MHV amplitudes 0 00000 One gluon emission in DY

P

Conclusions

# Next-eikonal approximation in the context of spinor helicity methods

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Supervisors: Eric Laenen, Giovanni Ridolfi

### 28th IMPRS Workshop



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Conclusions

### **Motivations**

• Eikonal (E) and Next-to-eikonal (NE) exponentiation  $M = M^0 e^{M_E + M_{NE}} (1 + M_r) + \mathbf{O}(NNE)$ 

E and NE approximation for propagators

$$rac{1}{(p+q)^2} pprox rac{1}{2pq} - rac{q^2}{(2pq)^2} + \mathbf{O}(NNE)$$

 Helicity spinor methods are an efficient way to compute cross sections.

# Let's use both!

MHV amplitudes 0 00000 One gluon emission in DY

Conclusions

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 Helicity spinor methods are an efficient way to compute cross sections.

# Let's use both!

MHV amplitudes

One gluon emission in DY

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Conclusions

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Introduction

# Only take into account massless particles *spinors*

$$ar{u}_{\pm}(ec{
ho}) = ar{v}_{\mp}(ec{
ho}) := \langle 
ho, \pm \mid \ u_{\pm}(ec{
ho}) = v_{\mp}(ec{
ho}) := \mid 
ho, \pm 
angle$$

spinor products

$$ar{u}_-(ec{p})u_+(ec{k}) = \langle p - |k+
angle := \langle pk
angle$$
  
 $ar{u}_+(ec{p})u_-(ec{k}) = \langle p + |k-
angle := [pk]$ 

$$\langle {\it pk} 
angle = e^{i \phi_{{\it pk}}} \sqrt{2 {\it pk}}$$

### Goal $\rightarrow$ write everything in terms of spinor products!

MHV amplitudes

One gluon emission in DY

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Conclusions

MQ (P

Introduction

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spinor products

$$ar{u}_{-}(ec{p})u_{+}(ec{k})=\langle p-er{k}+
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angle\ ar{u}_{+}(ec{p})u_{-}(ec{k})=\langle p+er{k}-
angle:=[pk]$$

$$\langle pk 
angle = e^{i \phi_{pk}} \sqrt{2pk}$$

Goal  $\rightarrow$ write everything in terms of spinor products!

Introduction	MHV amplitudes	One gluon emission in DY	Conclusions
0000	0 00000	0000 000	
Introduction			

fermion propagators

$$k = |k+\rangle\langle k+|+|k-\rangle\langle k-|$$

polarization vectors

$$\left[\epsilon_{\pm}(p)^{\mu}
ight]^{*} := \epsilon_{\pm}(p, p_{ref}) = \pm rac{\langle p \pm |\gamma^{\mu}| \, p_{ref} \pm 
angle}{\sqrt{2} \langle p \mp | p_{ref} \pm 
angle}$$

slashed polarization vectors

$${{}^{\!\!\!\!/}_{\!\!\!\!\!\!\!/}}_{\pm}(k,q)=rac{\pm\sqrt{2}}{\langle k\mp|q\pm
angle}\Big(|k\mp
angle\langle q\mp|+|q\pm
angle\langle k\pm|\Big)$$

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All outgoing momenta

$$M(p_1+, p_2-, q_1+, q_2-) = \frac{\langle p_2 q_2 \rangle^2}{\langle p_1 p_2 \rangle [q_1 q_2]}$$

Parity and Charge coniugation connect different cross sections

$$P[\langle pk \rangle] = [pk] \qquad \qquad C[\langle pk \rangle] = -[pk]$$

Introduction	MHV amplitudes	One gluon emission in DY	Conclusions
000●	0	0000	
Introduction			

If 
$$\vec{p} \parallel z$$
-axis



$$| k \pm \rangle = \sqrt{\frac{E_k}{E_p}} \left[ \cos \frac{\theta}{2} \mathbf{1} + \sin \frac{\theta}{2} \gamma^1 \gamma^3 \right] | p \pm \rangle$$

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- ▶ collinear limit →  $| k\pm \rangle = \sqrt{\frac{E_k}{E_p}} | p\pm \rangle$
- soft (eikonal )limit  $\rightarrow \mid k \pm \rangle = 0$
- soft (next-to-eikonal) limit  $\rightarrow$  keep all the terms

Introduction	MHV amplitudes	One gluon emission in DY	Conclusions
000●	0	0000	
Introduction			

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MHV amplitudes

One gluon emission in DY

A ► < 3

Conclusions

# Gluon scattering and MHV amplitudes

Introduction 0000	MHV amplitudes	One gluon emission in DY 0000 000	Conclusions
Color decomposition in gluon sc	attering		

At tree level:

$$A_n^{tree}(p_i, \lambda_i, a_i) = g^{n-2} \sum_{\sigma \in \frac{S_n}{Z_n}} Tr(t^{a_{\sigma(1)}} t^{a_{\sigma(2)}} \dots t^{a_{\sigma(n)}}) \cdot A_n(\sigma(1^{\lambda_1}), \dots \sigma(n^{\lambda_n}))$$

Colour is stripped off the amplitude, use colorless Feynman rules to compute color ordered amplitudes.

# Goal: calculating E and NE corrections to color ordered amplitudes .

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Introduction	MHV amplitudes	One gluon emission in DY	Conclusions
0000	0	0000	
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Simplest results			

$$A^{tree}_{n}(1^+,\ldots,\ldots,n^+)=0$$



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Introduction	MHV amplitudes	One gluon emission in DY	Conclusions
0000	0 0●000	0000	
Simplest results			

$$A^{tree}{}_{n}(1^{+},\ldots,\ldots,n^{+}) = 0$$
  
 $A^{tree}{}_{n}(1^{+},\ldots,i^{-},\ldots,n^{+}) = 0$ 



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Introduction	MHV amplitudes	One gluon emission in DY	Conclusions
0000	00000	0000	
Simplest results			

$$A^{tree}{}_{n}(1^{+},\ldots,\ldots,n^{+}) = 0$$
  
 $A^{tree}{}_{n}(1^{+},\ldots,i^{-},\ldots,n^{+}) = 0$ 

$$A^{tree MHV}_{n}(1^+,\ldots,i^-,\ldots,j^-,\ldots,n^+) = i \frac{\langle ij \rangle^4}{\langle 12 \rangle \langle 23 \rangle \cdots \langle n-1,n \rangle \langle n1 \rangle}$$



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Introduction	MHV amplitudes	One gluon emission in DY	Conclusions
0000	0 00000	0000	
Simplest results			

▶ Plus-helicity soft gluon →factorization (E level)

$$A^{tree MHV}_{n} \left(1^+, \dots, j^-, \dots, (i-1)^+, i^+, (i+1)^+, \dots, k^-, \dots, n^+\right) =$$

$$= \frac{\langle i-1, i+1 \rangle}{\langle i, i-1 \rangle \langle i, i+1 \rangle} \cdot A^{tree MHV}_{n-1} \left(1^+, \dots j^- \dots, (i-1)^+, (i+1)^+ \dots, k^-, \dots, n^+\right)$$

#### factor is purely eikonal, no NE contributions

▶ Negative-helicity soft gluon → no NE contributions

Numerator is at NNE level ,  $\langle ij \rangle^4 = \mathfrak{o}(\lambda^2)$ 

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Introduction	MHV amplitudes	One gluon emission in DY	Conclusions
0000	0 00000	0000	
Simplest results			

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$$A^{tree\,MHV}_{n}\left(1^{+},\ldots j^{-}\ldots,(i-1)^{+},i^{+},(i+1)^{+}\ldots,k^{-},\ldots,n^{+}\right)=$$

$$= \frac{\langle i-1, i+1 \rangle}{\langle i, i-1 \rangle \langle i, i+1 \rangle} \cdot A^{tree MHV}_{n-1} \left(1^+, \dots j^- \dots, (i-1)^+, (i+1)^+ \dots, k^-, \dots, n^+\right)$$

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Introduction	MHV amplitudes	One gluon emission in DY	Conclusions
0000	0 00000	0000	
Simplest results			

 $\blacktriangleright$  4-gluon scattering  $\rightarrow$  three independent amplitude

$$A(1\pm,2+,3+,4+) = 0$$
$$A(1-,2-,3+,4+) = i \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle}$$

• if gluon 3 soft  $\rightarrow \langle 3j \rangle \approx \lambda$  with  $\lambda \ll 1$ 

$$A(1-,2-,3+,4+) = \frac{\langle 24 \rangle}{\langle 23 \rangle \langle 34 \rangle} \cdot i \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 24 \rangle \langle 41 \rangle}$$

• if gluon 2 soft  $ightarrow \langle 2j 
angle pprox \lambda$  with  $\lambda \ll 1$ 

$$A(1-,2-,3+,4+) = rac{\langle 12 
angle^4}{\langle 12 
angle \langle 23 
angle} \cdot i rac{1}{\langle 34 
angle \langle 41 
angle} pprox 0$$

Introduction	MHV amplitudes	One gluon emission in DY	Conclusions
0000	0 00000	0000	
Simplest results			

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Introduction	MHV amplitudes	One gluon emission in DY	Conclusions
0000	0 00000	0000	
Simplest results			

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angle^4}{\langle 12 
angle \langle 23 
angle} \cdot i rac{1}{\langle 34 
angle \langle 41 
angle} pprox 0 \end{aligned}$$

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MHV amplitudes 0 00000 One gluon emission in DY

Conclusions

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# One gluon emission in Drell-Yan $q + \overline{q} \rightarrow \gamma \rightarrow I + \overline{I}$



Introduction 0000	MHV amplitudes 0 00000	One gluon emission in DY •ooo •oo	Conclusions
Real gluon emission			



$$M(q_1^+q_2^-g^+p_1^+p_2^-) \propto rac{\langle q_1-|p_2+
angle^2}{\langle p_1-|p_2+
angle\langle g-|q_2+
angle\langle g-|q_1+
angle}$$

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MHV amplitude

One gluon emission in DY ○●○○ ○○○

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Conclusions

Real gluon emission

#### Correction to the truncated amplitude.

$$M_{E}^{\mu} = \left[g_{s}\sqrt{2}rac{C}{C'}rac{\langle q_{1} - |q_{2}+
angle}{\langle g - |q_{1}+
angle\langle g - |q_{2}+
angle}
ight]\left[eQC'\langle q_{2} + |\gamma^{\mu}|\,q_{1}+
angle
ight]$$



$$M^{E}(q_{1}^{+},q_{2}^{-},g^{+},p_{1}^{+},p_{2}^{-}) = M^{0}\left[g_{s}\sqrt{2}rac{C}{C'}rac{\langle q_{1}-|q_{2}+
angle}{\langle g-|q_{1}+
angle\langle g-|q_{2}+
angle}
ight]$$

MHV amplitudes

One gluon emission in DY 000 000

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Conclusions

Real gluon emission

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angle}{\langle g-|q_{2}+
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MHV amplitude

One gluon emission in DY

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Conclusions

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Real gluon emission

#### Correction to the truncated amplitude

$$\begin{split} M^{\mu}_{NE} &= S(g,q_1,q_2) e Q C' \langle q_2 + |\Gamma^{\mu}| q_1 + \rangle \\ \text{where} \qquad \Gamma^{\mu} &= \frac{\eta^{\mu\nu} (2q_1g) - 2q_1^{\mu}g^{\nu}}{2q_1q_2} \gamma_{\nu} \end{split}$$



$$M^{NE}(q_{1}^{+}q_{2}^{-}g^{+}p_{1}^{+}p_{2}^{-}) = M^{0}\left[g_{s}\sqrt{2}\frac{C}{C'}\frac{\langle g + |p_{1}-\rangle}{\langle g - |q_{2}+\rangle\langle q_{2}+|p_{1}-\rangle}\right]$$

MHV amplitude

One gluon emission in DY

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Conclusions

Real gluon emission

#### Correction to the truncated amplitude

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$$M^{NE}(q_1^+q_2^-g^+p_1^+p_2^-) = M^0 \left[ g_s \sqrt{2} \frac{\zeta}{C'} \frac{\langle g + | p_1 - \rangle}{\langle g - | q_2 + \rangle \langle q_2 + | p_1 - \rangle} \right]$$

Introduction 0000	MHV amplitudes 0 00000	One gluon emission in DY	Conclusions
Real gluon emission			

#### Summing results:

$$M_{E+NE}(q_1^+q_2^-g^+p_1^+p_2^-) = M^0 \left[ g_s \sqrt{2} \frac{C}{C'} \frac{\langle q_1 q_2 \rangle}{\langle gq_1 \rangle \langle gq_2 \rangle} \right] \left[ 1 - \frac{\langle q_1 g \rangle \left[ gp_1 \right]}{\langle q_1 q_2 \rangle \left[ q_2 p_1 \right]} \right]$$

K-factor 
$$K(z) = \frac{1}{\sigma^0} \frac{d\sigma}{dz}$$
  
 $K_{E+NE}^{real}(z) = \frac{\alpha_s}{4\pi} C_f \left(\frac{2}{\epsilon^2} \delta(1-z) - \frac{4}{\epsilon} \left(\frac{1}{[1-z]_+} - 1\right) + \text{ finite terms}\right)$ 

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Introduction 0000	MHV amplitudes 0 00000	One gluon emission in DY ○○○● ○○○	Conclusions
Real gluon emission			

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Introduction 0000	MHV amplitudes 0 00000	One gluon emission in DY	Conclusions
Virtual gluon exchange in DY			

$$M(q_1^+q_2^-p_1^+p_2^-) = M^0(q_1^+, q_2^-, p_1^+, p_2^-) \cdot \cdot (i2g_s^2 C_f) \int \frac{d^n k}{(2\pi)^n} \frac{2q_1q_2}{k^2 k_1^2 k_2^2} \left[1 - \frac{k_1^2 - k^2}{2q_1q_2}\right] \left[1 - \frac{k_2^2 - k^2}{2q_1q_2}\right]$$



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Introduction 0000	MHV amplitudes 0 00000	One gluon emission in DY ○○○○ ○●○	Conclusions
Virtual gluon exchange in DY			

#### Computation of the amplitude

- 4-dim helicity scheme instead of standard dimensional regularization
- E and NE contributions from numerator and denominator.
- E and NE approximation are done *before* integration
- Certain kind of integrals require "Glasgow" precription

#### K-factor:

$$\mathcal{K}_{E+NE}^{\textit{virtual}}(z) = \frac{\alpha_s}{4\pi} C_f \left[ -\frac{2}{\epsilon^2} \delta(1-z) - \frac{4}{\epsilon} \delta(1-z) + \mathfrak{o}(\epsilon^0) \right]$$

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Introduction 0000	MHV amplitudes 0 00000	One gluon emission in DY ○○○○ ○●○	Conclusions
Virtual gluon exchange in DY			

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Introduction	MHV amplitudes	One gluon emission in DY	Conclusions
0000	0	0000	
	00000	000	
Virtual gluon exchange in DY			

#### Summing virtual and real contributions

$$\mathcal{K}_{E+NE}^{virtual+real}(z) = -\frac{\alpha_s C_f}{4\pi} \frac{4}{\epsilon} \left( \frac{1}{[1-z]_+} + \frac{\delta(1-z) - 1}{\epsilon} + \text{ finite terms} \right)$$

Expansion of Altarelli-Parisi splitting function:

$$P_{qq}(z) = \left[\frac{1+z^2}{1-z}\right]_+ \approx \frac{2}{[1-z]_+} + 2\delta(1-z) - 2 + \text{NNE terms}$$

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MHV amplitudes 0 00000 One gluon emission in DY

Conclusions

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## Conclusions

Results:

- Found no NE corrections to MHV amplitudes
- ► Derived effective NE rules for gluon emission from quark-line
- ► Was able to reproduce DY logs that diverge at treshold
- extending NE effects and DY logs to all orders

MHV amplitudes 0 00000 One gluon emission in DY

Conclusions

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## Conclusions

Results:

- Found no NE corrections to MHV amplitudes
- ► Derived effective NE rules for gluon emission from quark-line
- ► Was able to reproduce DY logs that diverge at treshold Still to do:
  - extending NE effects and DY logs to all orders

MHV amplitudes 0 00000 One gluon emission in DY

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Conclusions

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# Thanks for your attention!

MHV amplitudes 0 00000 One gluon emission in DY

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Conclusions

# "Glasgow" prescription

$$I_{Gl}(d,n) := \int_0^1 dx \ x^{-1-d+n} (1-x)^{-1+d-n}$$

Because of a UV/IR cancellation around  $d \approx n$  this should be zero, but UV divergencies have counterterms that we have to include