Two loop corrections to the masses of the Higgs bosons of the complex MSSM

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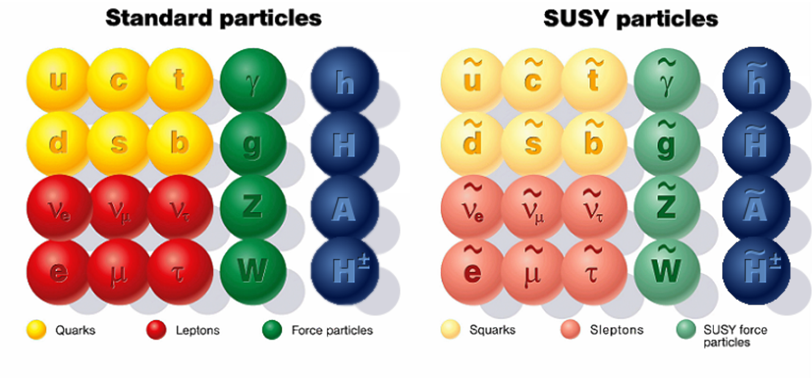
content

MSSM content

extension of the SM by Supersymmetry



Higgs bosons in the MSSM

two complex $SU(2)$-Higgs doublets necessary,

$$\mathcal{H}_1 = \begin{pmatrix} \frac{1}{\sqrt{2}}\left(v_1 + \phi_1^0 - \mathrm{i}\gamma_1^0\right) \\ -\phi_1^- \end{pmatrix} \quad \text{and} \quad \mathcal{H}_2 = \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}}\left(v_2 + \phi_2^0 - \mathrm{i}\gamma_2^0\right) \end{pmatrix},$$

$\Rightarrow$ 8 bosonic degrees of freedom:

3 Goldstone bosons, 5 physical Higgs bosons,

at tree level:

Goldstone bosons	$G^0, G^\pm$,
physical CP even bosons	h^0, H^0,
physical CP odd bosons	A^0,
physical charged Higgs bosons	$H^\pm$.

Higgs potential

- Higgs potential:

$$V_{\text{Higgs}} = m_1^2 h_1^\dagger h_1 + m_2^2 h_2^\dagger h_2 - m_{12}^2 \epsilon_{ab} \left(h_1^a h_2^b + h_1^{a\dagger} h_2^{b\dagger} \right) + \frac{1}{8}\left(g_1^2 + g_2^2\right)\left(h_2^\dagger h_2 - h_1^\dagger h_1\right)^2 + \frac{1}{2} g_2^2 h_1^\dagger h_1 h_2^\dagger h_2,$$

- tree-level masses correlated:

$$m_{H^0,h^0}^2 = \frac{1}{2}\left(m_{A^0}^2 + m_Z^2 \pm \sqrt{\left(m_{A^0}^2 + m_Z^2\right)^2 - \left(2 m_Z m_{A^0} \cos 2\beta\right)^2} \right),$$

$$m_{H^\pm}^2 = m_{A^0}^2 + m_W^2,$$

- two free parameters: conventionally $\tan\beta = \frac{v_2}{v_1}$, m_{A^0},
- theoretical upper bound: $m_{h^0}^2 \leq \left(m_Z \cos 2\beta\right)^2$,
  but: significiant shift by higher orders.

Higgs boson masses at higher orders

$$\mathcal{M}^2 = \begin{pmatrix} m_{h^0}^2 - \hat{\Sigma}_{h^0h^0} & -\hat{\Sigma}_{h^0H^0} & -\hat{\Sigma}_{h^0A^0} \\ -\hat{\Sigma}_{H^0h^0} & m_{H^0}^2 - \hat{\Sigma}_{H^0H^0} & -\hat{\Sigma}_{H^0A^0} \\ -\hat{\Sigma}_{A^0h^0} & -\hat{\Sigma}_{A^0H^0} & m_{A^0}^2 - \hat{\Sigma}_{A^0A^0} \end{pmatrix},$$

(most general case also includes longitudinal G^0 and Z),

Higgs boson masses at higher orders

$$\mathcal{M}^2 = \begin{pmatrix} m_{h^0}^2 - \hat{\Sigma}_{h^0h^0} & -\hat{\Sigma}_{h^0H^0} & -\hat{\Sigma}_{h^0A^0} \\ -\hat{\Sigma}_{H^0h^0} & m_{H^0}^2 - \hat{\Sigma}_{H^0H^0} & -\hat{\Sigma}_{H^0A^0} \\ -\hat{\Sigma}_{A^0h^0} & -\hat{\Sigma}_{A^0H^0} & m_{A^0}^2 - \hat{\Sigma}_{A^0A^0} \end{pmatrix},$$

(most general case also includes longitudinal G^0 and Z),

renormalized self-energies $\hat{\Sigma}$:

$$\hat{\Sigma} = \Sigma\left(p^2\right) - \delta m^2 = \hat{\Sigma}\left(p^2\right),$$

$$\hat{\Sigma}\left(p^2\right) = \hat{\Sigma}^{\text{one loop}}\left(p^2\right) + \hat{\Sigma}^{\text{two loop}}\left(p^2\right) + \ldots$$

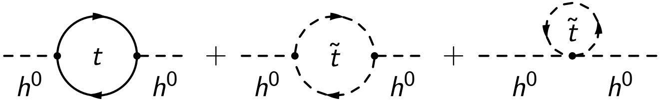
new mass eigenstates with eigenvalues:

$$\det\left(p^2\mathbf{1} - \mathcal{M}^2\left(p^2\right)\right) = 0.$$

one-loop corrections

- main contributions come from t and $\tilde{t}$ loops;
  order α_t, but proportional to m_t^4:

$\Sigma_{hh} =$



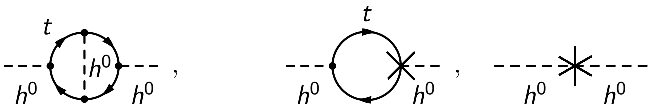
,

- additional parameters: $m_{\tilde{t}_1}, m_{\tilde{t}_2}, A_t, \mu$,
  complex case: ϕ_{A_t}, ϕ_μ, mixing of h^0, H^0, A^0,
- mass contribution: up to 40% of tree-level result,
- higher-order corrections necessary.

most important parts:
corrections to m_t-enhanced one-loop contributions
in the gauge-less limit,

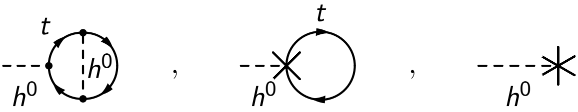
- corrections by gluons and gluinos in complex MSSM, order $\alpha_t \alpha_s$ in on-shell scheme,
  [Heinemeyer, Hollik, Rzehak, Weiglein, arXiv:hep-ph/0705.0746, 2007],
- corrections by Higgs and Higgsinos in real MSSM in effective potential approach, order α_t^2 in $\overline{\text{DR}}$ scheme,
  [Brignole, Degrassi, Slavich, Zwirner, arXiv:hep-ph/0112177, 2002],
- corrections by Higgs and Higgsinos in complex MSSM, this talk, [Hollik, SP, arXiv:1401.8275 [hep-ph], 2014],
- corrections by gluons and gluinos in real MSSM, momentum dependent parts, [Borowka, Heinemeyer, Heinrich, Hollik, in preparation].

- again: enhancement by additional m_t^2,
- Feynman-diagrammatic approach:



$$(\text{two-loop}) + (\text{one-loop})^2, \quad (\text{one-loop}) \cdot (\delta^{(1)}), \quad (\delta^{(2)}) + (\delta^{(1)})^2,$$

- $(\delta^{(2)}) + (\delta^{(1)})^2$ acquired by renormalizing the Higgs potential, additional Feynman-diagrams necessary:



procedure of calculation

- creation of Feynman-diagrams and amplitudes with FeynArts, [Hahn, arXiv:hep-ph/0012260, 2001],
- applying approximations,
- reducing one-loop diagrams to master integrals with FormCalc, [Hahn, arXiv:hep-ph/0901.1528, 2009],
- reducing two-loop diagrams to master integrals with TwoCalc, [Weiglein, Scharf, Böhm, arXiv:hep-ph/9310358, 1993],
- creating counterterms from the Higgs potential,
- applying renormalisation scheme,
- evaluating renormalisation constants.

applied approximations

(similar as for $\alpha_t\alpha_s$ corrections)

1. gauge-less limit: $g_1 = 0$, $g_2 = 0$,
   - only Yukawa-couplings kept,
   - $m_W = 0$, $m_Z = 0$,
     $\Rightarrow m_{h^0} = 0$, $m_{G^0} = 0$, $m_{G^\pm} = 0$, $m_{H^0} = m_{H^\pm}$, $m_{A^0} = m_{H^\pm}$,
     $\Rightarrow m_{\tilde{\chi}_3^0} = |\mu|$, $m_{\tilde{\chi}_4^0} = |\mu|$, $m_{\tilde{\chi}_2^\pm} = |\mu|$,
     other Charginos and Neutralinos decouple,
2. external momentum equal to zero,
   - only two-loop vacuum diagrams $\Rightarrow$ analytical calculation,
   - renormalisation constants for genuine two-loop counterterms calculated at zero momentum,
3. bottom mass equal to zero,
   - no mixing in sbottom sector,
   - only $\tilde{b}_L$ contributes,
   - $m^2_{\tilde{b}_L} = m^2_{\tilde{t}_L} - m^2_t$.

renormalisation scheme

required renormalisation constants:

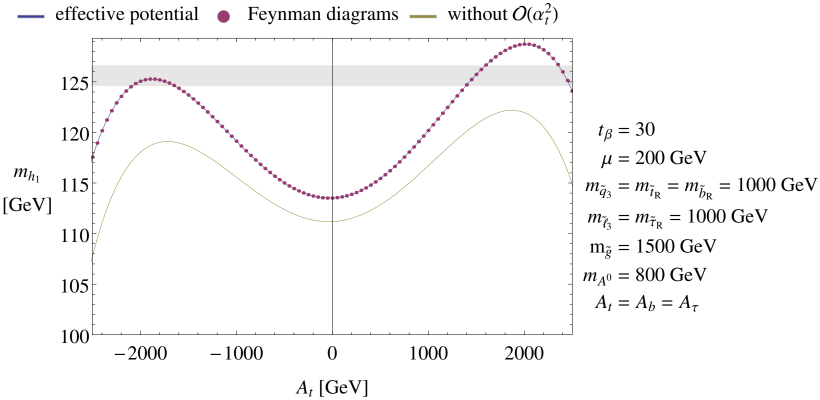
- δm_t, $\delta m_{\tilde{t}_1}$ and $\delta m_{\tilde{t}_2}$ on-shell or $\overline{\text{DR}}$,
- δA_t on-shell or $\overline{\text{DR}}$,
- $\delta\mu$ on-shell for $\tilde{\chi}_2^{\pm}$ or $\overline{\text{DR}}$,
- $\delta M_W/M_W$ and $\delta M_Z/M_Z$ on-shell,
- Higgs field renormalisation constants $\delta Z_{\mathcal{H}_1}\big|_{\text{div.}}$ and $\delta Z_{\mathcal{H}_2}\big|_{\text{div.}}$ $\overline{\text{DR}}$,
- $\delta \tan\beta = \frac{\tan\beta}{2}(\delta Z_{H_2} - \delta Z_{H_1})\big|_{\text{div.}}$ $\overline{\text{DR}}$,
- tadpoles $\delta t_{h^0}, \delta t_{H^0}, \delta t_{A^0}$ on-shell,
- $\delta m_{H^\pm}$ on-shell,

numerical agreement with existing result for real parameters, example:

$$M_2 = 200 \text{ GeV}, \; M_1 = \left(5 s_{\mathrm{w}}^2\right) / \left(3 c_{\mathrm{w}}^2\right) M_2 \,,$$

$$m_{\tilde{l}_{\mathrm{L}}} = m_{\tilde{q}_{\mathrm{L}}} = m_{\tilde{l}_{\mathrm{R}}} = m_{\tilde{q}_{\mathrm{R}}} = 2000 \text{ GeV}, \text{ for first two generations},$$

$$m_t = 173.2 \text{ GeV}, \; \alpha_s = 0.118.$$

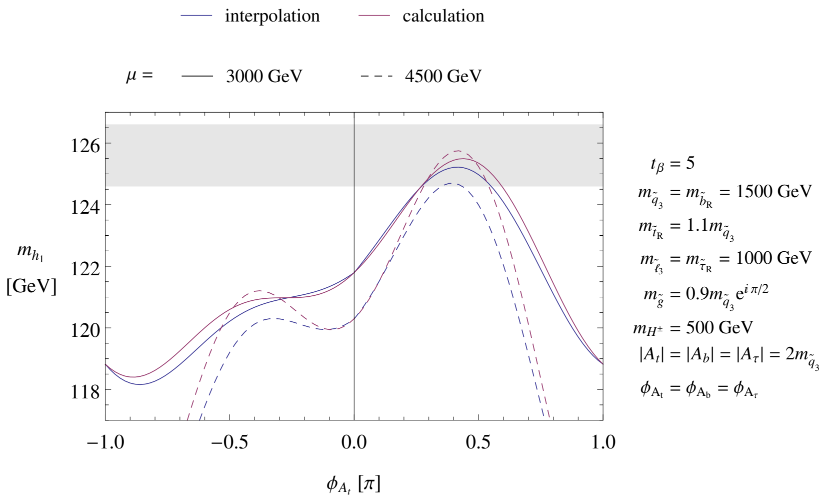


numerical results

$M_2 = 200$ GeV, $M_1 = \left(5s_w^2\right) / \left(3c_w^2\right) M_2$,

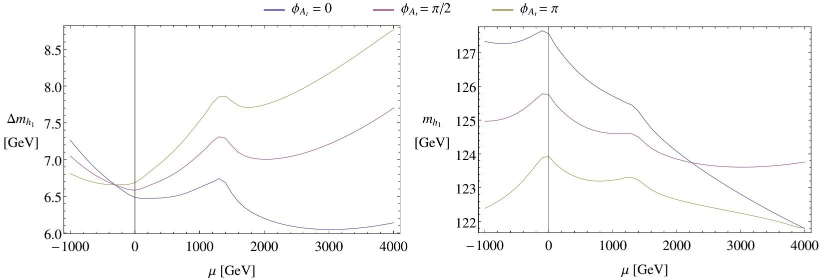
$m_{\tilde{l}_L} = m_{\tilde{q}_L} = m_{\tilde{l}_R} = m_{\tilde{q}_R} = 2000$ GeV, for first two generations,

$m_t = 173.2$ GeV, $\alpha_s = 0.118$.



dependence on μ for different phases ϕ_{A_t}:

$$M_2 = 200 \text{ GeV}, M_1 = \left(5s_{\rm w}^2\right) / \left(3c_{\rm w}^2\right) M_2 \ ,$$
$$m_{\tilde{l}_{\rm L}} = m_{\tilde{q}_{\rm L}} = m_{\tilde{l}_{\rm R}} = m_{\tilde{q}_{\rm R}} = 2000 \text{ GeV}, \text{ for first two generations},$$
$$m_t = 173.2 \text{ GeV}, \alpha_s = 0.118,$$
$$t_\beta = 7, m_{\tilde{g}} = 1500 \text{ GeV}, m_{H^\pm} = 500 \text{ GeV},$$
$$m_{\tilde{q}_3} = m_{\tilde{t}_{\rm R}} = m_{\tilde{b}_{\rm R}} = 1500 \text{ GeV}, m_{\tilde{l}_3} = m_{\tilde{\tau}_{\rm R}} = 1000 \text{ GeV},$$
$$A_t = A_b = A_\tau = 1.6 m_{\tilde{q}_3} \,,$$

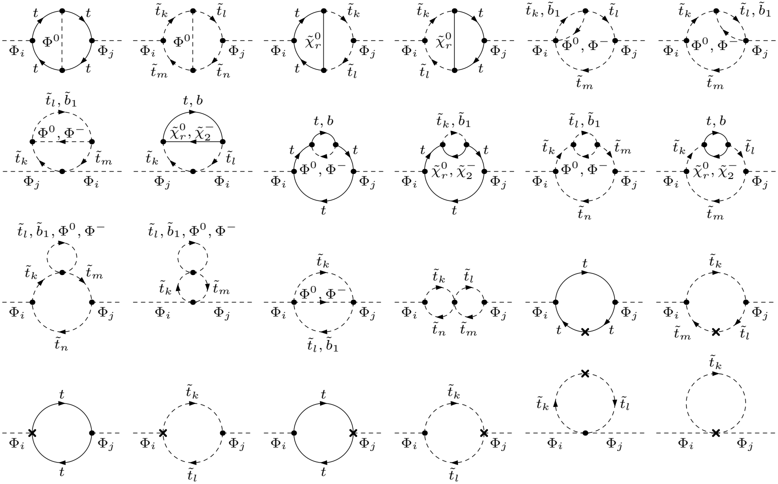


outlook

- comparison of analytic results in the limit of real parameters,
- investigation of full Higgs-boson spectrum,
- implementation into FeynHiggs,

  [Hahn, Heinemeyer, Hollik, Rzehak, Weiglein, arXiv:hep-ph/1007.0956, 2010],
- investigation of the momentum dependent parts.

Feynman diagrams



full list of two-loop self-energies for neutral Higgs bosons;
a cross denotes a one-loop counterterm insertion;
$\Phi_i = h, H, A$; $\Phi^0 = h, H, A, G$; $\Phi^- = H^-, G^-$.

shift by Δr

top-Yukawa coupling $h_t = m_t/v_2 = m_t/(v s_\beta)$ expressed in terms of

$$\frac{1}{v} = \frac{g_2}{\sqrt{2}M_W} = \frac{e}{\sqrt{2}M_W s_\mathrm{w}},$$

also convenient to use Fermi constant G_F for parametrization of one-loop self-energies:

$$\sqrt{2}G_\mathsf{F} = \frac{e^2}{4 s_\mathrm{w}^2 M_W^2}\left(1 + \Delta^{(k)} r\right),$$

in the gaugeless limit at one-loop order:

$$\Delta^{(1)} r = -\frac{c_\mathrm{w}^2}{s_\mathrm{w}^2}\left(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2}\right) = -\frac{\delta s_\mathrm{w}^2}{s_\mathrm{w}^2}\,.$$

input sectors

squarks:

$$\mathbf{M}_{\tilde{q}} = \begin{pmatrix} m_{\tilde{q}_\mathrm{L}}^2 + m_q^2 + M_Z^2 c_{2\beta}(T_q^3 - Q_q s_\mathrm{w}^2) & m_q \left(A_q^* - \mu\kappa_q\right) \\ m_q \left(A_q - \mu^*\kappa_q\right) & m_{\tilde{q}_\mathrm{R}}^2 + m_q^2 + M_Z^2 c_{2\beta} Q_q s_\mathrm{w}^2 \end{pmatrix},$$

$$\kappa_t = \frac{1}{t_\beta}, \quad \kappa_b = t_\beta,$$

neutralinos:

$$\mathbf{Y} = \begin{pmatrix} M_1 & 0 & -M_Z s_\mathrm{w} c_\beta & M_Z s_\mathrm{w} s_\beta \\ 0 & M_2 & M_Z c_\mathrm{w} c_\beta & M_Z c_\mathrm{w} s_\beta \\ -M_Z s_\mathrm{w} c_\beta & M_Z c_\mathrm{w} c_\beta & 0 & -\mu \\ M_Z s_\mathrm{w} s_\beta & M_Z c_\mathrm{w} s_\beta & -\mu & 0 \end{pmatrix},$$

charginos:

$$\mathbf{X} = \begin{pmatrix} M_2 & \sqrt{2} M_W s_\beta \\ \sqrt{2} M_W c_\beta & \mu \end{pmatrix}.$$