Next-eikonal approximation in the context of spinor helicity methods

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NIKHEF

Motivations

- Eikonal (E) and Next-to-eikonal (NE) exponentiation

  $$M = M^0 e^{M_E + M_{NE}}(1 + M_r) + \mathbf{O}(NNE)$$

  E and NE approximation for propagators

  $$\frac{1}{(p+q)^2} \approx \frac{1}{2pq} - \frac{q^2}{(2pq)^2} + \mathbf{O}(NNE)$$

- Helicity spinor methods are an efficent way to compute cross sections.

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Only take into account massless particles

spinors

$$\bar{u}_\pm(\vec{p}) = \bar{v}_\mp(\vec{p}) := \langle p, \pm \,|$$
$$u_\pm(\vec{p}) = v_\mp(\vec{p}) := |\, p, \pm\rangle$$

spinor products

$$\bar{u}_-(\vec{p})u_+(\vec{k}) = \langle p - | k+\rangle := \langle pk\rangle$$
$$\bar{u}_+(\vec{p})u_-(\vec{k}) = \langle p + | k-\rangle := [pk]$$

$$\langle pk\rangle = e^{i\phi_{pk}}\sqrt{2pk}$$

Goal $\rightarrow$write everything in terms of spinor products!

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fermion propagators

$$\not{k} = |\,k+\rangle\langle k+\,| + |\,k-\rangle\langle k-\,|$$

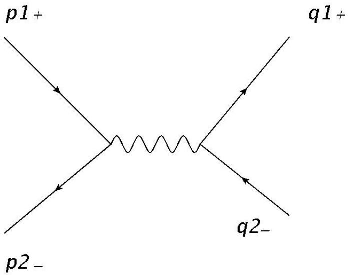
polarization vectors

$$[\epsilon_\pm(p)^\mu]^* := \epsilon_\pm(p, p_{ref}) = \pm\frac{\langle p\pm|\gamma^\mu|\,p_{ref}\pm\rangle}{\sqrt{2}\langle p\mp|p_{ref}\pm\rangle}$$

slashed polarization vectors

$$\not{\epsilon}_\pm(k,q) = \frac{\pm\sqrt{2}}{\langle k\mp|q\pm\rangle}\Big(|k\mp\rangle\langle q\mp| + |q\pm\rangle\langle k\pm|\Big)$$

Example!



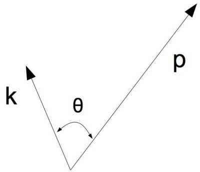
All outgoing momenta

$$M(p_1+, p_2-, q_1+, q_2-) = \frac{\langle p_2 q_2\rangle^2}{\langle p_1 p_2\rangle\,[q_1 q_2]}$$

Parity and Charge coniugation connect different cross sections

$$P\Big[\langle pk\rangle\Big] = [pk] \qquad C\Big[\langle pk\rangle\Big] = -[pk]$$

If $\vec{p} \parallel z$-axis



$$| k\pm\rangle = \sqrt{\frac{E_k}{E_p}} \left[\cos\frac{\theta}{2}\mathbf{1} + \sin\frac{\theta}{2}\gamma^1\gamma^3 \right] | p\pm\rangle$$

- collinear limit $\rightarrow | k\pm\rangle = \sqrt{\frac{E_k}{E_p}} | p\pm\rangle$
- soft (eikonal)limit $\rightarrow | k\pm\rangle = 0$
- soft (next-to-eikonal) limit $\rightarrow$ keep all the terms

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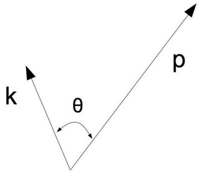

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Gluon scattering and MHV amplitudes

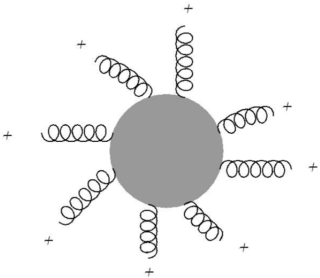
At tree level:

$$A_n^{tree}(p_i, \lambda_i, a_i) = g^{n-2} \sum_{\sigma \epsilon \frac{S_n}{Z_n}} Tr\left(t^{a_{\sigma(1)}} t^{a_{\sigma(2)}} \dots t^{a_{\sigma(n)}}\right) \cdot \cdot A_n(\sigma(1^{\lambda_1}), \dots \sigma(n^{\lambda_n}))$$

Colour is stripped off the amplitude, use colorless Feynman rules to compute color ordered amplitudes.

Goal: calculating E and NE corrections to color ordered amplitudes .

$$A^{tree}{}_n(1^+, \ldots, \ldots, n^+) = 0$$



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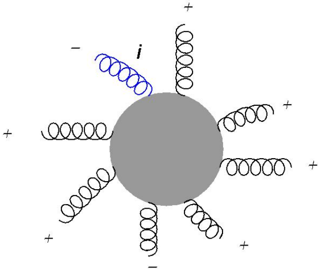

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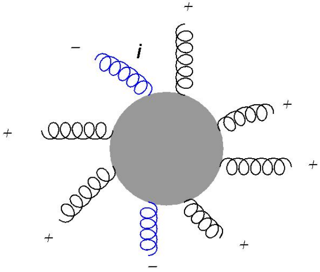
$$A^{tree}{}^{MHV}_n(1^+, \ldots, i^-, \ldots, j^-, \ldots, n^+) = i \frac{\langle ij \rangle^4}{\langle 12 \rangle \langle 23 \rangle \cdots \langle n-1, n \rangle \langle n1 \rangle}$$



- Plus-helicity soft gluon →factorization (E level)

$$A^{tree\,MHV}_{n}\left(1^+,\dots j^-\dots,(i-1)^+,i^+,(i+1)^+\dots,k^-,\dots,n^+\right)=$$

$$=\frac{\langle i-1,i+1\rangle}{\langle i,i-1\rangle\langle i,i+1\rangle}\cdot$$
$$\cdot A^{tree\,MHV}_{n-1}\left(1^+,\dots j^-\dots,(i-1)^+,(i+1)^+\dots,k^-,\dots,n^+\right)$$

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Numerator is at NNE level , $\langle ij\rangle^4=o(\lambda^2)$

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- 4-gluon scattering → three independent amplitude

$$A(1\pm,2+,3+,4+)=0$$

$$A(1-,2-,3+,4+)=i\frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle}$$

- if gluon 3 soft → $\langle 3j\rangle \approx \lambda$ with $\lambda \ll 1$

$$A(1-,2-,3+,4+)=\frac{\langle 24\rangle}{\langle 23\rangle\langle 34\rangle}\cdot i\frac{\langle 12\rangle^4}{\langle 12\rangle\langle 24\rangle\langle 41\rangle}$$

- if gluon 2 soft → $\langle 2j\rangle \approx \lambda$ with $\lambda \ll 1$

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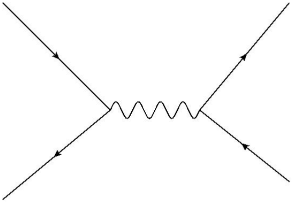
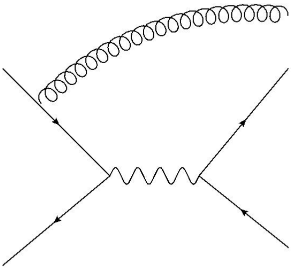
One gluon emission in Drell-Yan

$$q + \bar{q} \to \gamma \to l + \bar{l}$$



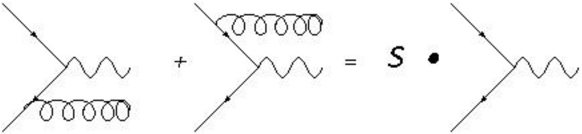
Real gluon emission



$$M(q_1^+ q_2^- g^+ p_1^+ p_2^-) \propto \frac{\langle q_1 - | p_2 + \rangle^2}{\langle p_1 - | p_2 + \rangle \langle g - | q_2 + \rangle \langle g - | q_1 + \rangle}$$

Correction to the truncated amplitude.

$$M_E^\mu = \left[g_s \sqrt{2} \frac{C}{C'} \frac{\langle q_1 - | q_2 + \rangle}{\langle g - | q_1 + \rangle \langle g - | q_2 + \rangle} \right] \left[eQC' \langle q_2 + | \gamma^\mu | q_1 + \rangle \right]$$



Including the leptonic part

$$M^E(q_1^+, q_2^-, g^+, p_1^+, p_2^-) = M^0 \left[g_s \sqrt{2} \frac{C}{C'} \frac{\langle q_1 - | q_2 + \rangle}{\langle g - | q_1 + \rangle \langle g - | q_2 + \rangle} \right]$$

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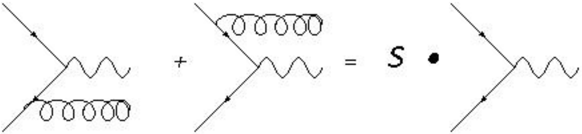

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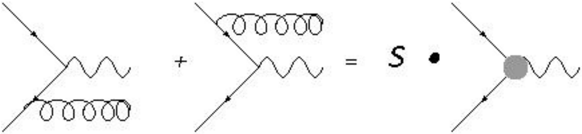
$$M^{\mu}_{NE} = S(g, q_1, q_2) eQC' \langle q_2 + | \Gamma^{\mu} | q_1 + \rangle$$

$$\text{where} \qquad \Gamma^{\mu} = \frac{\eta^{\mu\nu}(2q_1 g) - 2q_1^{\mu} g^{\nu}}{2q_1 q_2} \gamma_{\nu}$$



Including the leptonic part

$$M^{NE}(q_1^+ q_2^- g^+ p_1^+ p_2^-) = M^0 \left[g_s \sqrt{2} \frac{C}{C'} \frac{\langle g + | p_1 - \rangle}{\langle g - | q_2 + \rangle \langle q_2 + | p_1 - \rangle} \right]$$

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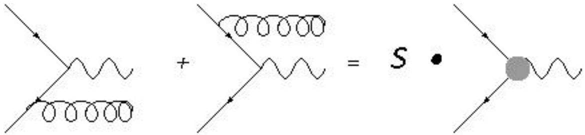

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Summing results:

$$M_{E+NE}(q_1^+ q_2^- g^+ p_1^+ p_2^-) = M^0 \left[g_s \sqrt{2} \frac{C}{C'} \frac{\langle q_1 q_2 \rangle}{\langle g q_1 \rangle \langle g q_2 \rangle} \right] \left[1 - \frac{\langle q_1 g \rangle [g p_1]}{\langle q_1 q_2 \rangle [q_2 p_1]} \right]$$

K-factor $K(z) = \frac{1}{\sigma^0} \frac{d\sigma}{dz}$

$$K^{real}_{E+NE}(z) = \frac{\alpha_s}{4\pi} C_f \left(\frac{2}{\epsilon^2} \delta(1-z) - \frac{4}{\epsilon} \left(\frac{1}{[1-z]_+} - 1 \right) + \text{ finite terms } \right)$$

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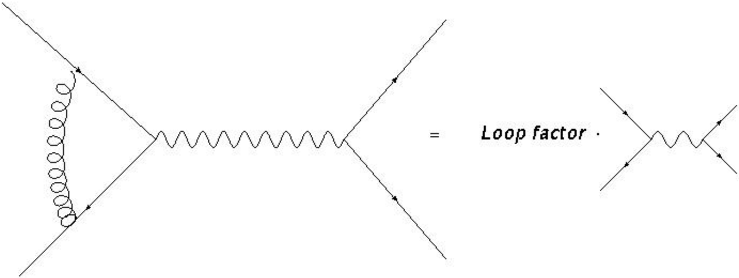
$$M(q_1^+ q_2^- p_1^+ p_2^-) = M^0(q_1^+, q_2^-, p_1^+, p_2^-) \cdot$$
$$\cdot (i2g_s^2 C_f) \int \frac{d^n k}{(2\pi)^n} \frac{2q_1 q_2}{k^2 k_1^2 k_2^2} \left[1 - \frac{k_1^2 - k^2}{2q_1 q_2}\right] \left[1 - \frac{k_2^2 - k^2}{2q_1 q_2}\right]$$



Computation of the amplitude

- 4-dim helicity scheme instead of standard dimensional regularization
- E and NE contributions from numerator and denominator.
- E and NE approximation are done *before* integration
- Certain kind of integrals require "Glasgow" precription

K-factor:

$$K^{virtual}_{E+NE}(z) = \frac{\alpha_s}{4\pi} C_f \left[-\frac{2}{\epsilon^2}\delta(1-z) - \frac{4}{\epsilon}\delta(1-z) + \mathcal{O}(\epsilon^0) \right]$$

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Summing virtual and real contributions

$$K^{virtual+real}_{E+NE}(z) = -\frac{\alpha_s C_f}{4\pi}\frac{4}{\epsilon}\left(\frac{1}{[1-z]_+} + \delta(1-z) - 1 + \text{ finite terms }\right)$$

Expansion of Altarelli-Parisi splitting function:

$$P_{qq}(z) = \left[\frac{1+z^2}{1-z}\right]_+ \approx \frac{2}{[1-z]_+} + 2\delta(1-z) - 2 + \text{NNE terms}$$

Conclusions

Results:

- Found no NE corrections to MHV amplitudes
- Derived effective NE rules for gluon emission from quark-line
- Was able to reproduce DY logs that diverge at treshold

Still to do:

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Thanks for your attention!

"Glasgow" prescription

$$I_{Gl}(d,n) := \int_0^1 dx\; x^{-1-d+n}(1-x)^{-1+d-n}$$

Because of a UV/IR cancellation around $d \approx n$ this should be zero, but UV divergencies have counterterms that we have to include