Two-Loop Corrections to the Muon Magnetic Moment from Fermion/Sfermion Loops in the MSSM

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in collaboration with

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arXiv:1309.0980 [hep-ph], arXiv:1311.1775 [hep-ph].

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content

What is a_μ?

experimentally:

low energetic muon in an external homogeneous magnetic field:

- circular motion due to the electrical charge,
  frequency: $\omega_\mathrm{c} = -\frac{e}{m}B$,
- precession due to the spin,
  frequency: $\omega_\mathrm{s} = -\frac{e}{m}B \cdot \frac{g}{2}$,

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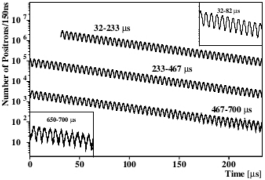
experimentally accesible: $$\omega_\mathrm{a} := \omega_\mathrm{s} - \omega_\mathrm{c} = -\frac{e}{m}B \cdot \frac{g-2}{2},$$

measurement: $$\omega_\mathrm{a} \neq 0,$$

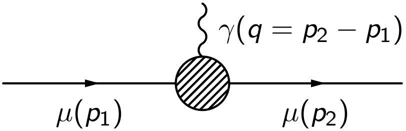
anomalous magnetic moment: $$\Rightarrow a_\mu := \frac{g-2}{2},$$



[H. N. Brown, et. al., P. R. L. 86 (2001)]

What is a_μ?

theoretically:



$=$

$$i e \bar{u}(p_2) \left[\gamma^\mu F_\mathrm{E}(q^2) + i \frac{\sigma^{\mu\nu} q_\nu}{2 m_\mu} F_\mathrm{M}(q^2) \right] u(p_1),$$

$$F_\mathrm{M}(0) \equiv a_\mu,$$

tree-level: $F_\mathrm{M}(0) = 0,$

$\Rightarrow a_\mu$ induced by higher order corrections

contributions to a_μ in the Standard Model

QED	$(11\,658\,471.896 \pm 0.008) \cdot 10^{-10}$
[T. Aoyama, M. Hayakawa, T. Kinoshita and M. Nio, arXiv:1205.5370 (2012)]	
hadronic vac. pol., leading order	$(687.53 \pm 4.34) \cdot 10^{-10}$
[M. Benayoun, P. David, L. DelBuono, F. Jegerlehner (2012)]	
hadronic vac. pol., higher order	$(-9.97 \pm 0.09) \cdot 10^{-10}$
[F. Jegerlehner and R. Szafron (2011)]	
hadronic light by light	$(10.5 \pm 2.6) \cdot 10^{-10}$
[J. Prades, E. de Rafael, A. Vainshtein, Advanced Series on Directions in High Energy Physics – Vol. 20 (2009)]	
electroweak	$(15.36 \pm 0.10) \cdot 10^{-10}$
[C. Gnendiger, D. Stöckinger and H. Stöckinger-Kim, arXiv:1306.5546 [hep-ph]]	

theory: $a_\mu^{\text{SM}} = (11\,659\,175.3 \pm 5.1) \cdot 10^{-10}$

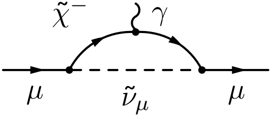
experiment: $a_\mu^{\text{BNL}} = (11\,659\,208.9 \pm 6.3) \cdot 10^{-10}$

[G. W. Bennett et al., Physical Review D 73 (2006)],[A. Hoecker, W. Marciano, Journal of Physics G 37 (2010)],

$$\Rightarrow a_\mu^{\text{BNL}} - a_\mu^{\text{SM}} = (33.6 \pm 8.1) \cdot 10^{-10} \quad (4.2\sigma)$$

MSSM contributions to a_μ: one-loop corrections

loops with Chargino $\tilde{\chi}^+$ or Neutralino $\tilde{\chi}^0$ and corresponding slepton



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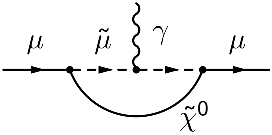
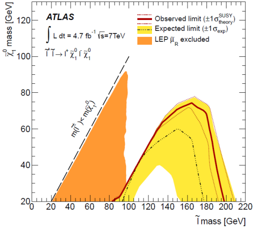


,

$$a_\mu^{\tilde{\chi}_i} = \frac{e^2}{16\pi^2 s_\mathrm{w}^2} \frac{m_\mu^2}{m_{\tilde{\ell}}^2}$$

$$\times \left[F_1(x_i) C_1^{\tilde{\chi}_i} + m_{\tilde{\chi}_i} F_2(x_i) C_2^{\tilde{\chi}_i} \right],$$



[arXiv:1208.2884v2]

with $C_j^{\tilde{\chi}_i}$: containing all couplings, some parts $\propto \mathrm{sgn}(\mu) \tan\beta$,

F_i : formfactors, depending on $x_i = \dfrac{m_{\tilde{\chi}_i}^2}{m_{\tilde{\ell}}^2}$,

MSSM contributions to a_μ: two-loop corrections

1. MSSM corrections to Standard Model diagrams,

   [S. Heinemeyer, D. Stöckinger, G. Weiglein, Nuclear Physics B 690 (2004)]

   [S. Heinemeyer, D. Stöckinger, G. Weiglein, Nuclear Physics B 699 (2004)]

2. corrections to diagrams with supersymmetric particles:
   - photonic corrections,

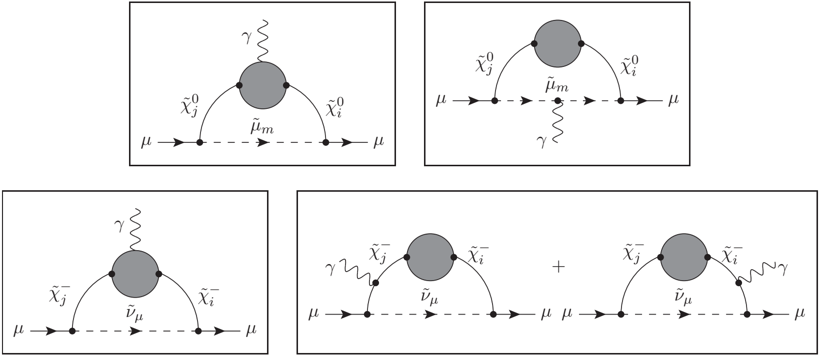
     [G. Degrassi, G. F. Giudice, Physical Review D 58 (1998)],

     [P. von Weitershausen, M. Schäfer, H. Stöckinger-Kim, D. Stöckinger, Phys.Rev. D81 (2010)]
   - corrections with SM fermions and superpartners: this talk,

     [H. Fargnoli, C. Gnendiger, SP, D. Stöckinger, H. Stöckinger-Kim, Phys. Lett. B726 (2013)],

     [H. Fargnoli, C. Gnendiger, SP, D. Stöckinger, H. Stöckinger-Kim, JHEP 1402 (2014)],

two-loop corrections with closed fermion–sfermion–loop



calculated by using two different methods:

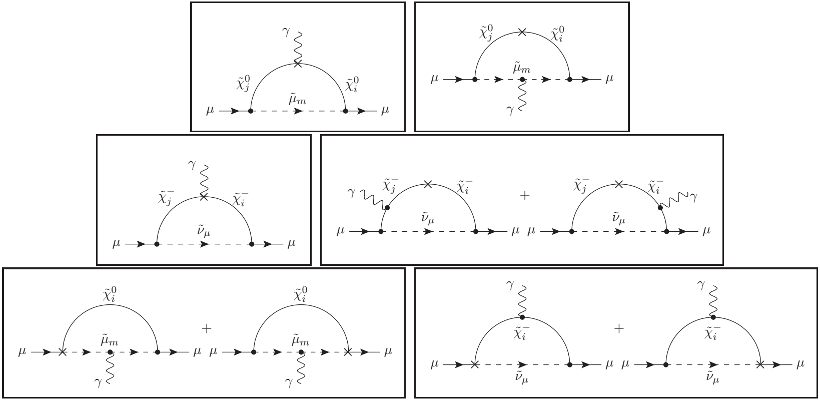
- usual two-loop evaluation,

  [S. Heinemeyer, D. Stöckinger and G. Weiglein, Nucl. Phys. B 690 (2004)]

- special method for Barr–Zee type diagrams
  (first compute inner one-loop; get integral representation)

  [S. M. Barr and A. Zee, Phys. Rev. Lett. 65 (1990)]

renormalization and counterterm contributions to a_μ



first two lines: absorbing divergencies,
last line: finite.

full analytic results: [arXiv:1311.1775 [hep-ph]]

renormalization constants

required renormalization constants:

- mass corrections to W and Z bosons: δM_W^2, δM_Z^2,
- mass corrections to μ, M_1 and M_2: $\delta\mu$, δM_1, δM_2,
- field corrections to γ and γ–Z–mixing: δZ_γ, $\delta Z_{Z\gamma}$,
- field corrections to Higgs fields $\mathcal{H}_1$ and $\mathcal{H}_2$: $\delta Z_{\mathcal{H}_1}$, $\delta Z_{\mathcal{H}_2}$, necessary for $\delta t_\beta = \frac{1}{2} t_\beta \left(\delta Z_{\mathcal{H}_2} - \delta Z_{\mathcal{H}_1}\right)$,

on-shell renormalization of the MSSM as far as possible:

- SM particles on-shell
- both Charginos and the lightest Neutralino on-shell
- $\overline{\text{DR}}$ definition of the Higgs renormalization constants (i. e. $\overline{\text{DR}}$ definition of $\tan\beta$)

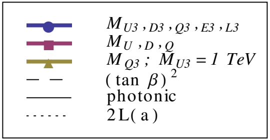
numerical results

benchmark scenarios:

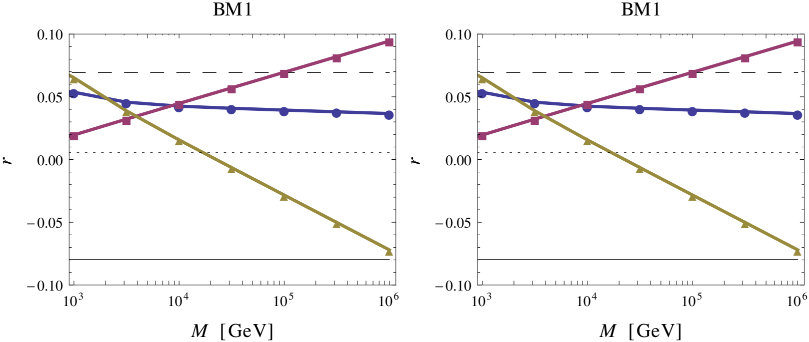
BM1: all one-loop masses similar,
BM4: M_2, M_L heavy, M_1, M_E light, μ negative,

	BM1	BM4
t_β	40	50
μ [GeV]	350	-160
M_1 [GeV]	150	140
M_2 [GeV]	300	2000
M_E [GeV]	400	200
M_L [GeV]	400	2000
$a_\mu^{1L,SUSY}[10^{-11}]$	440.2	159.8

comparison with known 2L contributions

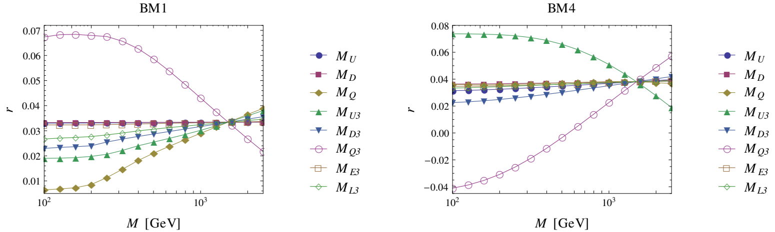


three different scenarios for running sfermion masses, standard value: $M = 1.5$ TeV.



$$r = a_\mu^{2L} \Big/ a_\mu^{1L,SUSY}$$

sfermion masses

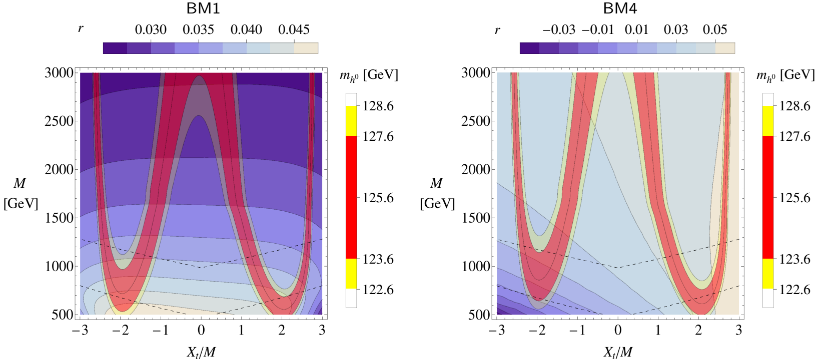


$$r = a_\mu^{2\text{L},f\tilde{f}} \Big/ a_\mu^{1\text{L,SUSY}}$$

standard value for sfermion masses: $M = 1.5$ TeV,
mixing parameters: $A_f = 0$,

logarithmic behavior for large masses,
threshold around 500 GeV.

stop mixing



$$r = a_\mu^{2\text{L},f\tilde{f}} \Big/ a_\mu^{1\text{L,SUSY}}\,,\; M = M_{Q3} = M_{U3} = M_{D3},\, X_t = A_t^* - \mu \Big/ t_\beta\,,$$

dashed lines: $m_{\tilde{t}_1} < 1000$ GeV (500 GeV),
other sfermion masses: $M = 1.5$ TeV,

Higgs mass symmetric for $X_t \leftrightarrow -X_t$, $a_\mu \approx$ linear in X_t.

summary

conclusions:

- two-loop corrections with a closed fermion–sfermion–loop have been evaluated analytically in the Feynman-diagrammatic approach
- logarithmically enhanced contributions by heavy sfermions
- possibly strongest two-loop contributions in the MSSM

outlook:

- full two-loop calculation in progress,

$\Rightarrow$ error on a_μ^{MSSM} below $1 \cdot 10^{-10}$

$\Rightarrow$ new experiment/more precise Standard Model prediction become more valuable (stronger restrictions on MSSM parameter space)

shift by $\Delta\alpha$

a_μ proportional to α at one-loop level,

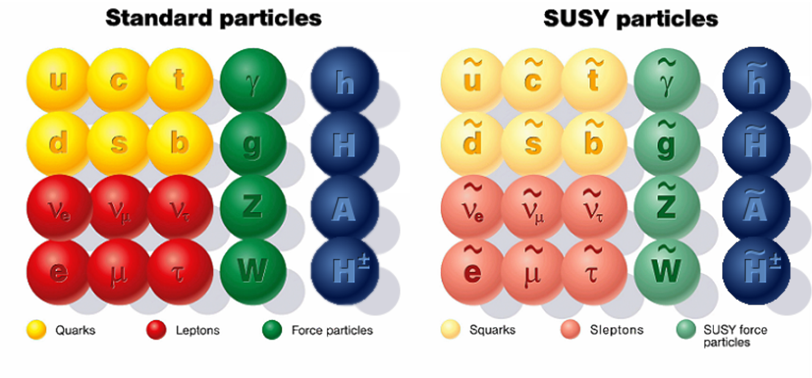
considered class of two-loop corrections: renormalization of α from photon vacuum polarization including light quarks,
α in the Thomson limit $\Rightarrow$ large intrinsic uncertainties,

choose $\alpha(M_Z) = \frac{\alpha(0)}{1-\Delta\alpha(M_Z)}$, $\Delta\alpha$ is measured,

renormalization: $\delta Z_\gamma = -\frac{1}{M_Z^2}\Re\left[\Sigma_\gamma^f(M_Z^2)\right] - \Re\left[\partial_{p^2}\Sigma_\gamma^{\text{others}}(p^2)\right]$.

MSSM content

extension of the SM by Supersymmetry



input sectors

squarks:

$$\mathbf{M}_{\tilde{q}} = \begin{pmatrix} m_{\tilde{q}_\mathrm{L}}^2 + m_q^2 + M_Z^2 c_{2\beta}(T_q^3 - Q_q s_\mathrm{w}^2) & m_q \left(A_q^* - \mu\kappa_q\right) \\ m_q \left(A_q - \mu^*\kappa_q\right) & m_{\tilde{q}_\mathrm{R}}^2 + m_q^2 + M_Z^2 c_{2\beta} Q_q s_\mathrm{w}^2 \end{pmatrix},$$

$$\kappa_t = \frac{1}{t_\beta}, \quad \kappa_b = t_\beta,$$

neutralinos:

$$\mathbf{Y} = \begin{pmatrix} M_1 & 0 & -M_Z s_\mathrm{w} c_\beta & M_Z s_\mathrm{w} s_\beta \\ 0 & M_2 & M_Z c_\mathrm{w} c_\beta & M_Z c_\mathrm{w} s_\beta \\ -M_Z s_\mathrm{w} c_\beta & M_Z c_\mathrm{w} c_\beta & 0 & -\mu \\ M_Z s_\mathrm{w} s_\beta & M_Z c_\mathrm{w} s_\beta & -\mu & 0 \end{pmatrix},$$

charginos:

$$\mathbf{X} = \begin{pmatrix} M_2 & \sqrt{2} M_W s_\beta \\ \sqrt{2} M_W c_\beta & \mu \end{pmatrix}.$$