Momentum dependent two-loop contributions to the neutral CP-even Higgs boson masses in the rMSSM

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IMPRS
EPP

Projects in collaboration with G. Heinrich & W. Hollik

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Comput.Phys.Commun. 184 396-408

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http://secdec.hepforge.org/

Official since July 2012: "We have it!"

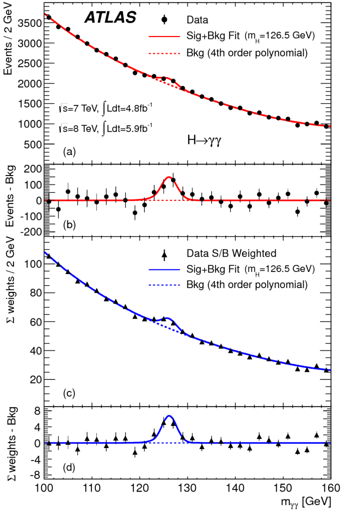
- But which one?

  Standard Model Higgs,
  supersymmetric Higgs,
  composite Higgs, ...

- Assumption: The found Higgs boson is just one out of several

$$H_1 = \begin{pmatrix} v_1 + \frac{1}{\sqrt{2}}(\phi_1^0 + i\chi_1^0) \\ -\phi_1^- \end{pmatrix}$$

$$H_2 = \begin{pmatrix} \phi_2^+ \\ v_2 + \frac{1}{\sqrt{2}}(\phi_2^0 + i\chi_2^0) \end{pmatrix}$$



Higgs potential in the MSSM

Further constraints on a 2 Higgs doublet model are given in the MSSM

- MSSM Higgs potential (incl. soft SUSY breaking terms):

$$
\begin{aligned}
V =& m_1|H_1|^2 + m_2|H_2|^2 - m_{12}(\epsilon_{ab}H_1^a H_2^b + h.c.) \\
&+ \frac{1}{8}(g_1^2 + g_2^2)(|H_1|^2 - |H_2|^2)^2 + \frac{1}{2}g_2^2|H_1^\dagger H_2|^2
\end{aligned}
$$

- m_1, m_2 and m_{12}: soft SUSY breaking mass terms
- MSSM Higgs potential is fixed by the SM gauge couplings g_1, g_2, the v.e.v.s in $\tan\beta = \frac{v2}{v1}$ and the soft SUSY breaking term in $M_A^2 = m_{12}^2(\tan\beta + \cot\beta)$

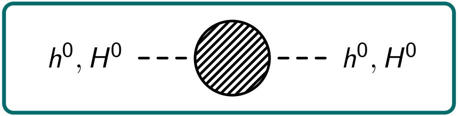
The neutral $\mathcal{CP}$-even Higgs boson masses

The tree-level neutral $\mathcal{CP}$-even Higgs boson masses

$$M^{2,\text{tree}}_{\text{Higgs}} = \begin{pmatrix} M_A^2 \sin^2\beta + M_Z^2 \cos^2\beta & -(M_A^2 + M_Z^2)\sin\beta\cos\beta \\ -(M_A^2 + M_Z^2)\sin\beta\cos\beta & M_A^2\cos^2\beta + M_Z^2\sin^2\beta \end{pmatrix}$$

are limited to $m_h \leq \min(M_Z, M_A)\,|\cos(2\beta)|$

$\Rightarrow$ We are interested in higher order self-energy corrections to the Higgs boson masses



$\Rightarrow$ These lead to maximal values for $m_{h_{max}} \approx 130$ GeV

Status: Radiative corrections in the real MSSM

Higher-order corrections to the Higgs mass sector in the rMSSM:

1-loop 2-loop 3-loop

- Ellis, Ridolfi, Zwirner '91; Okada, Yamaguchi, Yanagida '91; Haber & Hempfling '91; Brignole '92; Chankowski, Pokorski, Rosiek '92 '94; Dabelstein '95
- Hempfling & Hoang '94; Carena et al. '95 '96; Espinosa et al. '95 '00 '01; Heinemeyer, Hollik, Weiglein et al. '98 '99 '99 '00; Zhang '99; Degrassi, Slavich et al. '01 '03; Brignole, Degrassi, Slavich, Zwirner '02; Heinemeyer, Hollik, Rzehak, Weiglein '05; S. P. Martin '02 '03 '04 '05
- S.P. Martin '07; Harlander, Kant, Mihaila, Steinhauser '08 '10

Many more publications...

Public codes implementing the rMSSM corrections

FeynHiggs Frank, Hahn, Heinemeyer, Hollik, Rzehak, Weiglein '00 '03 '07

SoftSusy Allanach '02 SPheno Porod '03

CPsuperH Carena, Choi, Drees, Ellis, Lee, Pilaftsis, Wagner '04 '09

Suspect Djouadi, Kneur, Moultaka '07

H3m Kant, Harlander, Mihaila, Steinhauser '10

Summary of the implemented rMSSM corrections:

1-loop complete

2-loop $\mathcal{O}(\alpha_s\alpha_t)$, $\mathcal{O}(\alpha_t^2)$, $\mathcal{O}(\alpha_s\alpha_b)$, $\mathcal{O}(\alpha_t\alpha_b)$, $\mathcal{O}(\alpha_b^2)$, gaugeless limit and $p^2 = 0$

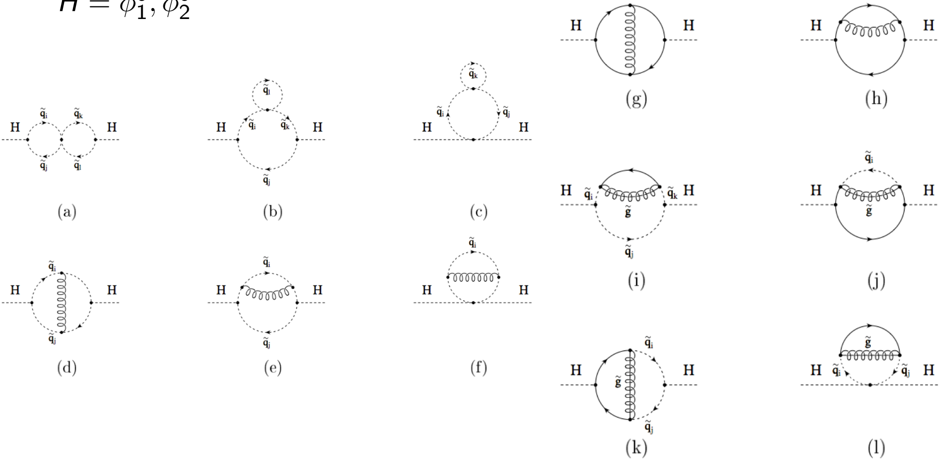
3-loop $\mathcal{O}(\alpha_s^2\alpha_t)$, gaugeless limit and $p^2 = 0$

dominant correction @ 2-loop: $\mathcal{O}(\alpha_s\alpha_t)$ $(p^2 = 0)$

$\longrightarrow$ **next improvement:** $\mathcal{O}(\alpha_s\alpha_t)$ for $p^2 \neq 0$

Higgs boson self-energy diagrams for $\mathcal{O}(\alpha_s\alpha_t)$

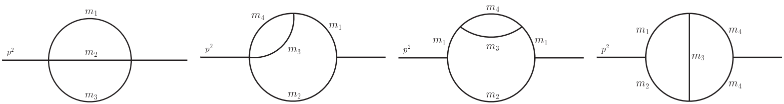
$H = \phi_1^0, \phi_2^0$



Treatment of loop integrals

Tensor reduction to only scalar integrals with the TwoCalc package possible G. Weiglein et al. '93

- Many of the resulting integrals (mainly 1-loop) are known analytically
- Full analytic results unknown for 4 different two-loop topologies



T_{234} T_{1234} T_{11234} T_{12345}

- These integrals are treated numerically with SecDec

The program SecDec 2.1

Idea and method of sector decomposition pioneered by
Hepp '66, Denner & Roth '96, Binoth & Heinrich '00

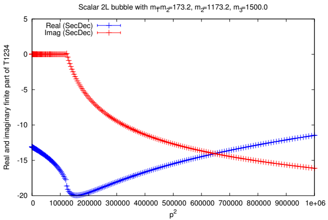
SecDec is a tool to numerically compute various sorts of integrals contributing to higher-order computations.

It can tackle:

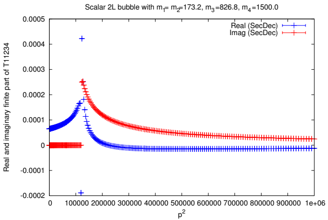
- General Feynman loop integrals, contracted **tensor integrals** to in principle arbitrary rank
- More general user-defined polynomial / parametric functions
- Since SecDec 2.0: Extension to general kinematics including mass thresholds! SB, Carter & Heinrich '12
- Fast evaluation of diverse integral topologies
- Widely used for numerical calculations/checks

Numerical evaluation of momentum dependent integrals

- 37 mass configurations are run with SecDec, e.g.



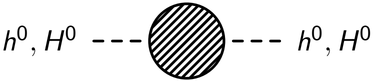
T_{1234}, finite part



T_{11234}, finite part

- differences of kinematic invariants of up to 14 orders of magnitude
- rel. accuracy better than 10^{-5}, timings range from $0.01 - 100$secs

Two-loop renormalization for neutral $\mathcal{CP}$-even Higgs-boson self-energies



Feynman diagrammatic calculation performed in the gaugeless limit

- Renormalization procedure same as the one used in FeynHiggs
- Mass renormalization in the OS scheme:

$$\delta M_A^{2(2)},\ \delta t_1^{(2)},\ \delta t_2^{(2)},\ \delta m_{\tilde{t}_1}^{(1)},\ \delta m_{\tilde{t}_2}^{(1)},\ \delta m_t^{(1)}$$

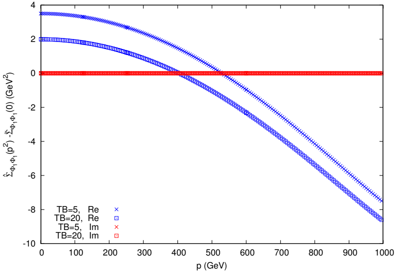
- Field renormalization in the $\overline{DR}$ scheme:

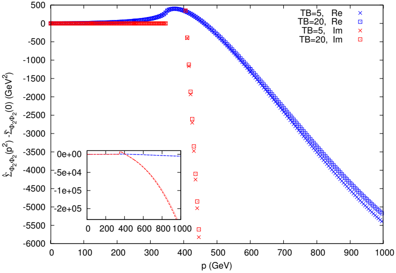
$$\delta Z_{H_1}^{(2)},\ \delta Z_{H_2}^{(2)},\ \delta \tan\beta^{(2)} = \frac{1}{2}\tan\beta\,(\delta Z_{H_2}^{(2)} - \delta Z_{H_1}^{(2)})$$

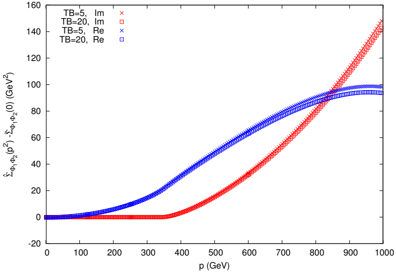
- Resulting input parameters: $m_t, \mu, X_t, M_{SUSY}, m_{\tilde{g}}, \tan\beta, m_A$

  $X_t = A_t - \mu\cot\beta$ and A_t the soft SUSY breaking parameters

Momentum dependence of the 2L self-energies







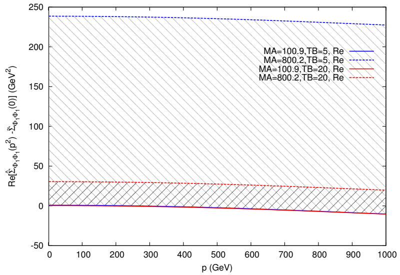
m_h^{max} benchmark scenario:

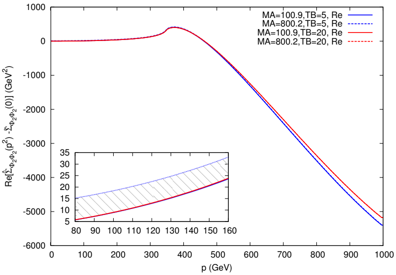
$m_t = 173.2\text{GeV},\ \mu = 200\text{GeV},$
$X_t = 2M_{SUSY},\ M_{SUSY} = 1\text{TeV},$
$m_{\tilde{g}} = 1.5\text{TeV},\ m_A = 250\text{GeV},$

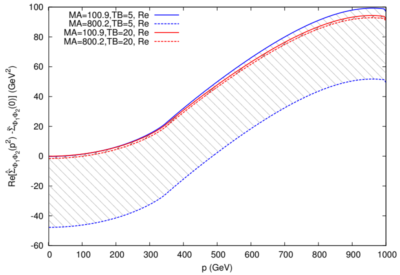
$\tan\beta = 5, 20$

(preliminary)

SUSY parameter dependence







m_h^{max} benchmark scenario:

$m_t = 173.2\text{GeV}$, $\mu = 200\text{GeV}$,
$X_t = 2M_{SUSY}$, $M_{SUSY} = 1\text{TeV}$,
$m_{\tilde{g}} = 1.5\text{TeV}$
$m_A, \tan\beta$ varied

(preliminary)

Corrections to the neutral $\mathcal{CP}$-even MSSM Higgs-boson masses

- The new self-energy corrections are included in the inverse Higgs-boson propagator matrix

$$\Gamma \equiv \Delta^{-1}_{\text{Higgs}} = \begin{pmatrix} p^2 - m^2_{H,\text{tree}} + \hat{\Sigma}_H(p^2) & \hat{\Sigma}_{hH}(p^2) \\ \hat{\Sigma}_{hH}(p^2) & p^2 - m^2_{h,\text{tree}} + \hat{\Sigma}_h(p^2) \end{pmatrix}$$

  with renormalized self-energies $\hat{\Sigma}$ up to the two-loop level
- The propagator poles m_H^2 and m_h^2 are solutions to $\text{Det}(\Gamma) = 0$
- correction to light Higgs: $-160\ \text{MeV} \lesssim \delta^{(2)} m_h \lesssim 10\ \text{MeV}$
  correction to heavy Higgs: $-260\ \text{MeV} \lesssim \delta^{(2)} m_H \lesssim 140\ \text{MeV}$

Summary & Outlook

Summary

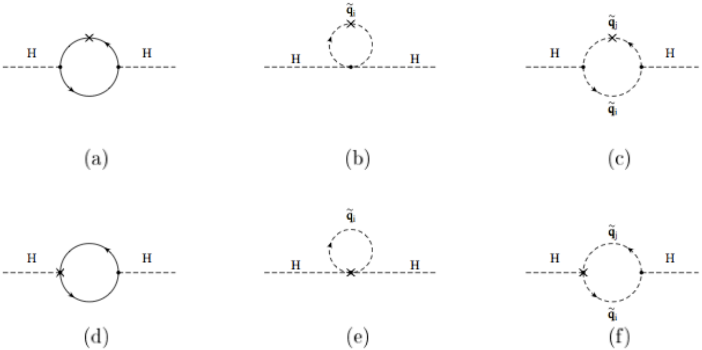
- We computed the momentum dependent 2-loop corrections to the $\mathcal{CP}$-even MSSM Higgs boson masses adopting the renormalization scheme used in FeynHiggs
- Numerical computation of integrals containing thresholds with the multi-feature program SecDec 2.1
- The light Higgs mass corrections are well below 1 GeV

Outlook

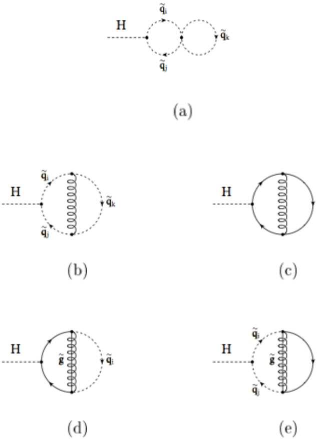
- Inclusion of the results into the public FeynHiggs code
- Further applications to other massive two-loop calculations using SecDec

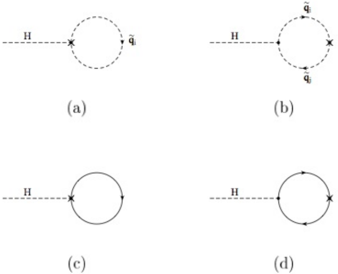
Backup

Diagrams for sub-loop renormalization I

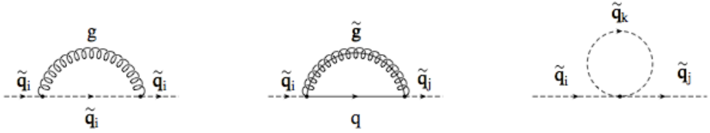


Tadpole diagrams needed in the renormalization





Counter term insertions for sub-loop renormalization



(a) (b) (c)

(d) (e)

Install SecDec 2.1

- **Download**:
  http://secdec.hepforge.org

- **Install**:
  tar xzvf SecDec.tar.gz
  cd SecDec-2.1
  ./install

- **Prerequisites**:
  Mathematica (version 6 or above), Perl, Fortran and/or C++ compiler

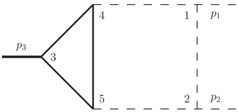
User Input I

- param.input: parameters for integrand specification and numerical integration

```
################ input parameters for sector decomposition ################
#----------------------------------------------------------------------------------------------------
# subdirectory for the mathematica output files (will be created if non-existent) :
# if not specified, a directory with the name of the graph given below will be created by default
subdir=2loop
#----------------------------------------------------------------------------------------------------
# if outputdir is not specified: default directory for
# the output will have integral name (given below) appended to directory above,
# otherwise specify full path for Mathematica output files here
outputdir=
#----------------------------------------------------------------------------------------------------
# graphname (can contain underscores, numbers, but should not contain commas)
graph=P126
#----------------------------------------------------------------------------------------------------
# number of propagators:
propagators=6
#----------------------------------------------------------------------------------------------------
# number of external legs:
legs=3
#----------------------------------------------------------------------------------------------------
# number of loops:
loops=2
#----------------------------------------------------------------------------------------------------
# construct integrand (F and U) via topological cuts (only possible for scalar integrals)
# default is 0 (no cut construction used)
cutconstruct=1
#########################################################
# parameters for subtractions and epsilon expansion
#########################################################
```

User Input II

- template.m: definition of the integrand (Mathematica syntax)




```
(******* USER INPUT for construction of integrand **************************)

(******* Use with cutconstruct=1 ********************************************)

proplist={{ms[1],{3,4}},{ms[1],{4,5}},{ms[1],{5,3}},
          {0,{1,2}},{0,{1,4}},{0,{2,5}}};

(******* Use with cutconstruct=0 ********************************************)
(*
momlist={k1,k2};
proplist={k1^2-ms[1],(k1+p3)^2-ms[1],(k1-k2)^2-ms[1],
         (k2+p3)^2,(k2+p1+p3)^2,k2^2};
numerator={1};
*)

(******* Propagator powers (optional) ***************************************)

powerlist=Table[1,{i,Length[proplist]}];

(******* On-shell conditions (optional) *************************************)
onshell={ssp[1]->0,ssp[2]->0,ssp[3]->sp[1,2],sp[1,3]->0,sp[2,3]->0};

(******* Set Dimension ******************************************************)

Dim=4-2*eps;
(*************************************************************************)
```

Program Test Run

- ./launch -p param.input -t template.m

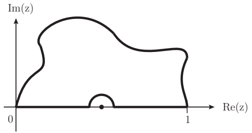
```
********** This is SecDec version 2.0 **********
Authors: Sophia Borowka, Jonathon Carter, Gudrun Heinrich
************************************************
graph = P126
primary sectors 1,2,3,4,5,6, will be calculated
calculating F and U . . .
done
written to /home/pcl335a/sborowka/Work/SecDecBeta/loop/2loop/P126/FUN.m

results of the decomposition will be written to
/home/pcl335a/sborowka/Work/SecDecBeta/loop/2loop/P126
doing sector decomposition . . .
done

working on pole structure: 2 logarithmic poles, 0 linear poles, 0 higher poles
C++ functions created for pole structure 2l0h0
compiling 2l0h0/epstothe0 ...
doing numerical integrations in P126/2l0h0/epstothe0
compiling 2l0h0/epstothe-1 ...
doing numerical integrations in P126/2l0h0/epstothe-1
compiling 2l0h0/epstothe-2 ...
doing numerical integrations in P126/2l0h0/epstothe-2
working on pole structure: 1 logarithmic pole, 0 linear poles, 0 higher poles
C++ functions created for pole structure 1l0h0
compiling 1l0h0/epstothe0 ...
doing numerical integrations in P126/1l0h0/epstothe0
compiling 1l0h0/epstothe-1 ...
doing numerical integrations in P126/1l0h0/epstothe-1
working on pole structure: 0 logarithmic poles, 0 linear poles, 0 higher poles
C++ functions created for pole structure 0l0h0
compiling 1l0h0/epstothe0 ...
doing numerical integrations in P126/0l0h0/epstothe0
Output written to /home/pcl335a/sborowka/Work/SecDecBeta/loop/2loop/P126/P126_pfull.res

To remove intermediate files, execute the command /home/pcl335a/sborowka/Work/SecDecBeta/loop/launchcleanP126
```

Get the Result

- resultfile P126_full.res

```
*************************************************
***OUTPUT: P126 p5 ***************
 point: 7.0
 ext. legs: 0.0 0.0 7.0
 prop. mass: 1.0 0. 0. 0. 0. 0.
Prefactor=-Exp[-2EulerGamma*eps]
*************************************************
****** eps^-2 coeff ******

result       =0.07563683
             +0.1003924148 I
error        =0.000493522517701388
             + 0.00139691015080074 I
CPUtime (all eps^-2 subfunctions)  =0.04
CPUtime (longest eps^-2 subfunction)  =0.01
.
.
.

****** eps^0 coeff ******

result       =0.906978296750816
             -0.908781551612644 I
error        =0.00754504726896407
             + 0.0442867373250588 I
CPUtime (all eps^0 subfunctions)  =2.44
CPUtime (longest eps^0 subfunction)  =0.51

*************************************************

Time taken for decomposition = 2.005725

Total time for subtraction and eps expansion = 41.5057 secs
Time taken for longest subtraction and eps expansion = 17.8613 secs
```

Deformation of the integration contour to integrate mass thresholds



- Integrand is analytically continued into the complex plane

$$\mathcal{F}(\vec{t}) \to \mathcal{F}(\vec{t} + \mathrm{i}\vec{y}(\vec{t})) = \mathcal{F}(\vec{t}) + \mathrm{i} \sum_j y_j(\vec{t}) \frac{\partial \mathcal{F}(\vec{t})}{\partial t_j} + \mathcal{O}(y(\vec{t})^2)$$

- The integration contour is deformed by

$$\vec{t} \to \vec{z} = \vec{t} + \mathrm{i}\vec{y} \ ,$$

$$y_j(\vec{t}) = -\lambda t_j (1 - t_j) \frac{\partial \mathcal{F}(\vec{t})}{\partial t_j}$$

Soper '99

Soper, Nagy, Binoth; Kurihara et al., Anastasiou et al., Freitas et al., Becker et al.