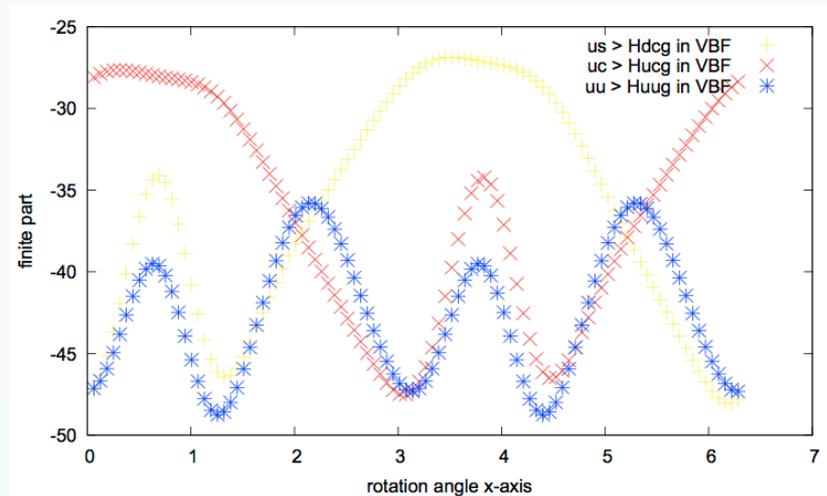
[HvD, Luisoni, Mastrolia, Mirabella, Ossola, Peraro (2013)]



# Vector Boson Fusion

| <b>H+2j</b>          | <b>240 NLO</b> |
|----------------------|----------------|
| $us \rightarrow Hdc$ | 24 NLO         |
| $uc \rightarrow Huc$ | 24 NLO         |
| $us \rightarrow Hus$ | 24 NLO         |
| $ds \rightarrow Hds$ | 24 NLO         |
| $ud \rightarrow Hud$ | 48 NLO         |
| $uu \rightarrow Huu$ | 48 NLO         |
| $dd \rightarrow Hdd$ | 48 NLO         |

| <b>H+3j</b>           | <b>2160 NLO</b> |
|-----------------------|-----------------|
| $us \rightarrow Hdcg$ | 216 NLO         |
| $uc \rightarrow Hucg$ | 216 NLO         |
| $us \rightarrow Husg$ | 216 NLO         |
| $ds \rightarrow Hdsg$ | 216 NLO         |
| $ud \rightarrow Hudg$ | 432 NLO         |
| $uu \rightarrow Huug$ | 432 NLO         |
| $dd \rightarrow Hddg$ | 432 NLO         |

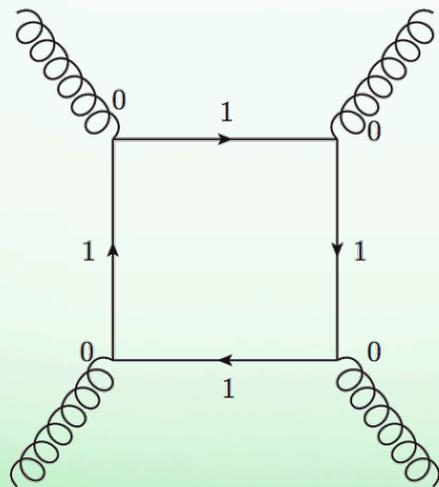
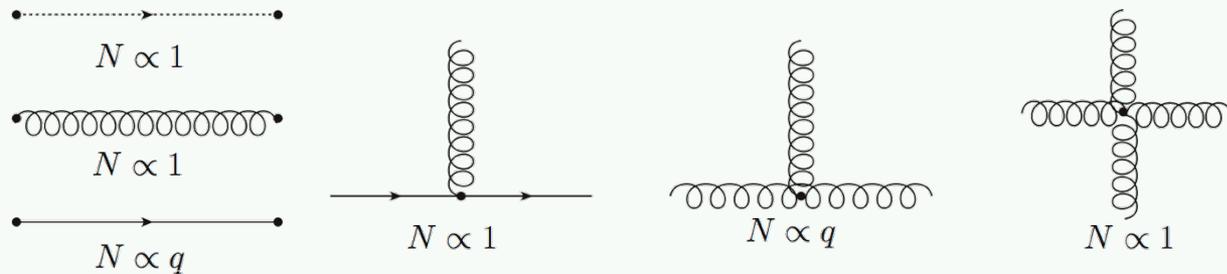


# Conclusions

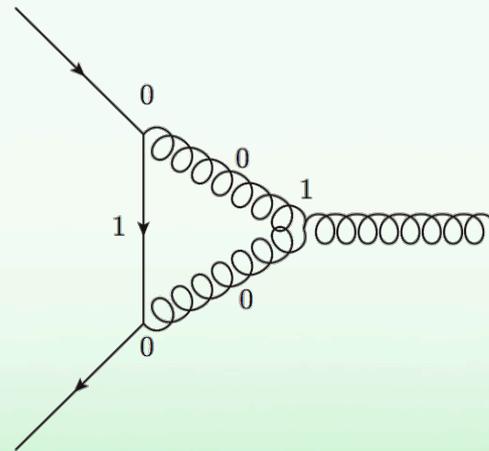
- Samurai has been extended to higher rank: Xsamurai
- Higgs plus two and three jets in GF have been calculated
- Ninja has been introduced as new algorithm
- $H\bar{t}tj$  has been calculated
- Higgs plus jets in VBF is no problem

# Backup slides

# Rank of the numerator

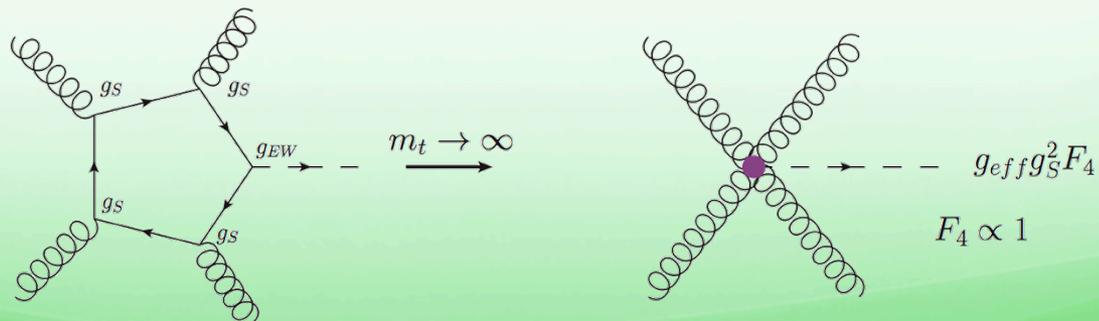
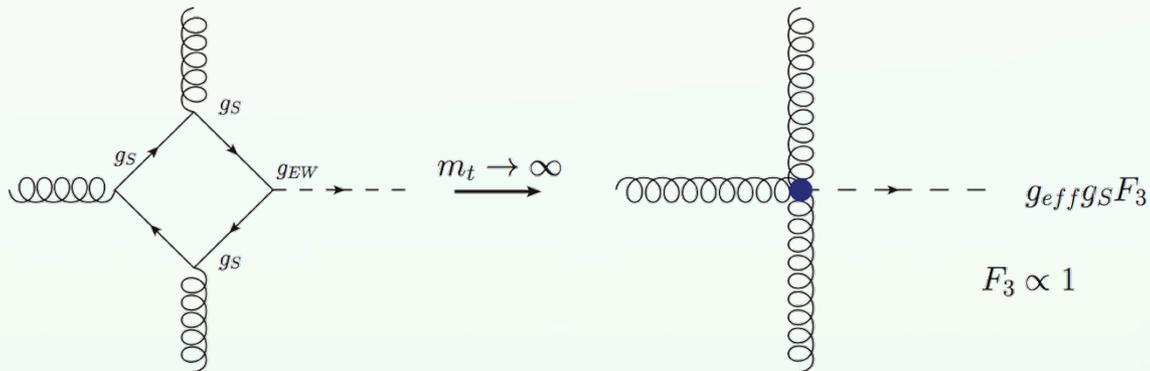
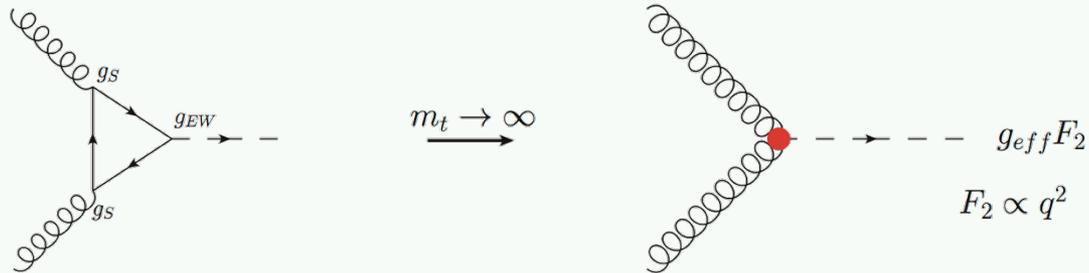


$$r_N = 4$$



$$r_N = 2$$

# Effective vertices



# Sampling strategy

$$\Lambda(\mu^2, q) \quad q^\mu = \sum x_i e_i^\mu \rightarrow (\mu^2, x_1, x_2, x_3, x_4)$$

- Five variables
  - Quintuple cut: 5 conditions on 5 variables  
→ Everything constrained
  - Quadruple cut: 4 conditions on 5 variables  
→ 1 free variable
- $$\Lambda(\mu^2)$$

[HvD, Acta Phys. Polon. B44 (2013) 11, 2223-2230]

# Sampling strategy

- Triple cut: 3 conditions on 5 variables  
→ 2 free variables
- One of those conditions constrains the product

$$x_3 x_4 = C(\mu^2)$$

$$\Lambda(\mu^2, x_3) \xrightarrow{\text{DFT}} c_i \propto \frac{1}{C}$$
$$\left. \begin{array}{l} \Lambda(\mu^2, x_3) \\ \Lambda(\mu^2, x_4) \end{array} \right\} \xrightarrow{\text{DFT}} c_i \propto \frac{1}{1-C}$$

[HvD, Acta Phys. Polon. B44 (2013) 11, 2223-2230]

# Sampling strategy

| Quadruple cut          |   | Triple cut                      | $C = 0$ | $C \neq 0$ |
|------------------------|---|---------------------------------|---------|------------|
| $\Lambda(0, q)$        | 2 | $\Lambda(0, x_3, C/x_3)$        | 5       | 9          |
|                        |   | $\Lambda(0, C/x_4, x_4)$        | 4       | 0          |
| $\Lambda(+\mu_s^2, q)$ | 2 | $\Lambda(+\mu_s^2, x_3, C/x_3)$ | 5       | 5          |
| $\Lambda(-\mu_s^2, q)$ | 2 | $\Lambda(-\mu_s^2, 1, C)$       | 1       | 1          |

[HvD, Acta Phys. Polon. B44 (2013) 11, 2223-2230]

# Sampling strategy

- Double cut: 2 conditions on 5 variables  
→ 3 free parameters
- One of those constraints the product

$$x_3 x_4 = F(\mu^2, x_1) = A x_1^2 + B x_1 + C(\mu^2)$$

- Numerous branchings: F can be zero/non-zero, equation can have 0, 1 or 2 solutions

[HvD, Acta Phys. Polon. B44 (2013) 11, 2223-2230]

# Sampling strategy

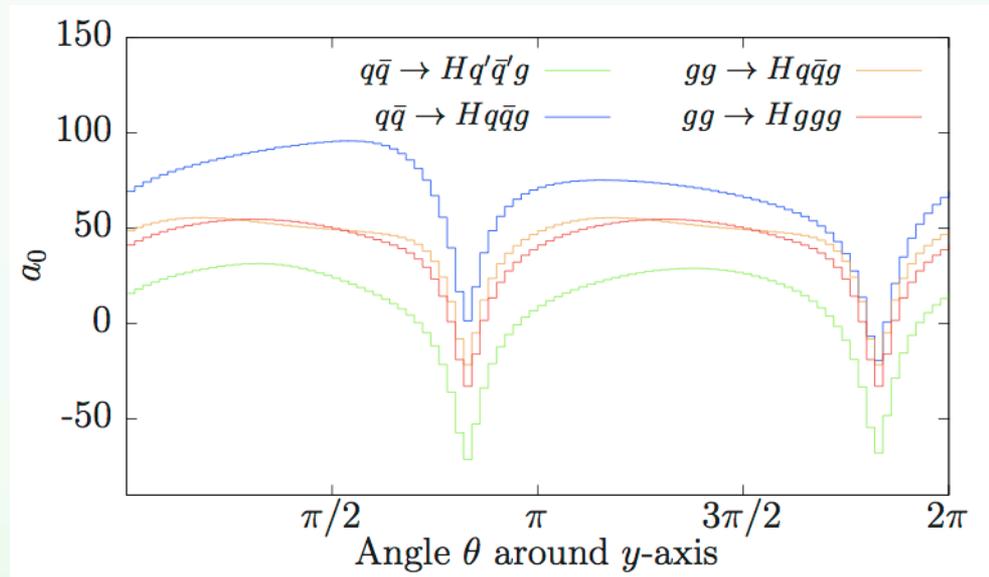
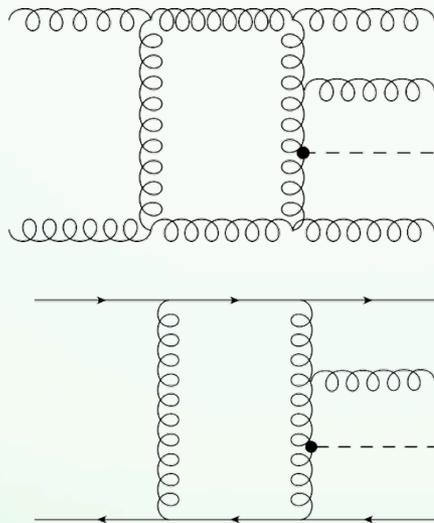
| Double cut                        | $F = 0$ | $F \neq 0$ | Single cut                      | $G = 0$ | $G \neq 0$ |
|-----------------------------------|---------|------------|---------------------------------|---------|------------|
| $\Lambda(0, 0, x_3, F/x_3)$       | 4       | 7          | $\Lambda(0, x_1, G/x_1, 0, 0)$  | 3       | 5          |
| $\Lambda(0, 0, F/x_4, x_4)$       | 3       | 0          | $\Lambda(0, G/x_2, x_2, 0, 0)$  | 2       | 0          |
| $\Lambda(0, x_{1a}, x_3, F/x_3)$  | 3       | 5          | $\Lambda(0, 0, 0, x_3, G/x_3)$  | 3       | 5          |
| $\Lambda(0, x_{1a}, F/x_4, x_4)$  | 2       | 0          | $\Lambda(0, 0, 0, G/x_4, x_4)$  | 2       | 0          |
| $\Lambda(0, x_{1b}, x_3, F/x_3)$  | 2       | 3          | $\Lambda(0, x_1, -G/x_1, 1, 0)$ | 1       | 2          |
| $\Lambda(0, x_{1b}, F/x_4, x_4)$  | 1       | 0          | $\Lambda(0, -G/x_2, x_2, 1, 0)$ | 1       | 0          |
| $\Lambda(0, x_{1c}, 1, F)$        | 1       | 1          | $\Lambda(0, x_1, -G/x_1, 0, 1)$ | 1       | 2          |
|                                   |         |            | $\Lambda(0, -G/x_2, x_2, 0, 1)$ | 1       | 0          |
| $\Lambda(\mu_s^2, 0, x_3, F/x_3)$ | 2       | 3          | $\Lambda(\mu_s^2, 0, 0, 0, 0)$  | 1       | 0          |
| $\Lambda(\mu_s^2, 0, F/x_4, x_4)$ | 1       | 0          | $\Lambda(\mu_s^2, 1, G, 0, 0)$  | 0       | 1          |
| $\Lambda(\mu_s^2, 1, 1, F)$       | 1       | 1          |                                 |         |            |

[HvD, Acta Phys. Polon. B44 (2013) 11, 2223-2230]

# Higgs + 3 jets in GF: virtual part

[Cullen, HvD, Greiner, Luisoni, Mastrolia, Mirabella, Ossola, Peraro, Tramontano (2013)]

- Virtual parts computed by GoSam



| SUBPROCESS                        | DIAGRAMS | TIME/PS-POINT [sec] |
|-----------------------------------|----------|---------------------|
| $q\bar{q} \rightarrow Hq'q'g$     | 467      | 0.29                |
| $q\bar{q} \rightarrow Hq\bar{q}g$ | 868      | 0.60                |
| $gg \rightarrow Hq\bar{q}g$       | 2519     | 3.9                 |
| $gg \rightarrow Hggg$             | 9325     | 20                  |

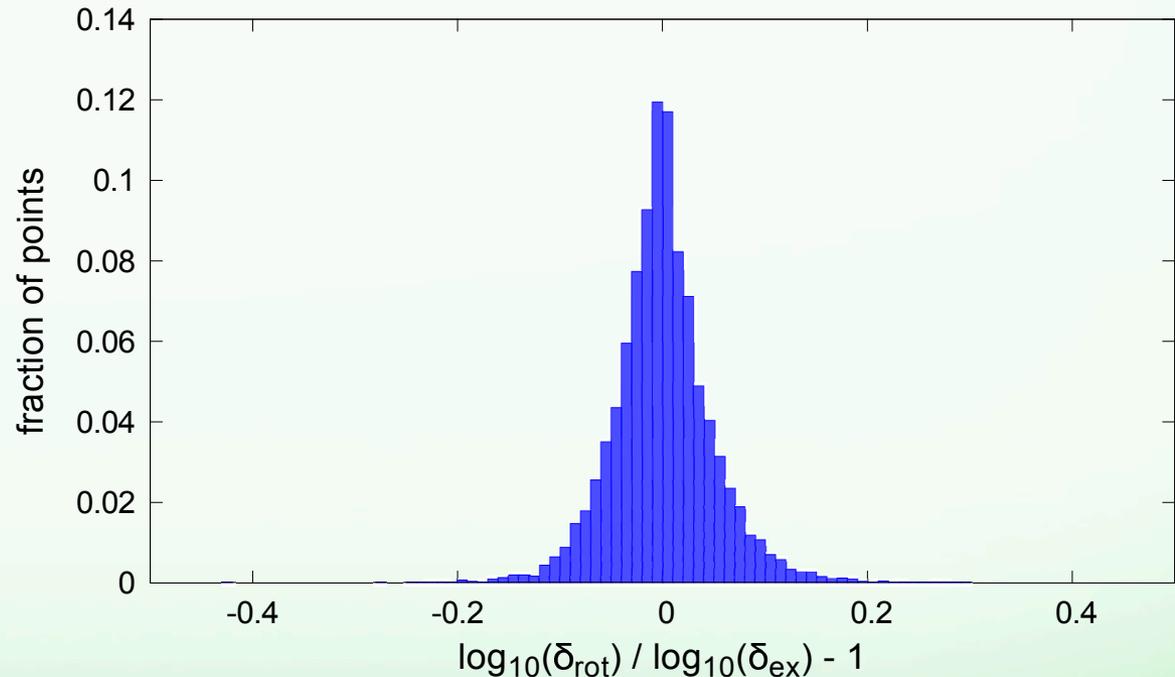
- gauge invariance
- IR poles

# Ninja: precision correlation

$$\delta_{ex} = \left| \frac{A_{quad} - A}{A_{quad}} \right|$$

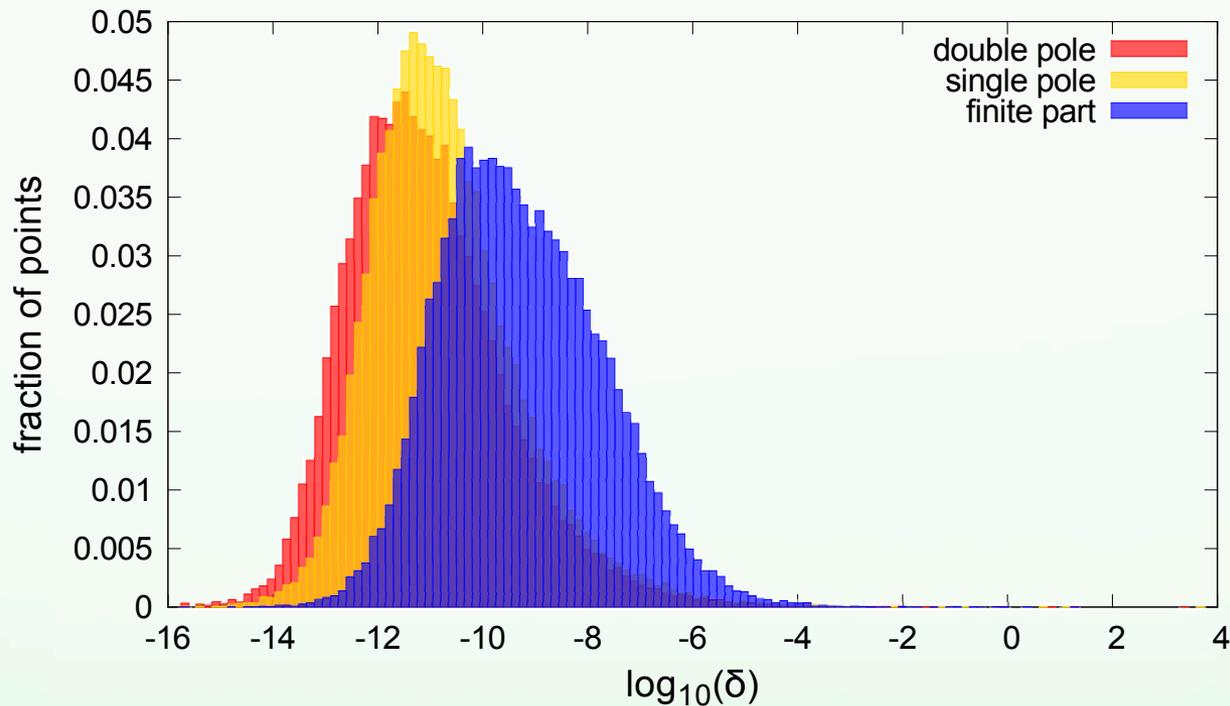
$$\delta_{rot} = 2 \left| \frac{A_{rot} - A}{A_{rot} + A} \right|$$

$$C = \frac{\log_{10}(\delta_{rot})}{\log_{10}(\delta_{ex})} - 1$$



[HvD, Luisoni, Mastrolia, Mirabella, Ossola, Peraro (2013)]

# Ninja: precision

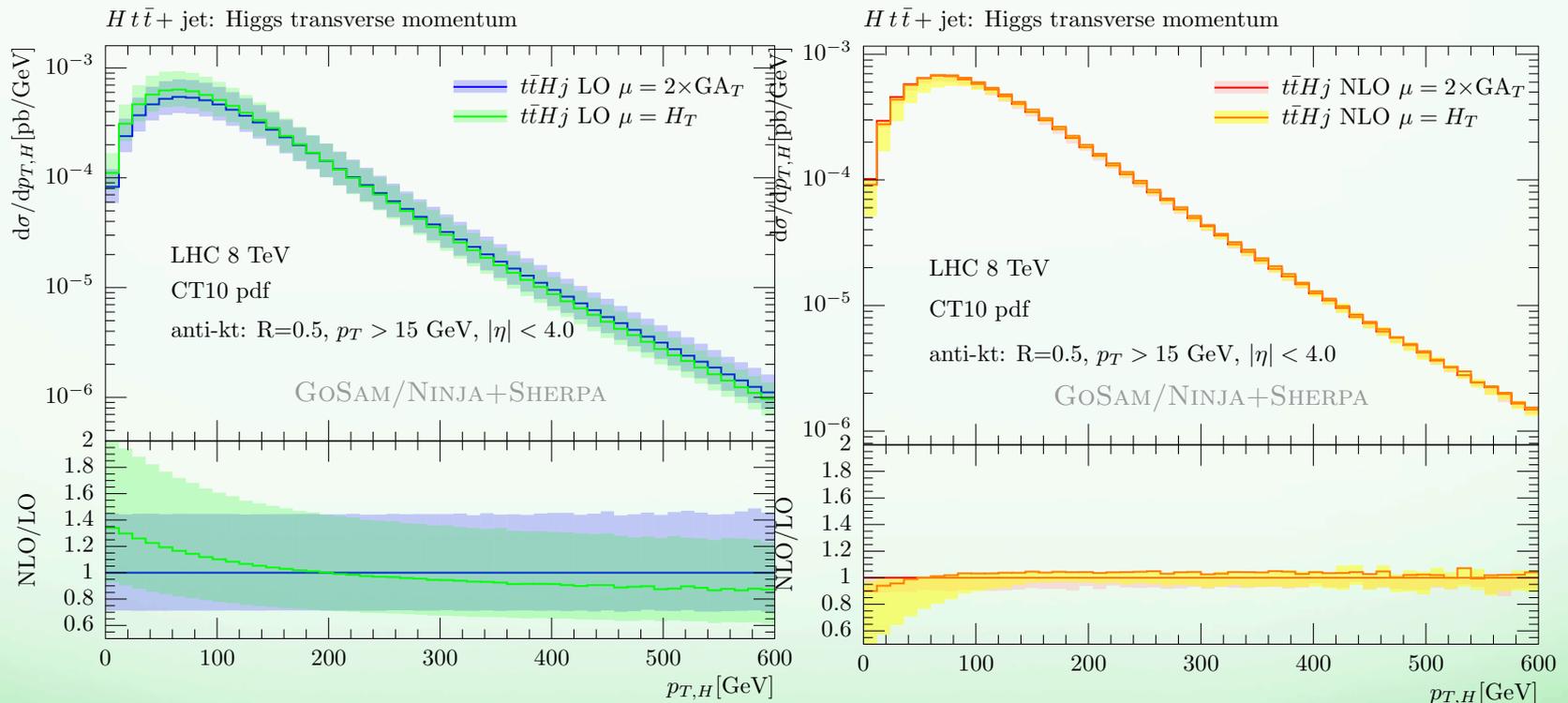


Precision Plot for  $gg \rightarrow t\bar{t}Hg$ : the distributions are obtained using  $5 \cdot 10^4$  randomly

[HvD, Luisoni, Mastrolia, Mirabella, Ossola, Peraro (2013)]

# $pp \rightarrow Ht\bar{t} + 1 \text{ jet results}$

[HvD, Luisoni, Mastrolia, Mirabella, Ossola, Peraro (2013)]



# NLO calculations

$$\sigma^{NLO} = \int_m \left[ d^{(4)}\sigma^B + \int_{loop} d^{(d)}\sigma^V + \int_1 d^{(d)}\sigma^S \right] + \int_{m+1} \left[ d^{(4)}\sigma^R - d^{(4)}\sigma^S \right]$$

- NLO calculation consists of:
  - LO: Born diagram
  - Virtual corrections: loop diagrams ← GoSam
  - Real corrections: additional radiation
  - Subtraction terms to regulate infinities

# Interfaces with external MC

- GoSam+MadGraph+MadDipole+MadEvent
  - ad-hoc interface
- GoSam+Sherpa
  - via BLHA
- GoSam+Powheg
  - via BLHA
- GoSam+Herwig
  - work in progress
- GoSam+aMC@NLO
  - Work in progress