Generalizations of the holographic Kondo model

Max-Niklas Newrzella

supervised by PD. Dr. Johanna Erdmenger

Max Planck Institute for Physics

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The Kondo model

Interaction between free electrons and spin at single impurity

$$\mathcal{H}_{int} = \hat{\lambda}_K \delta(\vec{x}) \left(\chi^\dagger \vec{T} \chi\right) \cdot \left(\psi^\dagger \vec{T} \psi\right)$$

The Kondo model

Interaction between free electrons and spin at single impurity

$$\mathcal{H}_{int} = \lambda_K \delta(\vec{x}) \left(\mathcal{O}^\dagger \mathcal{O}\right) + \ldots$$

$$\mathcal{O} = \psi^\dagger \chi$$

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Perturbation theory: logarithmic behavior of the resistivity at low T (J negative)

$$\rho = \rho_0 \left(1 + J \log \left(\frac{T}{T_F} \right) \right)$$

J. Kondo, 1964

At even lower temperatures, this behavior breaks down:

$$T_K \approx T_F e^{-1/|J|}$$

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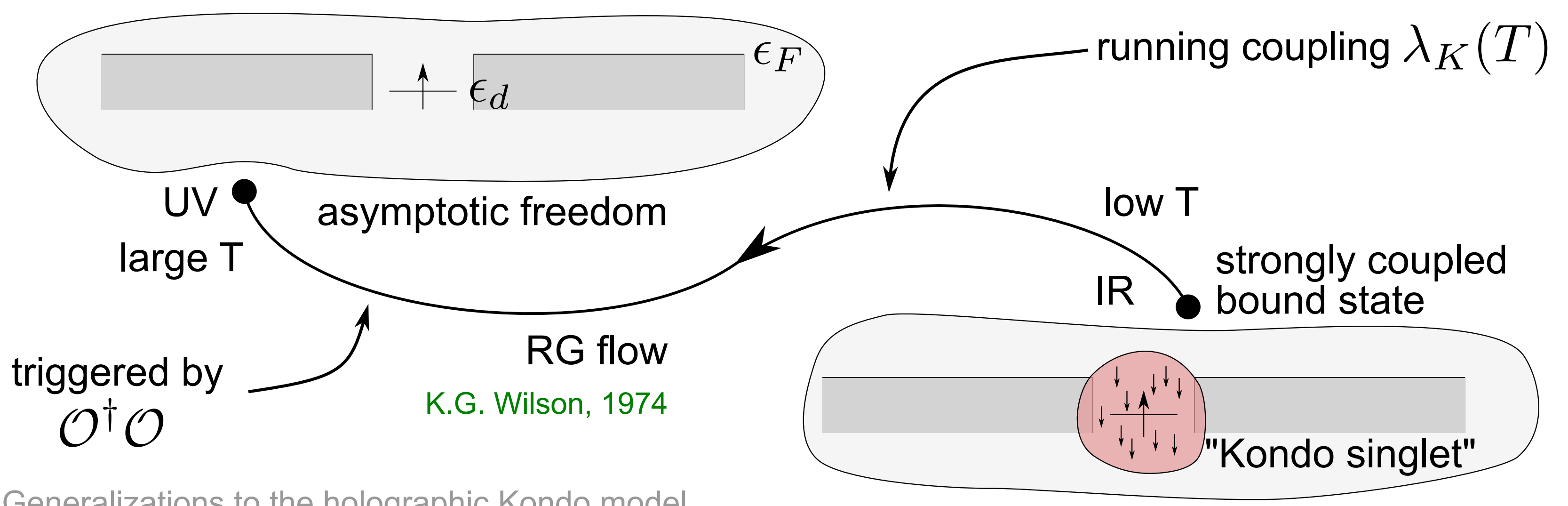
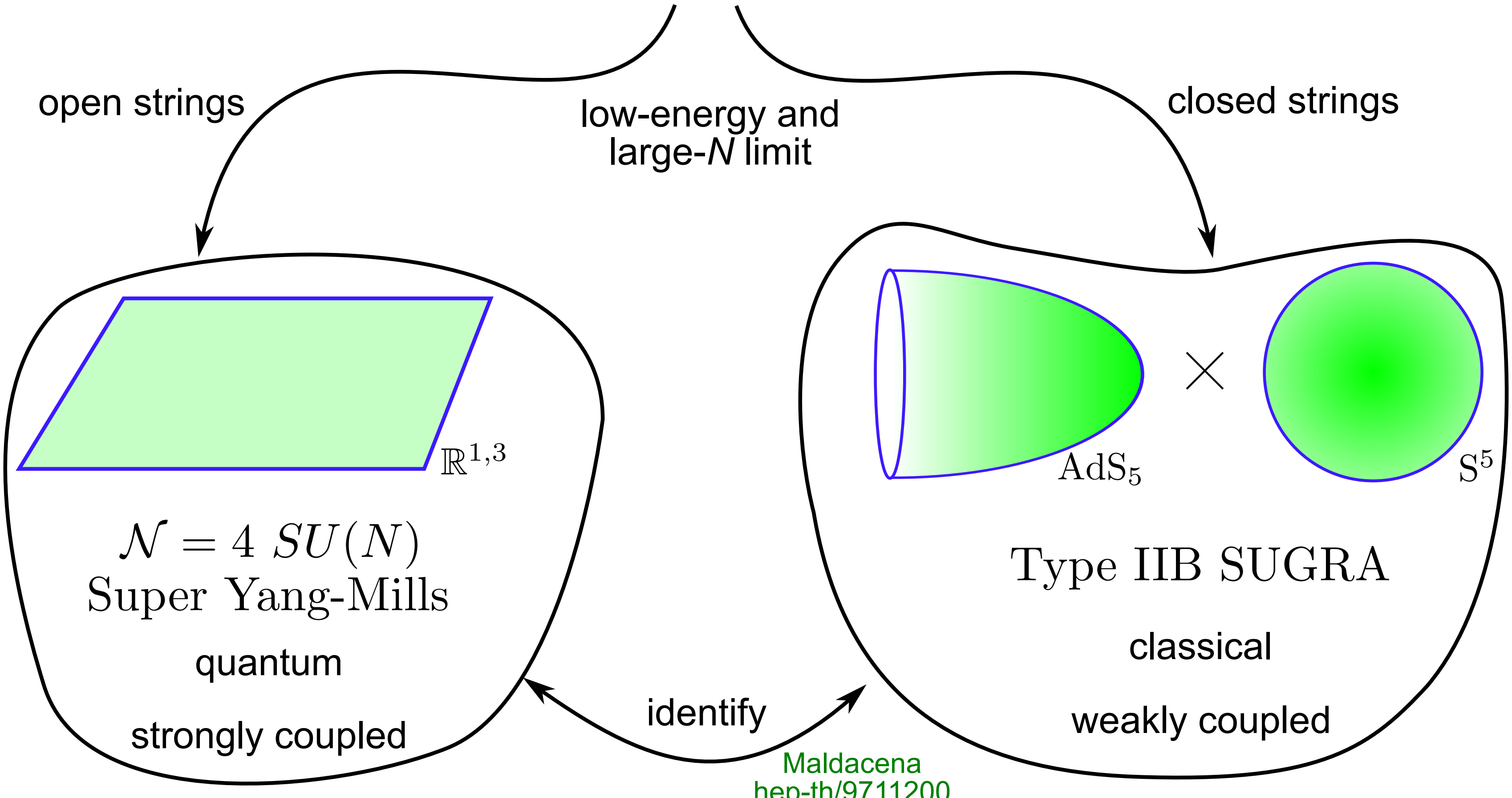


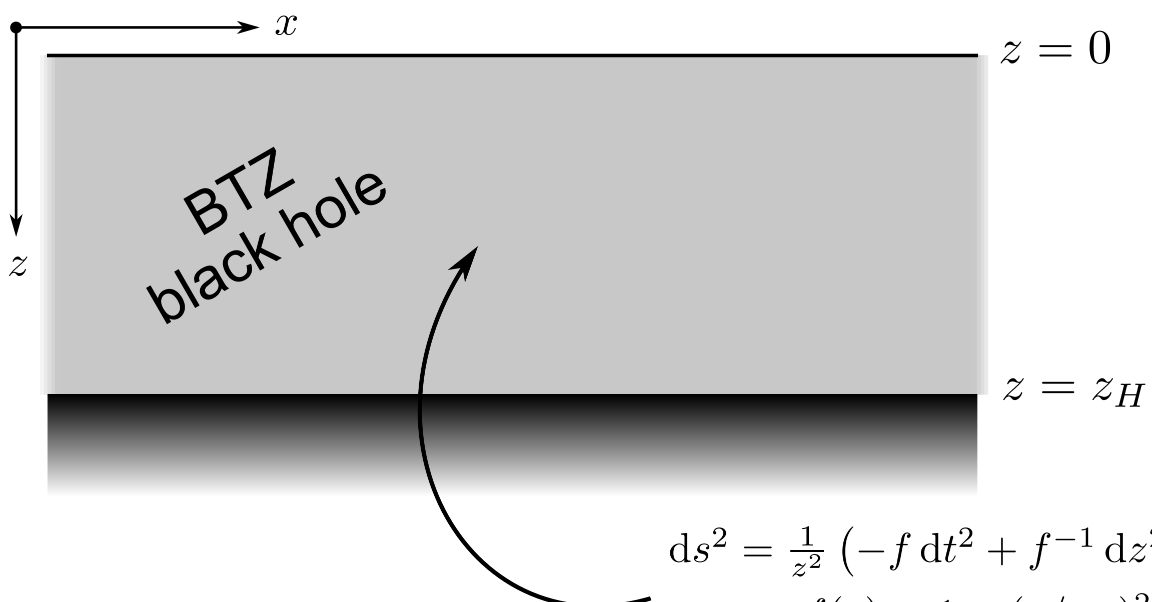
The original AdS/CFT conjecture

Two different interpretations of D3-branes due to open and closed string excitations



The holographic Kondo model

Essential ingredients from the top-down model:



Erdmenger, Hoyos, O'Bannon, Wu arXiv:1310.3271

$$\mathrm{d}s^2 = \frac{1}{z^2}\left(-f\,\mathrm{d}t^2 + f^{-1}\,\mathrm{d}z^2 + \mathrm{d}x^2\right)$$

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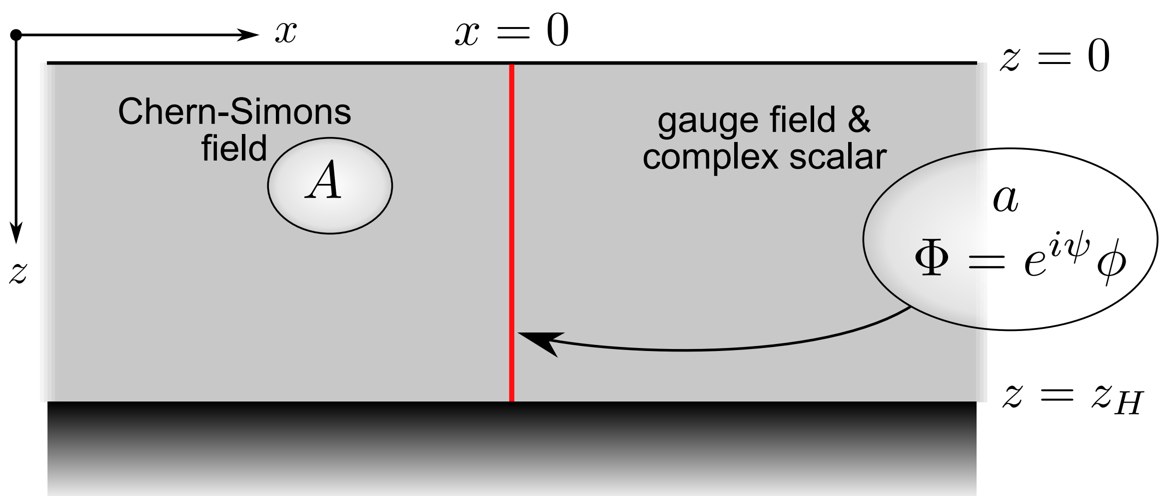

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$$ds^2 = \frac{1}{z^2}\left(-f\,dt^2 + f^{-1}\,dz^2 + dx^2\right)$$

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$$S = S_{CS} + S_{AdS_2}$$

$$S_{CS} = -\frac{N}{4\pi} \int_{AdS_3} A \wedge \mathrm{d}A + A \wedge A \wedge A$$

Erdmenger, Hoyos, O'Bannon, Wu arXiv:1310.3271

$$S_{AdS_2} = -N \int_{AdS_2} \big(f \wedge (*_2 f) + (D\Phi)^\dagger \wedge (*_2 D\Phi) + V \big)$$

$$\mathrm{d}s^2 = \tfrac{1}{z^2} \left(-f \, \mathrm{d}t^2 + f^{-1} \, \mathrm{d}z^2 + \mathrm{d}x^2 \right)$$
$$f(z) = 1 - (z/z_H)^2$$

Boundary expansions

Non-vanishing field components: $A_x,\, a_t,\, \phi$

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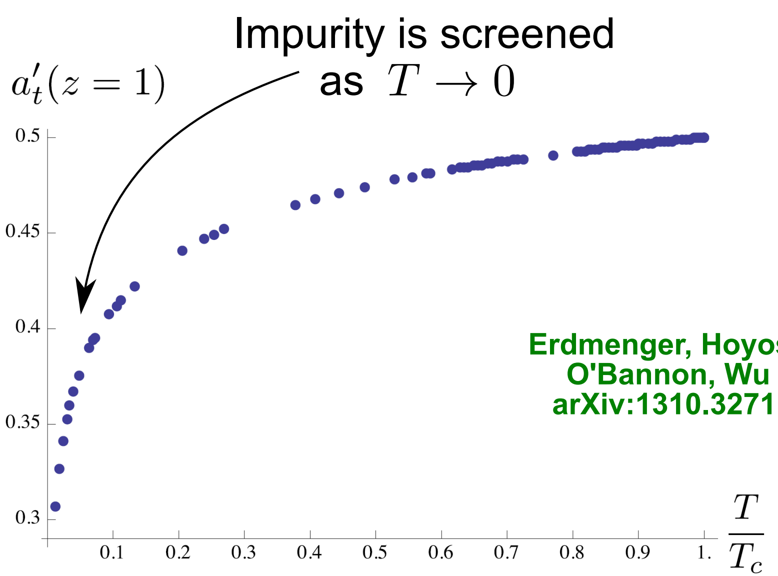
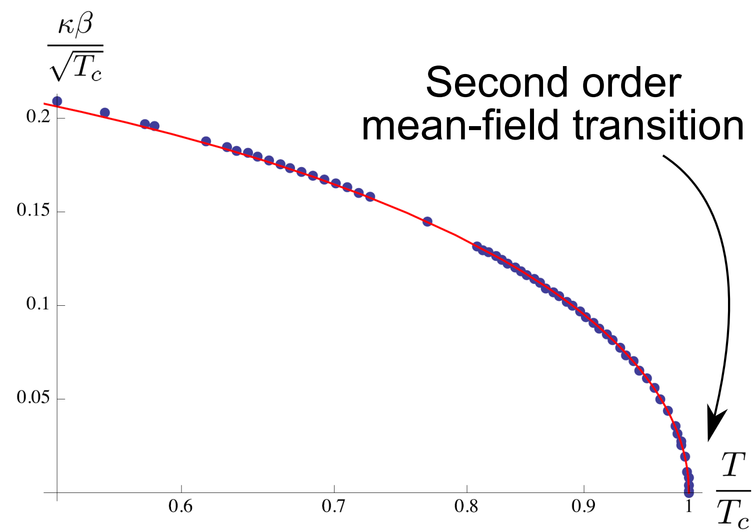
as $\Lambda_0 \to \Lambda$

diverges at $\Lambda_K := \Lambda_0 e^{-1/\kappa_0}$

Results from the static case

- Phase transition at critical temperature
- Impurity is screened at low temperature
- Phase shift in the Chern-Simons field





Erdmenger, Hoyos, O'Bannon, Wu
arXiv:1310.3271

Generalizations

look at two possibilities

time dependence

metric back-reaction

Get in touch with experiments

Compute entanglement entropy

Generalizations (1) - time dependent scenarios

Include time dependence into the model:
The expansion coefficients are now functions of time

$$\phi = \sqrt{z}(\alpha(t)\log z + \beta(t)) + \ldots$$

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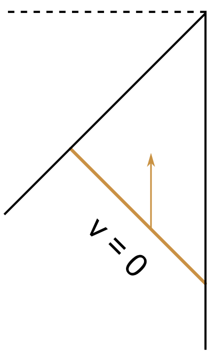
Basic idea:

- Start with some initial configuration
- Impose a time dependent behavior on the expansion coefficients
- Compute the evolution of the system

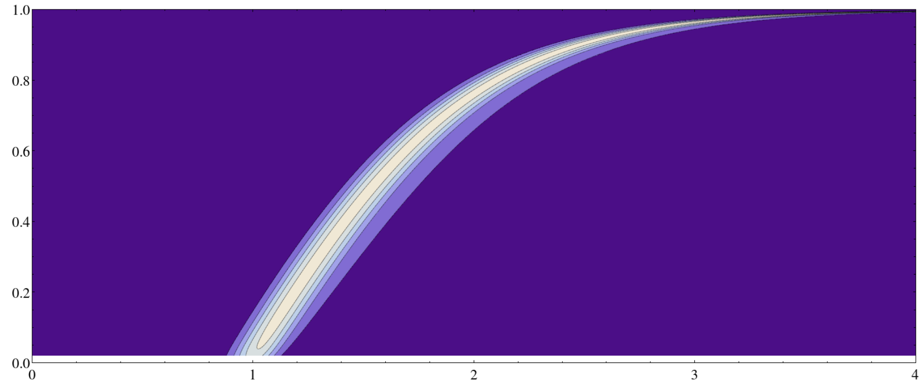


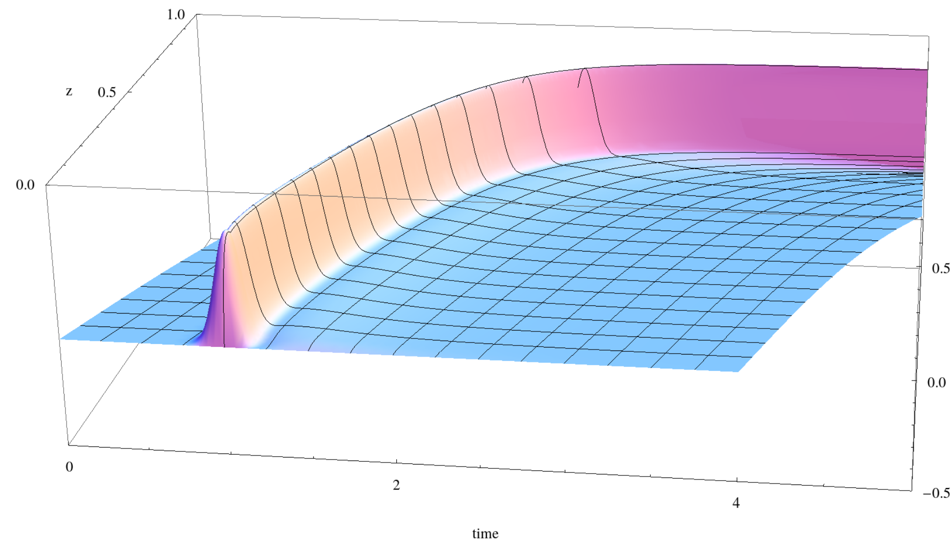
Computational methods: finite differences, implicit time discretization

Results from quench scenario

Quench scalar via boundary conditions

The excitation propagates towards
the event horizon





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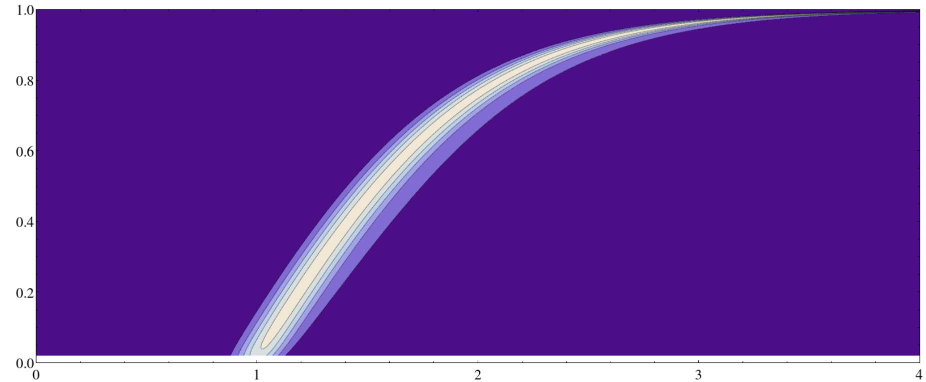

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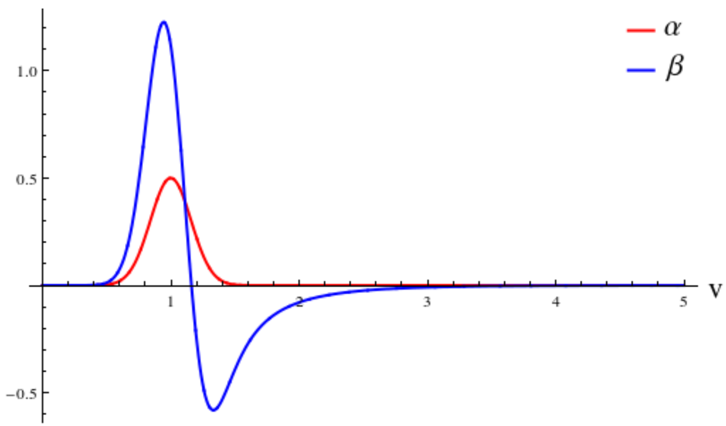
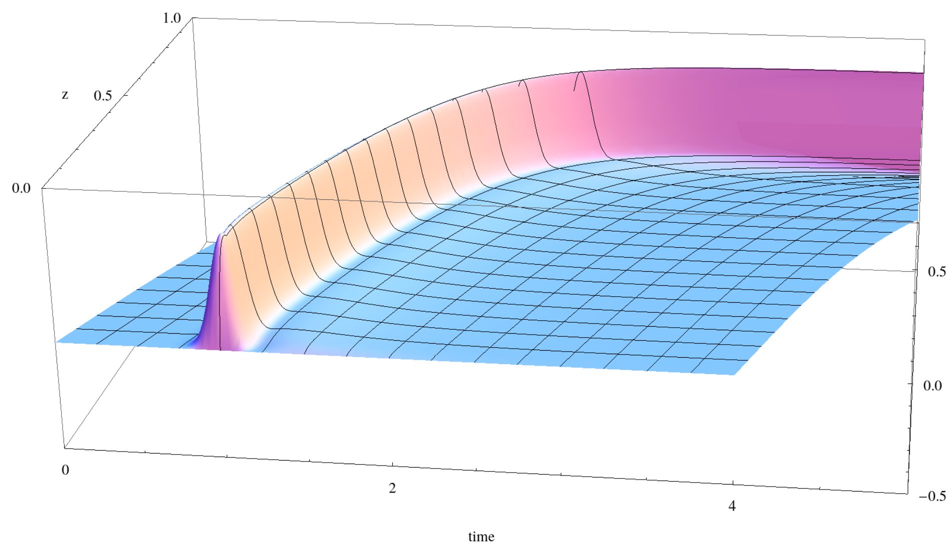
Read off behavior of other coefficients



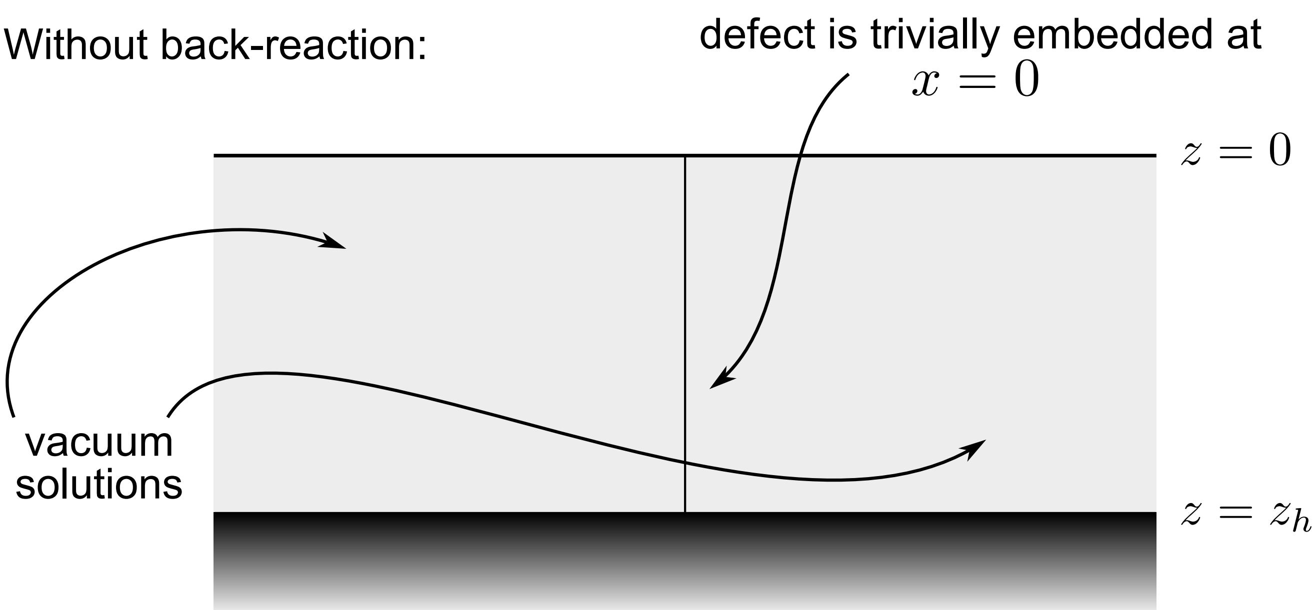


Generalizations (2) - metric back-reaction

Gravity in 1+2 dim is "trivial" $\mathcal{R} \sim Sc + Ricc$

Only the energy-momentum on the brane can change the geometry

Without back-reaction:



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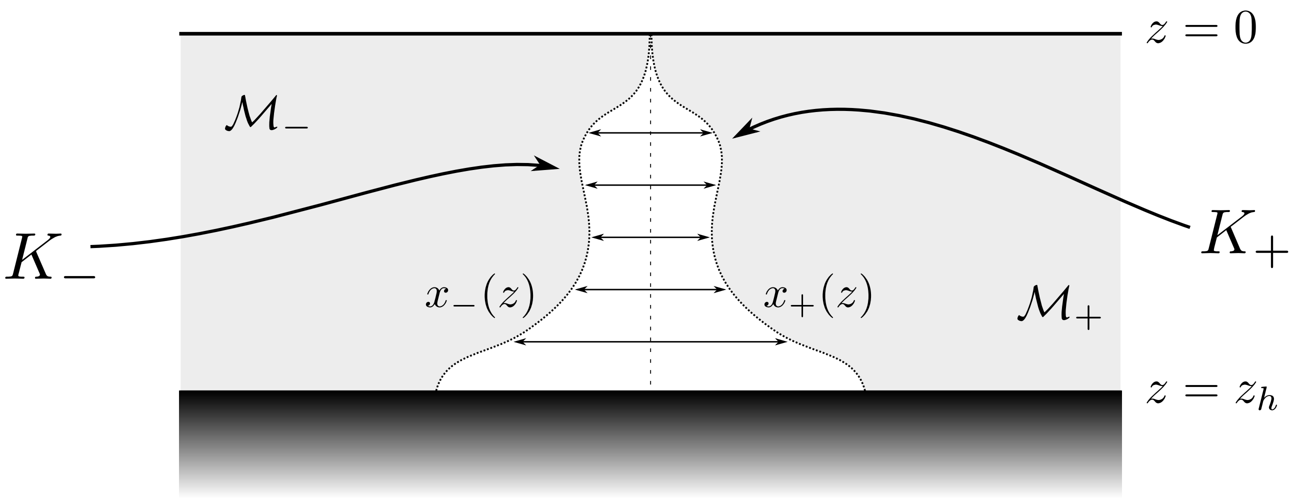
Israel junction conditions $[K - \gamma\, \mathrm{tr}(K)] = -\kappa_g S \qquad [A] := A_+ - A_-$

Extrinsic curvature *K* is given by embedding of the defect

Israel, 1965



Summary

Holographic model provides basic features of the Kondo model

- Running coupling
- Phase transition at critical temperature near Kondo temperature
- Screening of the impurity as $T \to 0$

Generalizations of holographic model for the Kondo effect

- Time dependencies to simulate quench scenarios
- Framework for metric back-reaction

Outlook

- Different quenching scenarios
- Computation of metric back-reaction
- Entanglement entropy in condensed phase

Thank you for your attention

Holographic entanglement entropy

Compute holographic entanglement entropy

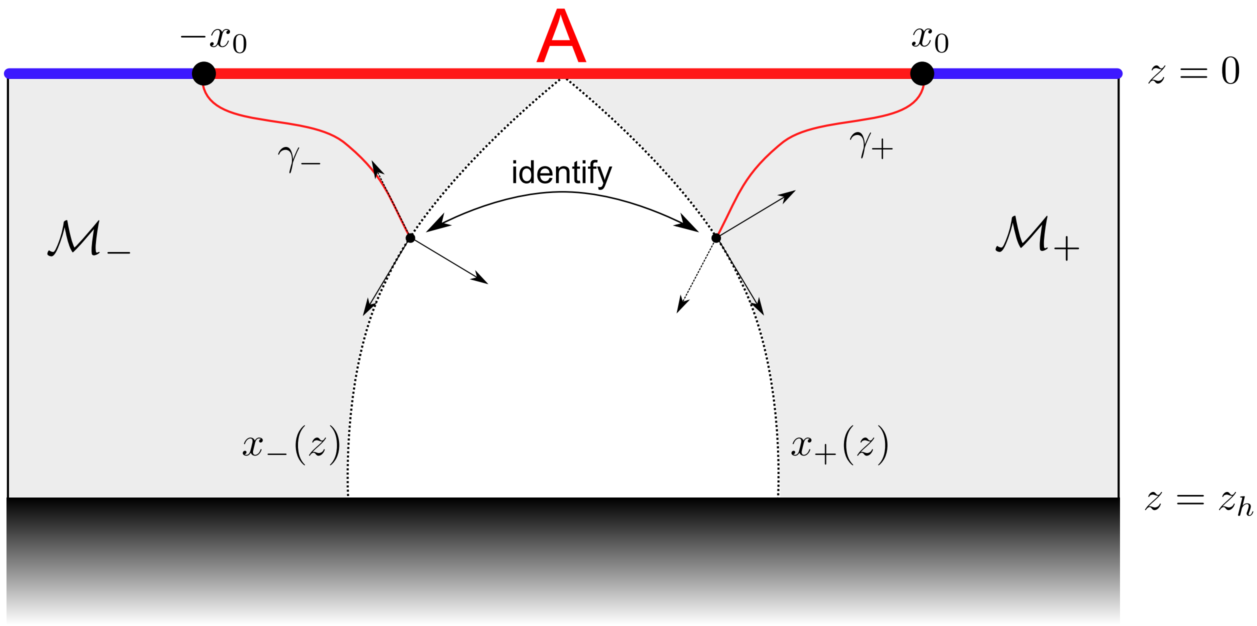
$$S_A = \frac{A_{min}}{4G_N}$$

Ryu, Takayanagi
hep-th/0603001

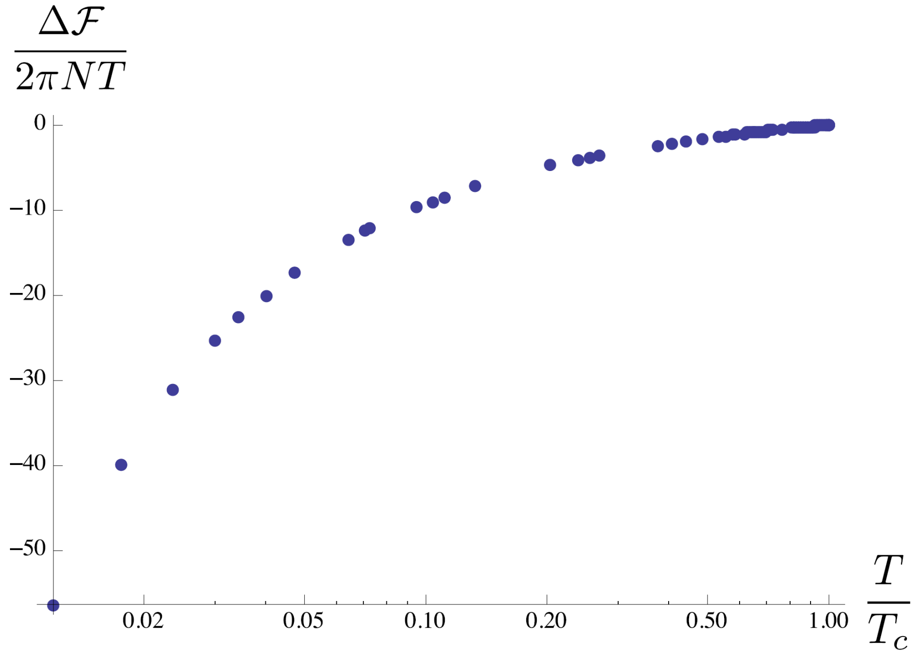
Possible extention to time dependent case

$$\rho_A = \mathrm{Tr}_B(\rho)$$
$$S_A = -\mathrm{Tr}_A\left(\rho_A \log(\rho_A)\right)$$

Hubeny, Rangamani, Takayanagi
0705.0016



Free energy



$\frac{\Delta \mathcal{F}}{2\pi N T}$
0
−10
−20
−30
−40
−50
0.02
0.05
0.10
0.20
0.50
1.00
$\frac{T}{T_c}$

The top-down model

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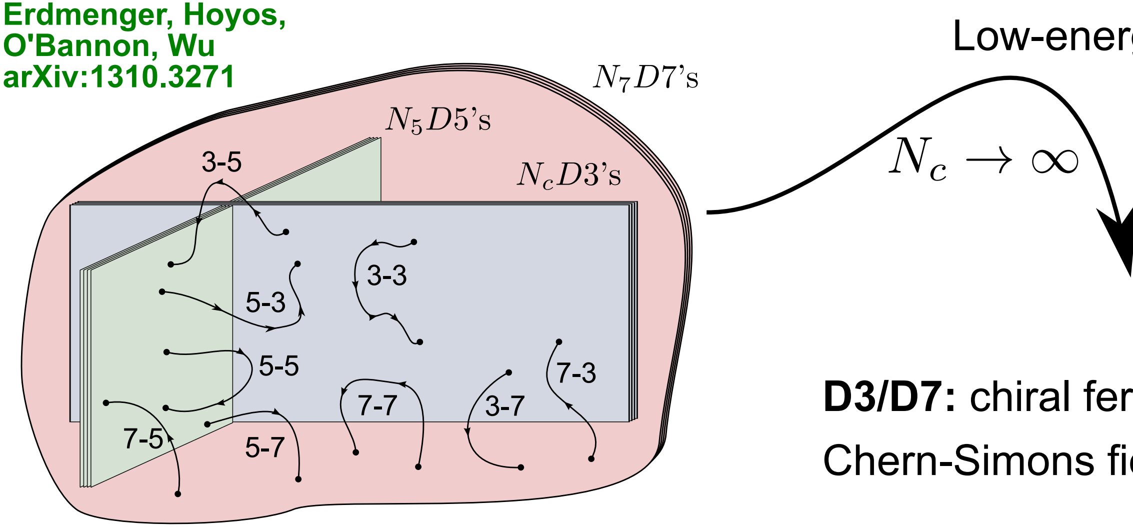

N_5, N_7 finite

	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
$D3$	•	•	•	•	–	–	–	–	–	–
$D7$	•	•	–	–	•	•	•	•	•	•
$D5$	•	–	–	–	•	•	•	•	•	–

D3/D7: chiral fermions dual to Chern-Simons field on $\mathrm{AdS}_3 \times \mathrm{S}^5$

D3/D5: slave fermions at defect dual to Yang Mills field on $\mathrm{AdS}_2 \times \mathrm{S}^4$

D5/D7: bifundamental scalar at defect dual to complex scalar on $\mathrm{AdS}_2 \times \mathrm{S}^4$