Spatial resolved radiation length (X/X_0) measurements of DEPFET pixel sensors using EUDET tracks

Ariane Frey, Benjamin Schwenker and Ulf Stolzenberg*

University of Göttingen, II. Physikalisches Institut

May 27th 2014, 16th International DEPFET Workshop

GEORG-AUGUST-UNIVERSITÄT GÖTTINGEN

VolkswagenStiftung

Method for measuring X/X_0

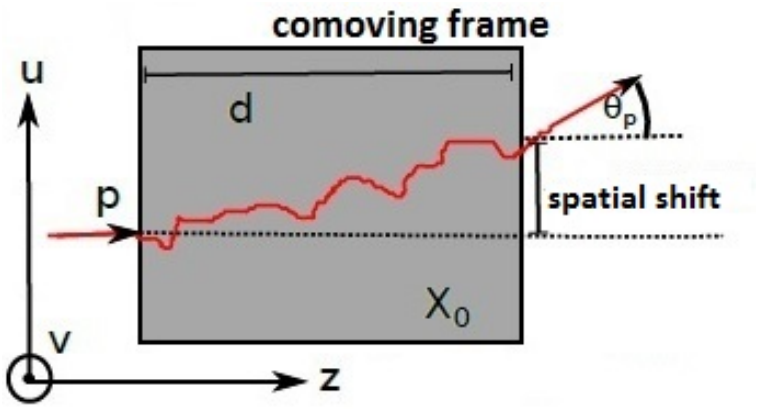
Measurement Procedure

- Use tracks from the EUDET pixel telescope to reconstruct angle distributions from multiple scattering (MSC) on a central plane
- The width of these distributions depend on the radiation length X/X_0 of the crossed material

- Is a spatial resolved X/X_0 measurement (with errors below 10%) possible, when using a EUDET telescope and a beam energy of a few GeV?
- What problems do occur during the measurement process?

Multiple scattering

- A particle with momentum p and charge q crosses matter with the radiation length X/X_0
- MSC projected angular distribution (u-z plane) $\approx$ Gaussian function with σ_{p} given by Highland Formula (HL)
  - Analog: v-z plane



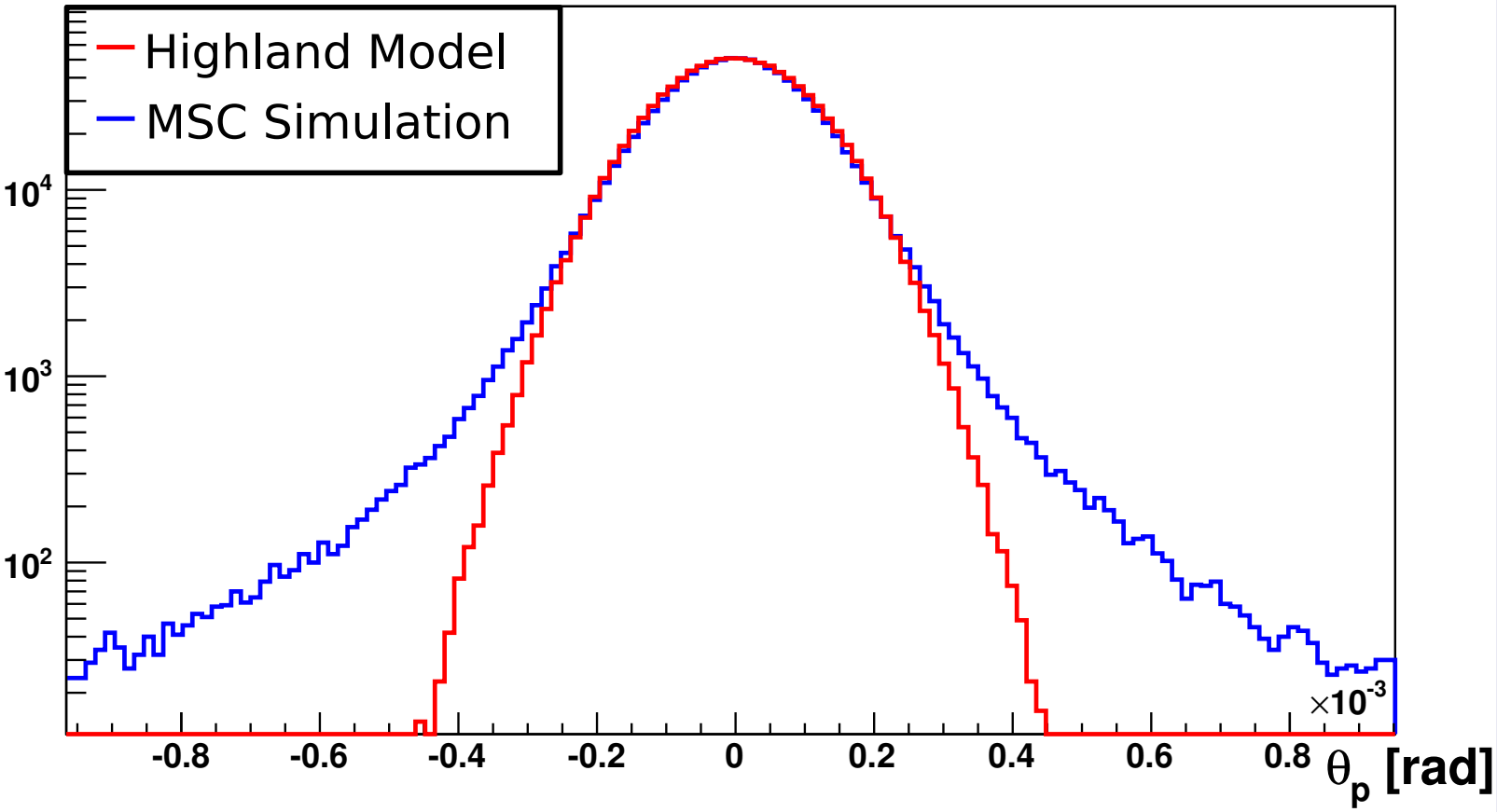
Highland Formula(only small scattering angles)

$$\sigma_{\mathrm{p}} = \frac{0.0136 \cdot q[e]}{\beta \cdot p\,[\mathrm{GeV}]} \cdot \sqrt{\frac{X}{X_0}} \left(1 + 0.0038 \ln\left(\frac{X}{X_0}\right)\right) \mathrm{rad}$$

V. L. Highland, *Some practical remarks on multiple scattering*, Nuclear Instruments and Methods,1975

MSC Models

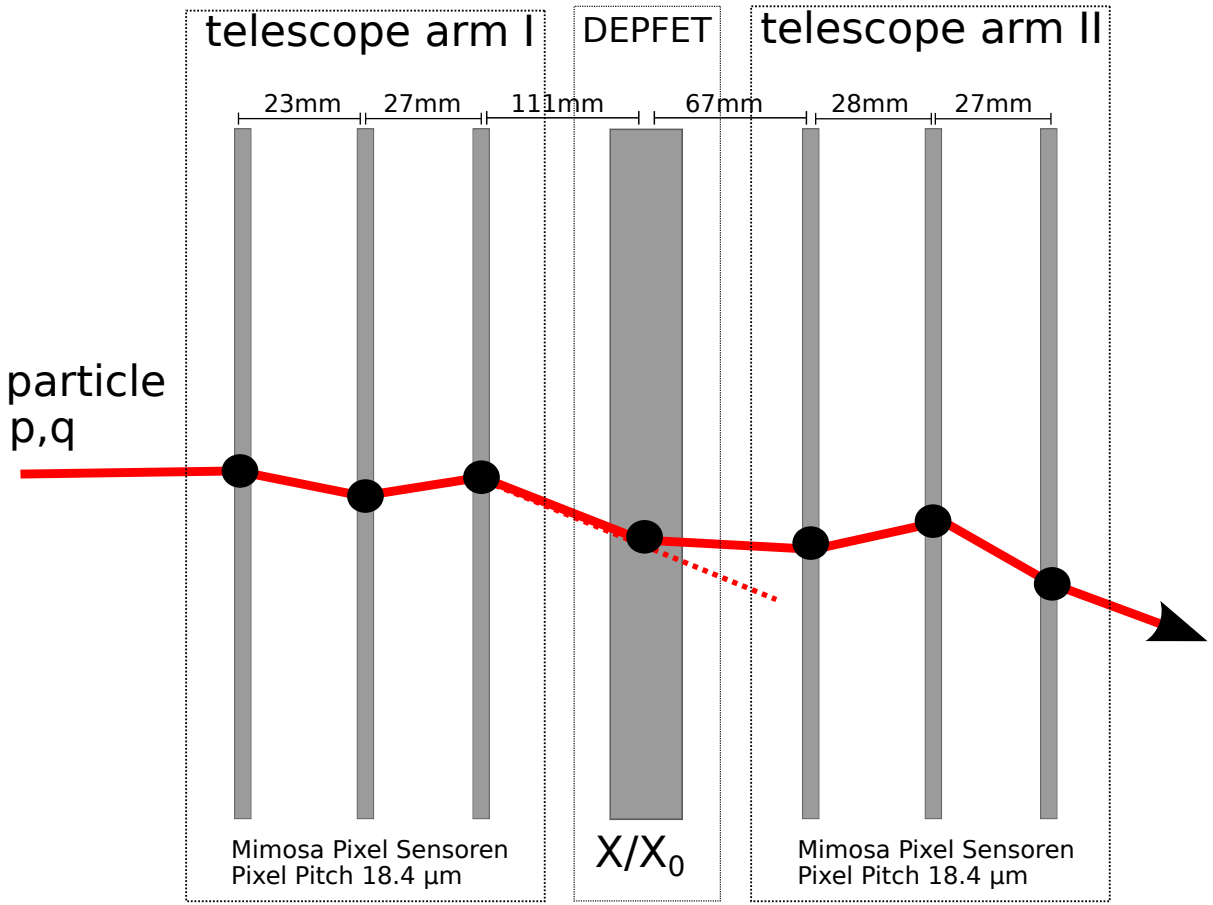
Highland model and Single Scattering model



MSC simulation based on single scatterings: See *R. Frühwirth, Nuclear Instruments and Methods, 2001*

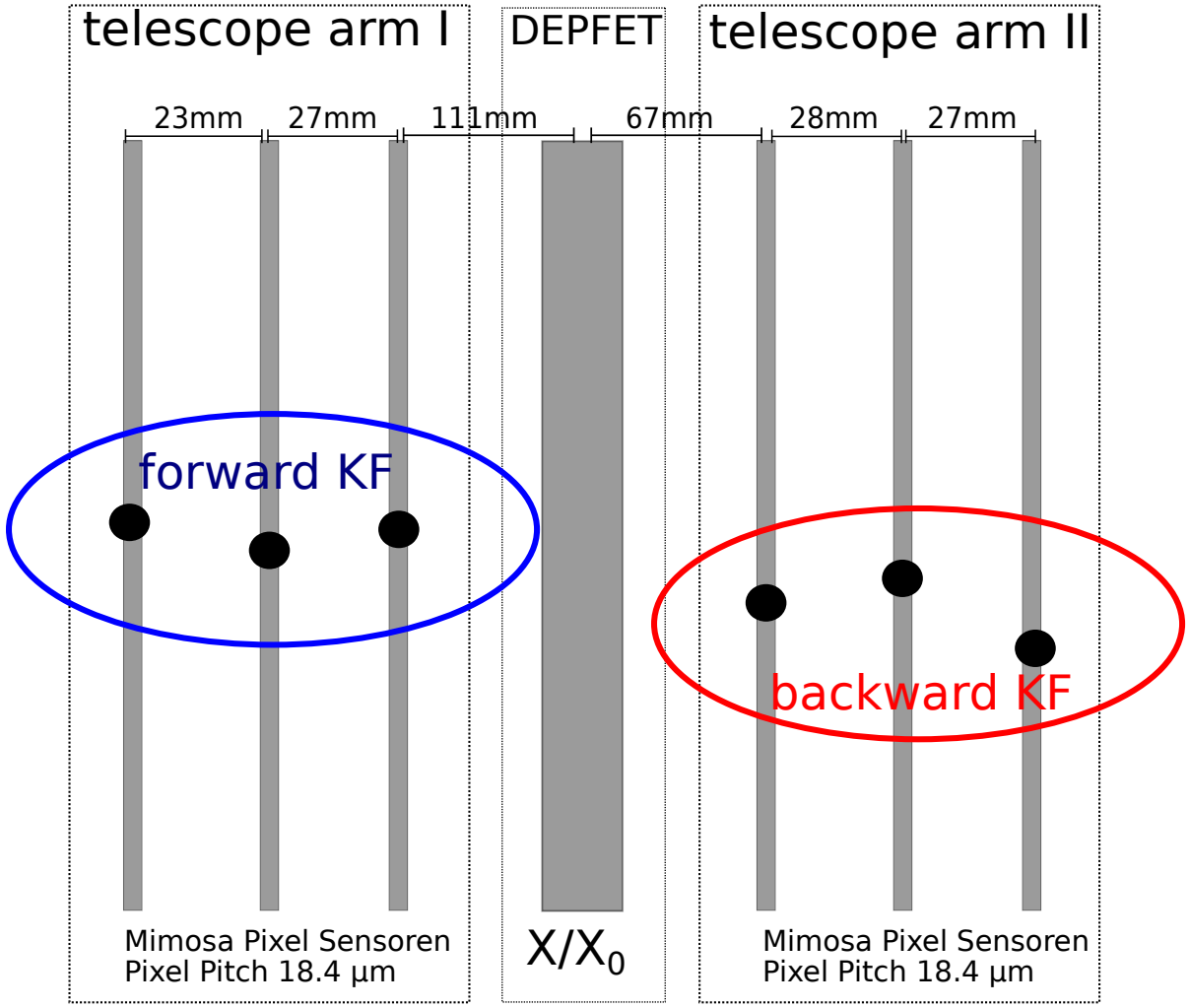
Reconstruction of MSC angles in a EUDET teleskop

- Reconstruct angles on the DEPFET
- Particle crosses sensor $\rightarrow$ hits



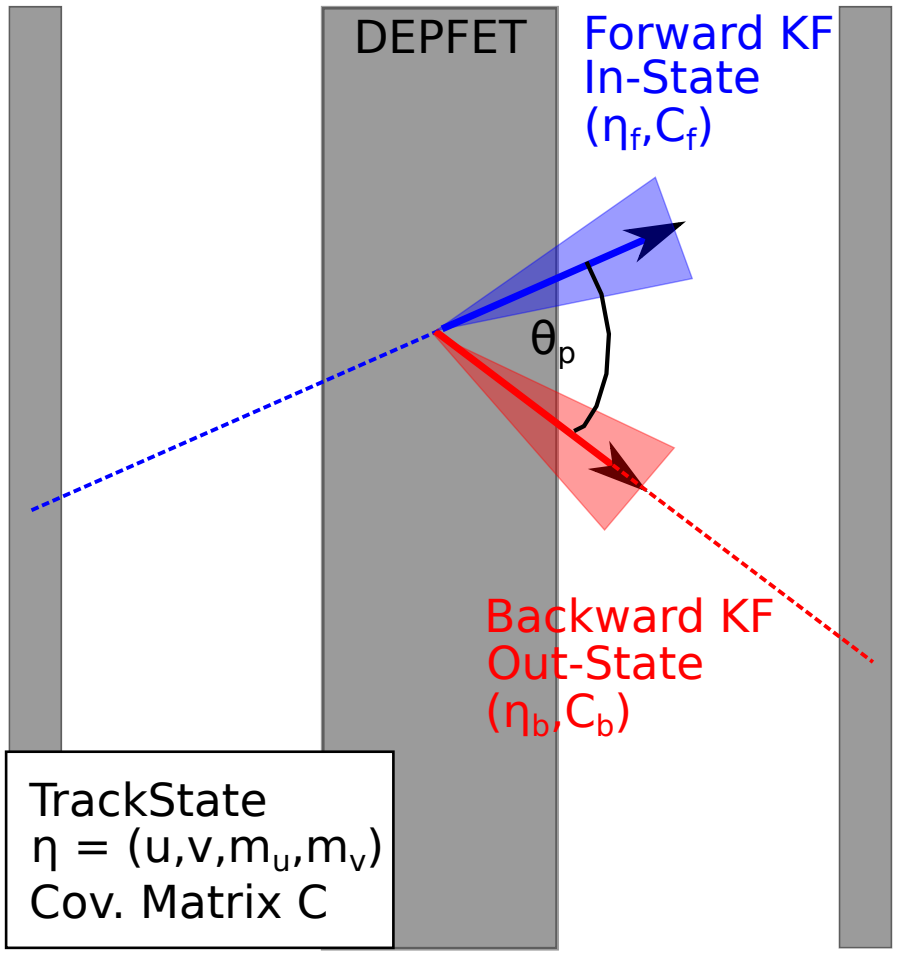
Reconstruction of MSC angles in a EUDET teleskop

- Reconstruct angles on the DEPFET
- Particle crosses sensor $\rightarrow$ hits
- Forward- backward Kalman Filter (KF) pair on hits
- hit on DEPFET not needed $\rightarrow$ maps
- Take MSC in air gaps into account

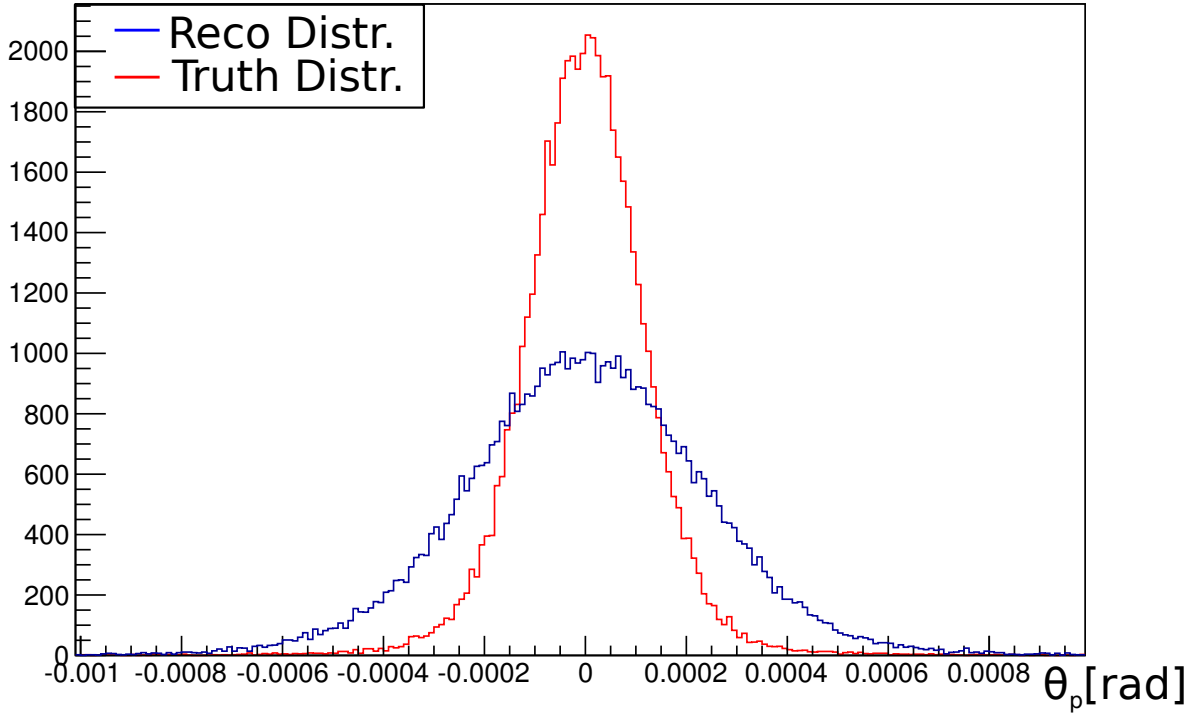


Reconstruction of MSC angles in a EUDET teleskop

- Reconstruct angles on the DEPFET
- Particle crosses sensor $\rightarrow$ hits
- Forward- backward Kalman Filter (KF) pair on hits
- hit on DEPFET not needed $\rightarrow$ maps
- Take MSC in air gaps into account
- θ_p calculated from (m_u, m_v)
- Reco error σ_{reco} from error propagation



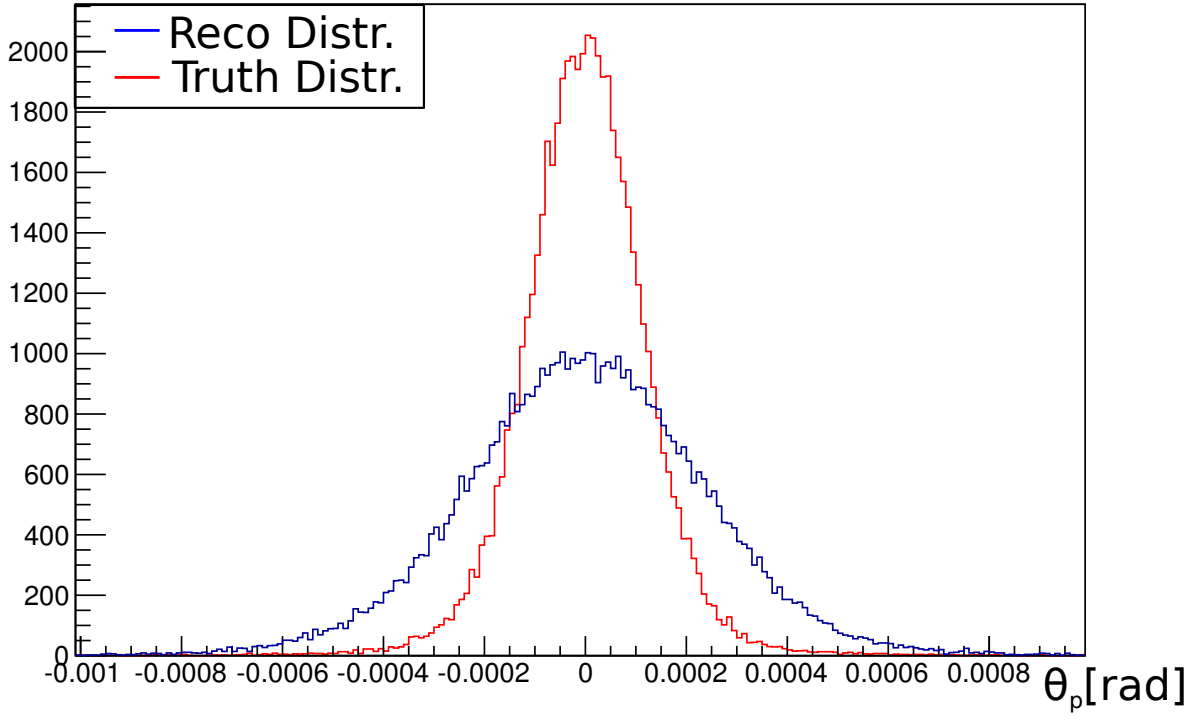
Example of a reconstructed angle distribution



Composition of the Reco Distribution

Reconstructed MSC angle distribution is a convolution between the truth MSC distribution and a Gaussian noise distribution caused by the reconstruction errors

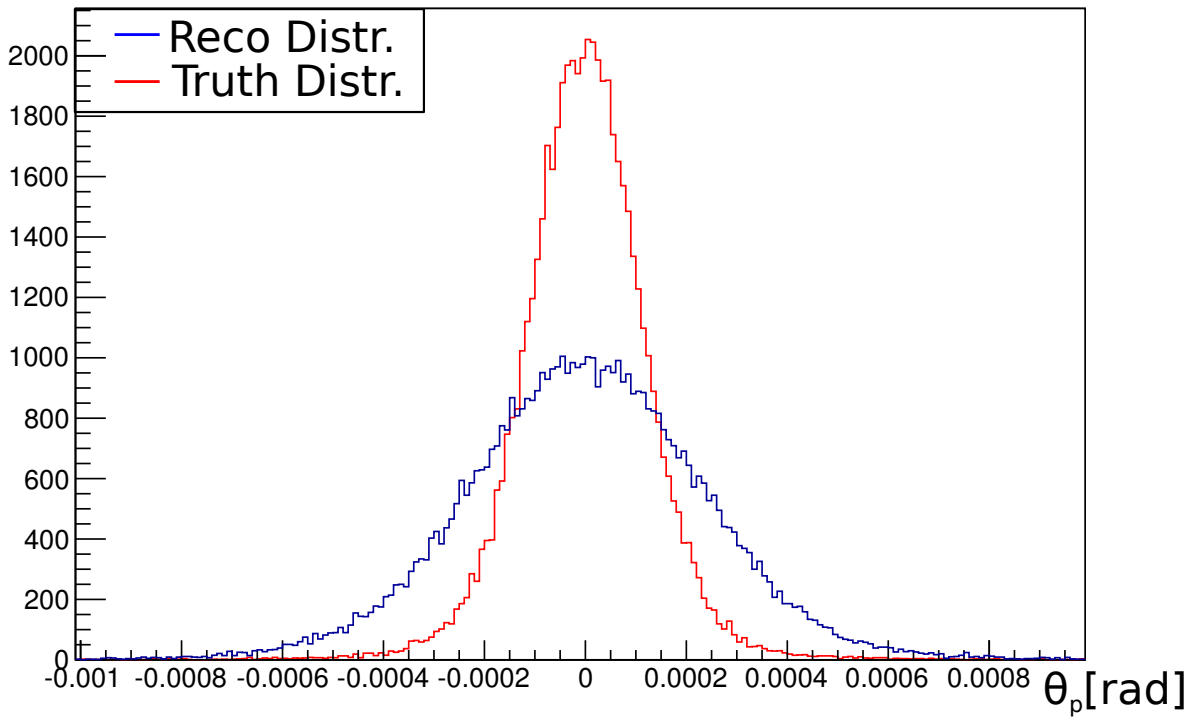
Example of a reconstructed angle distribution



Problem 1: Fitting the Reco Distribution

Non-Gaussian tails and convolution with Gaussian noise function complicate the fit
$\rightarrow$ A stable fitting method is needed

Example of a reconstructed angle distribution



Problem 2: Gaussian noise is dominant for thin materials

→ relative errors of σ_{reco} must be known well ($< 10\%$). To ensure this, we need:

- Accurate knowledge of telescope geometry, beam energy, good tracking model
- Calibration measurements for the reconstruction errors
  → $\sigma^*_{\mathrm{reco}} = \lambda \cdot \sigma_{\mathrm{reco}}$ with calibration factor λ

Solution of the 1. problem: Template fits

Templates

- Template: Distribution of MSC angles from simulations
- Add Gaussian noise due to σ_{reco}

Template Fit

- Produce several templates based on different X/X_0 or σ_{reco} values in a certain range
- Compare the templates and the distribution via Kolmogorov-Smirnov-Test (KS)

Problems

- Large track sample needed
- Only feasible in small X/X_0 ranges

σ_{reco} calibration

Best calibration scenario

- Test beam (TB) calibration runs with material plane of precisely known X/X_0
- σ_{reco} determined from error propagation
- use template fits with fixed X/X_0 to determine σ^*_{reco}

$\rightarrow$ Calibration factor $\lambda = \sigma^*_{\mathrm{reco}}/\sigma_{\mathrm{reco}}$

σ_{reco} calibration

Here: Calibration via Monte Carlo (MC) data

- Use TB Run MC Simulation $\rightarrow X/X_0$ known
- MSC model of the real particle: MSC simulation based on single scatterings
- Tracking MSC model: Highland model

$\rightarrow$ Correction of the errors (on σ_{reco}) coming from HL Model in the tracking

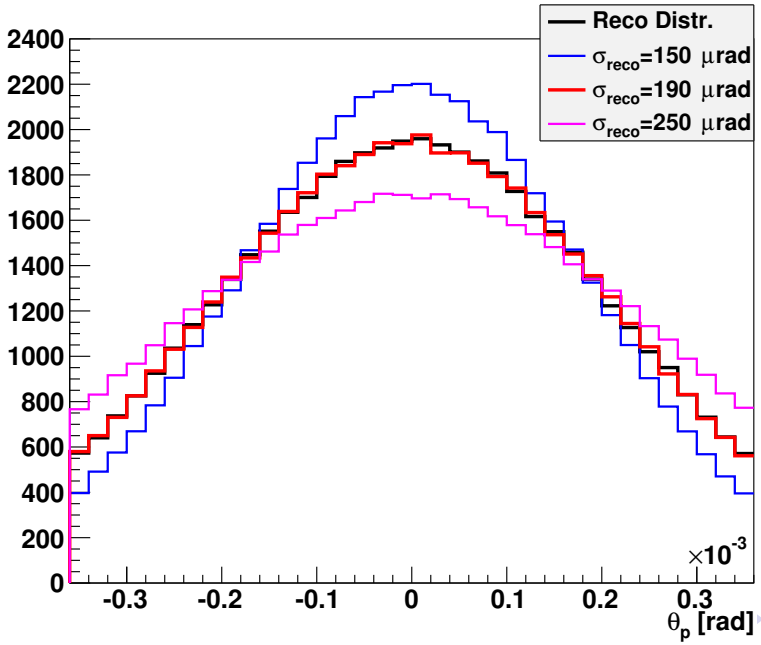
σ_{reco} calibration

Here: Calibration via Monte Carlo (MC) data

- Use TB Run MC Simulation $\rightarrow X/X_0$ known
- MSC model of the real particle: MSC simulation based on single scatterings
- Tracking MSC model: Highland model

$\rightarrow$ Correction of the errors (on σ_{reco}) coming from HL Model in the tracking

- Simu of a TB run
- 3.75 GeV electrons, σ_{reco}=173μrad
- Hybrid 4 module



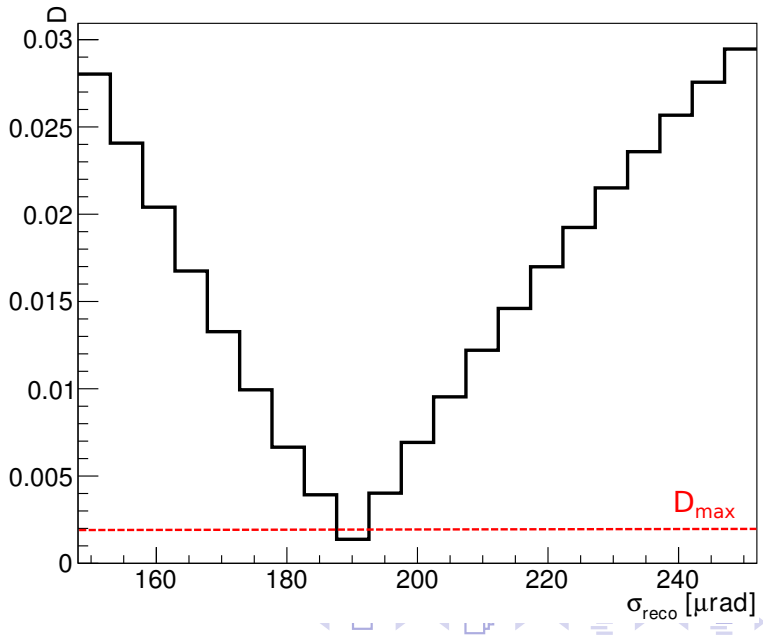
σ_{reco} calibration

Here: Calibration via Monte Carlo (MC) data

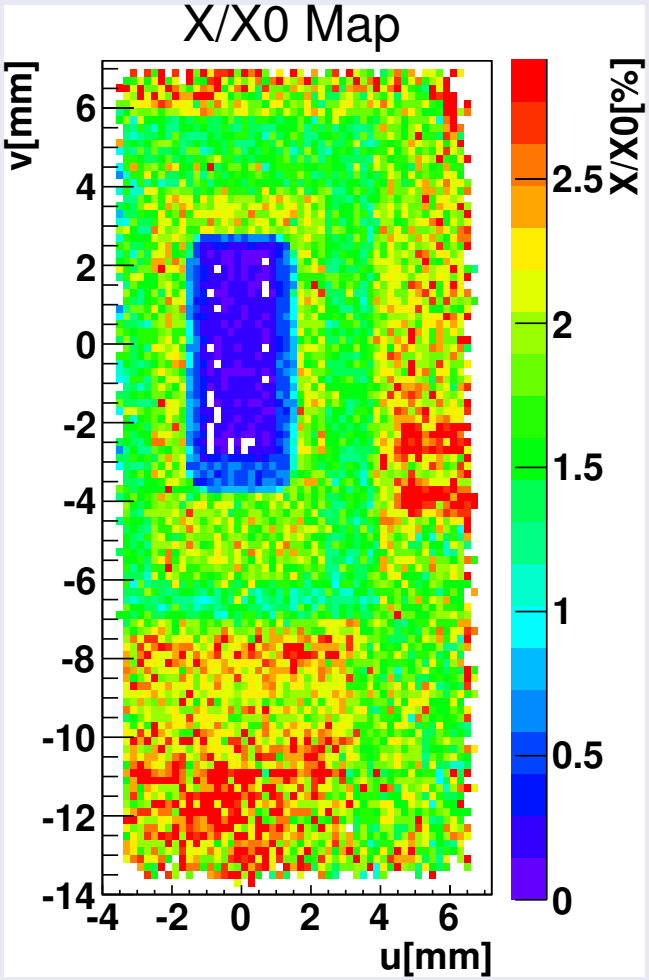
- Use TB Run MC Simulation $\rightarrow X/X_0$ known
- MSC model of the real particle: MSC simulation based on single scatterings
- Tracking MSC model: Highland model

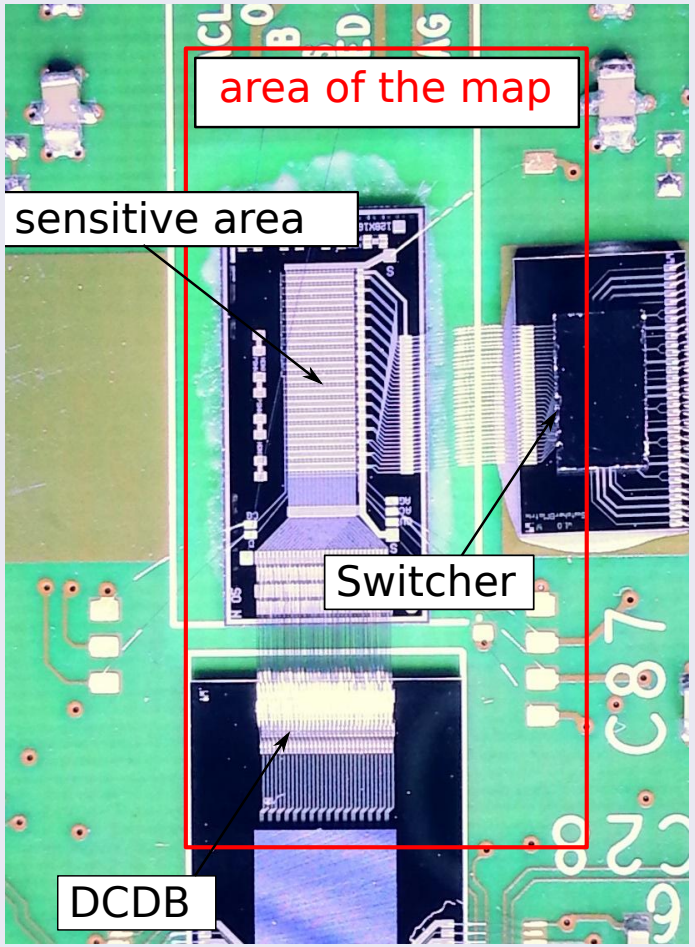
$\rightarrow$ Correction of the errors (on σ_{reco}) coming from HL Model in the tracking

- Simu of a TB run
- 3.75 GeV electrons, σ_{reco}=173μrad
- Hybrid 4 module
- Best fit: σ^*_{reco}=190 μrad
- $\lambda = \frac{\sigma_{\mathrm{reco;m}}}{\sigma_{\mathrm{reco,exp}}} \approx 1.1$

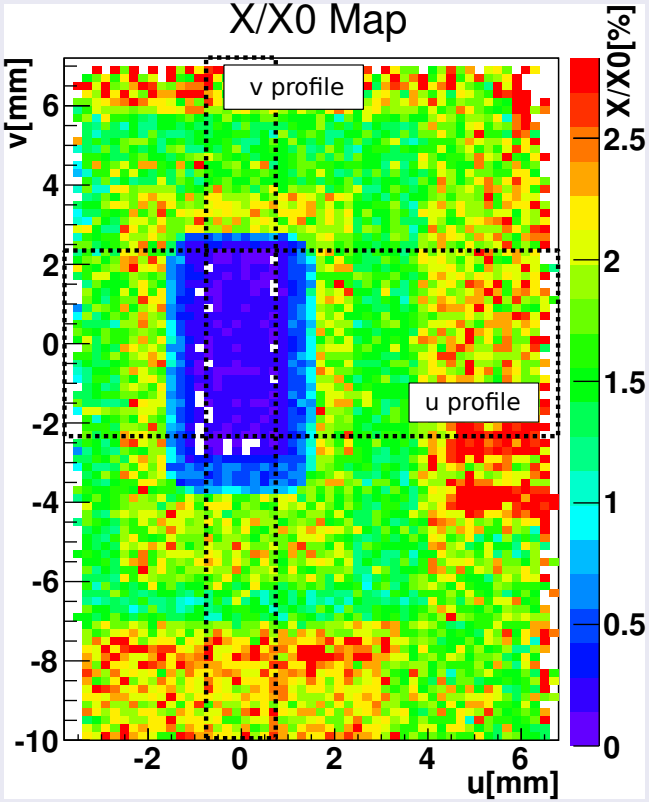


DEPFET Hybrid 4 Map



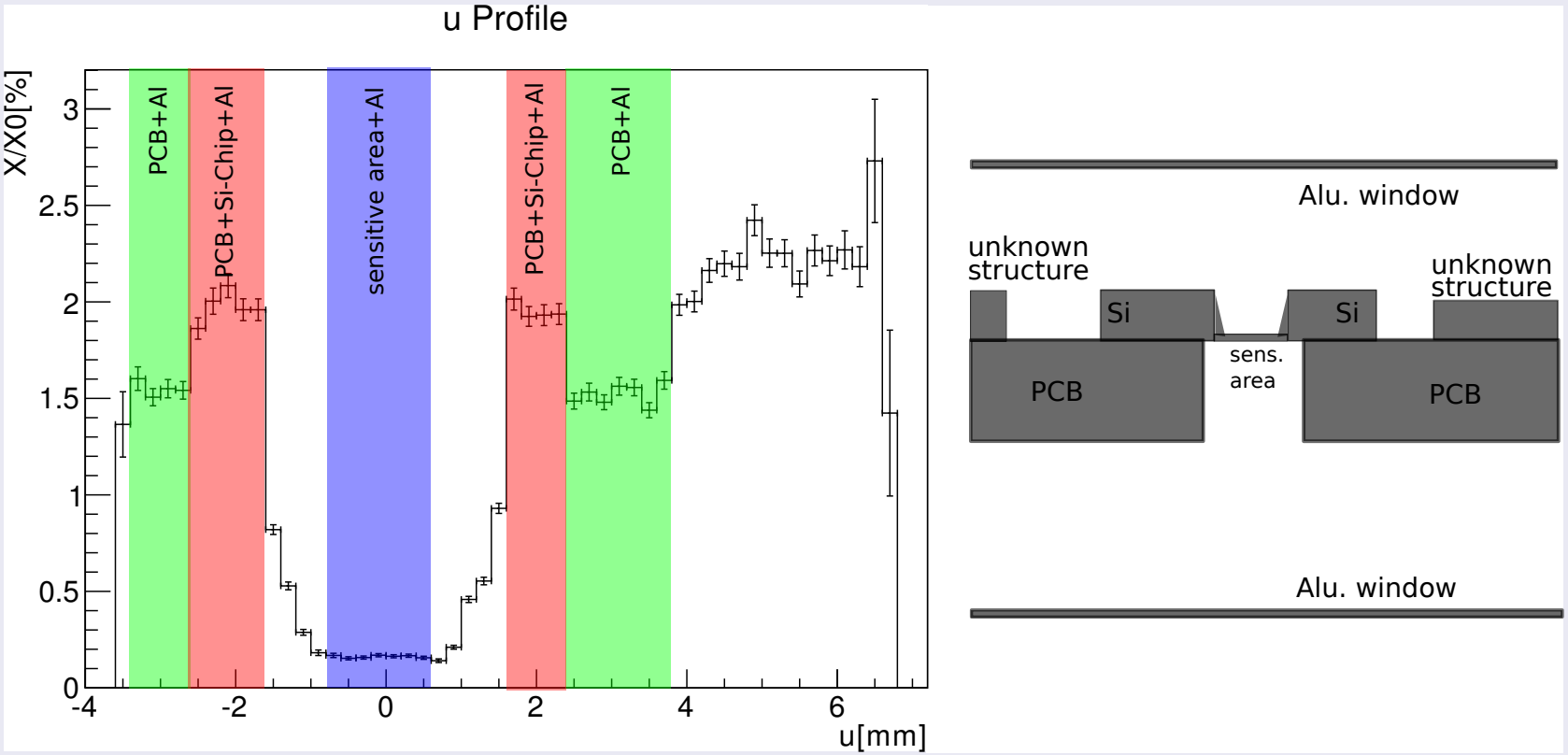


DEPFET Hybrid 4 Map

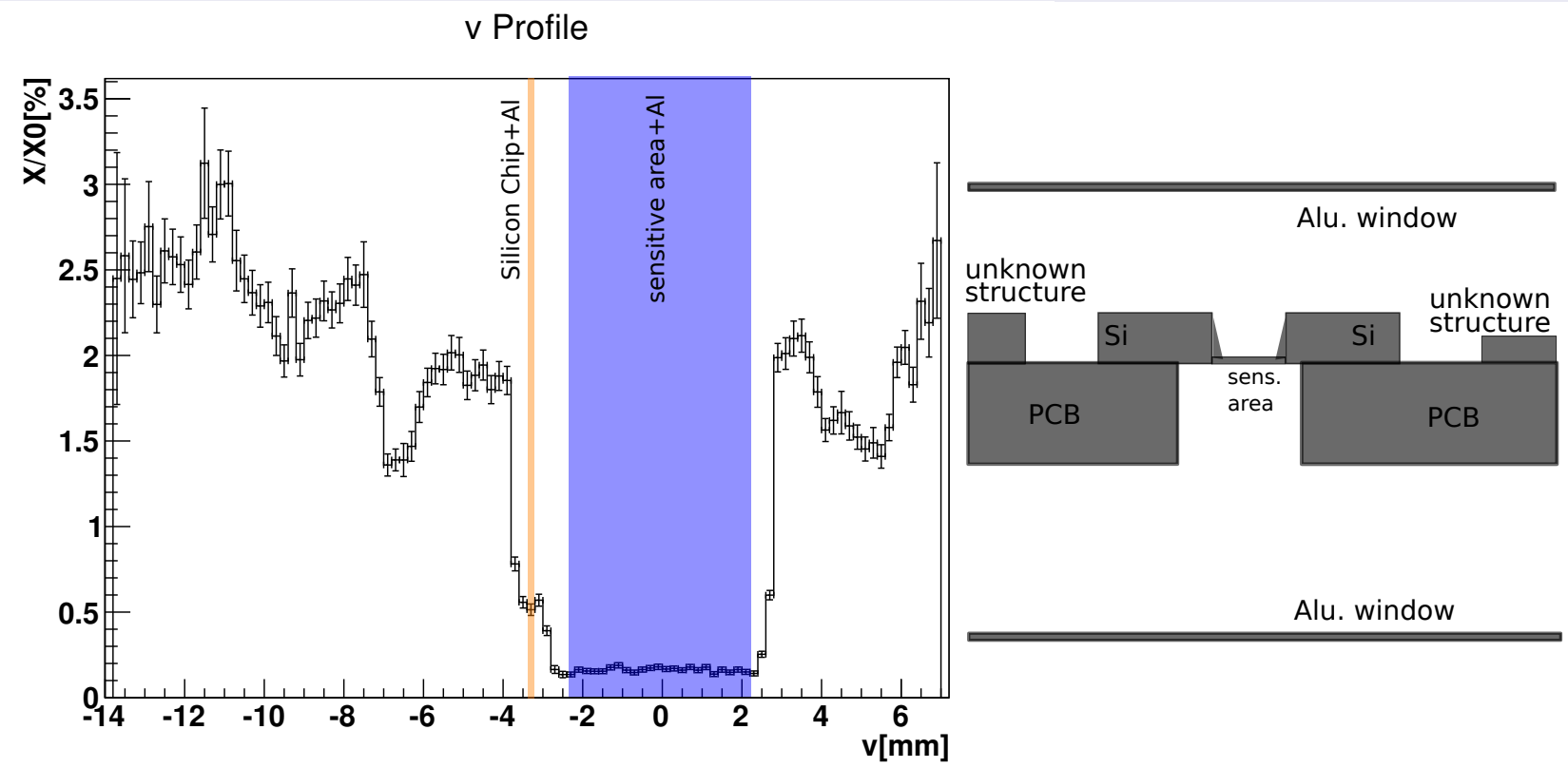


- tracksample: 1.8 Mio Tracks, pixel size: 200μm x 200μm
- fitting: Gaussian fit on distribution without tails (6% cuts on either side)
- calibration factor $\lambda = 1.1$
- use indicated profiles to compute X/X_0 in different regions of the map
- u profile: sum over 24 pixels
- v profile: sum over 8 pixels

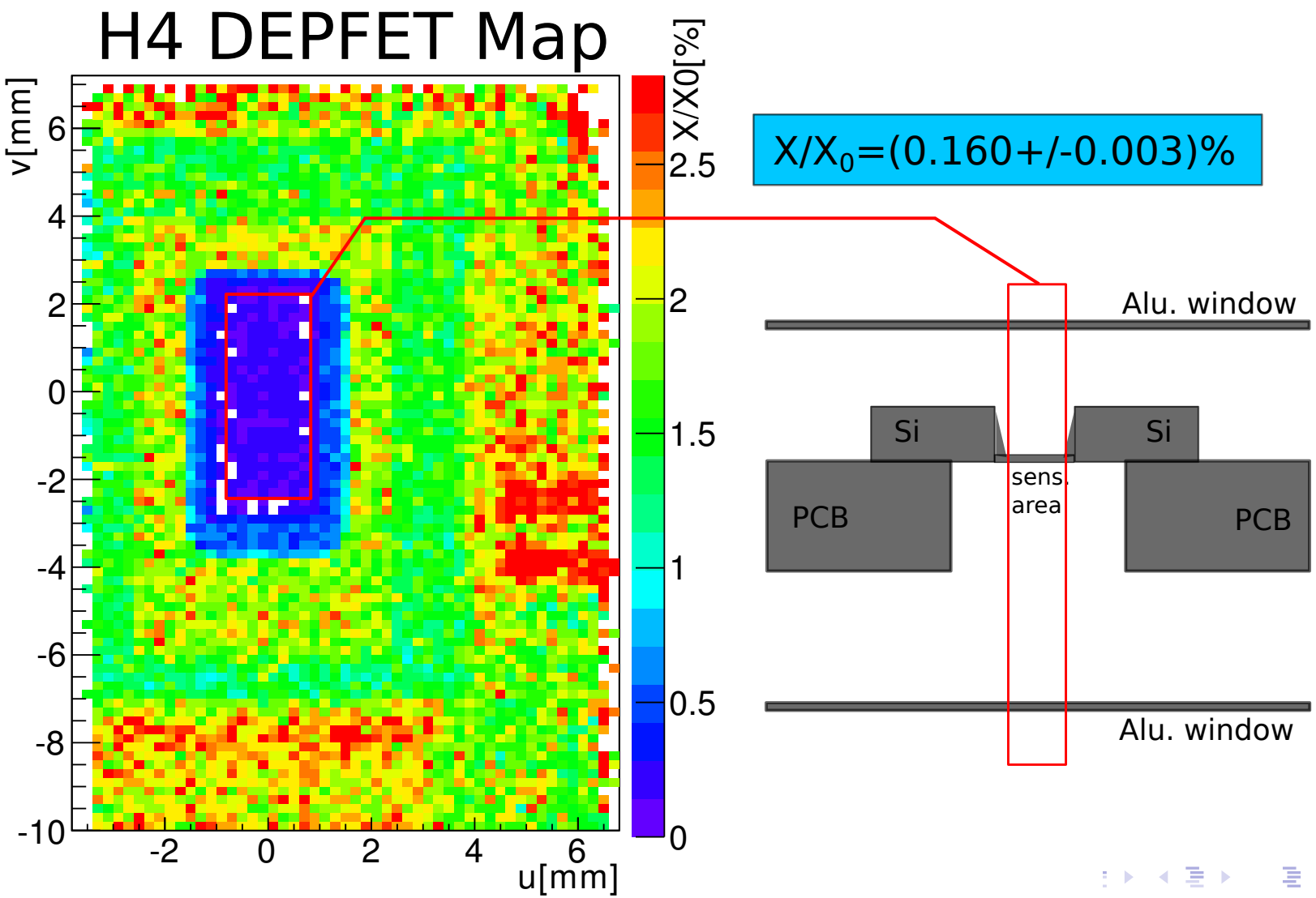
DEPFET Hybrid 4 Map



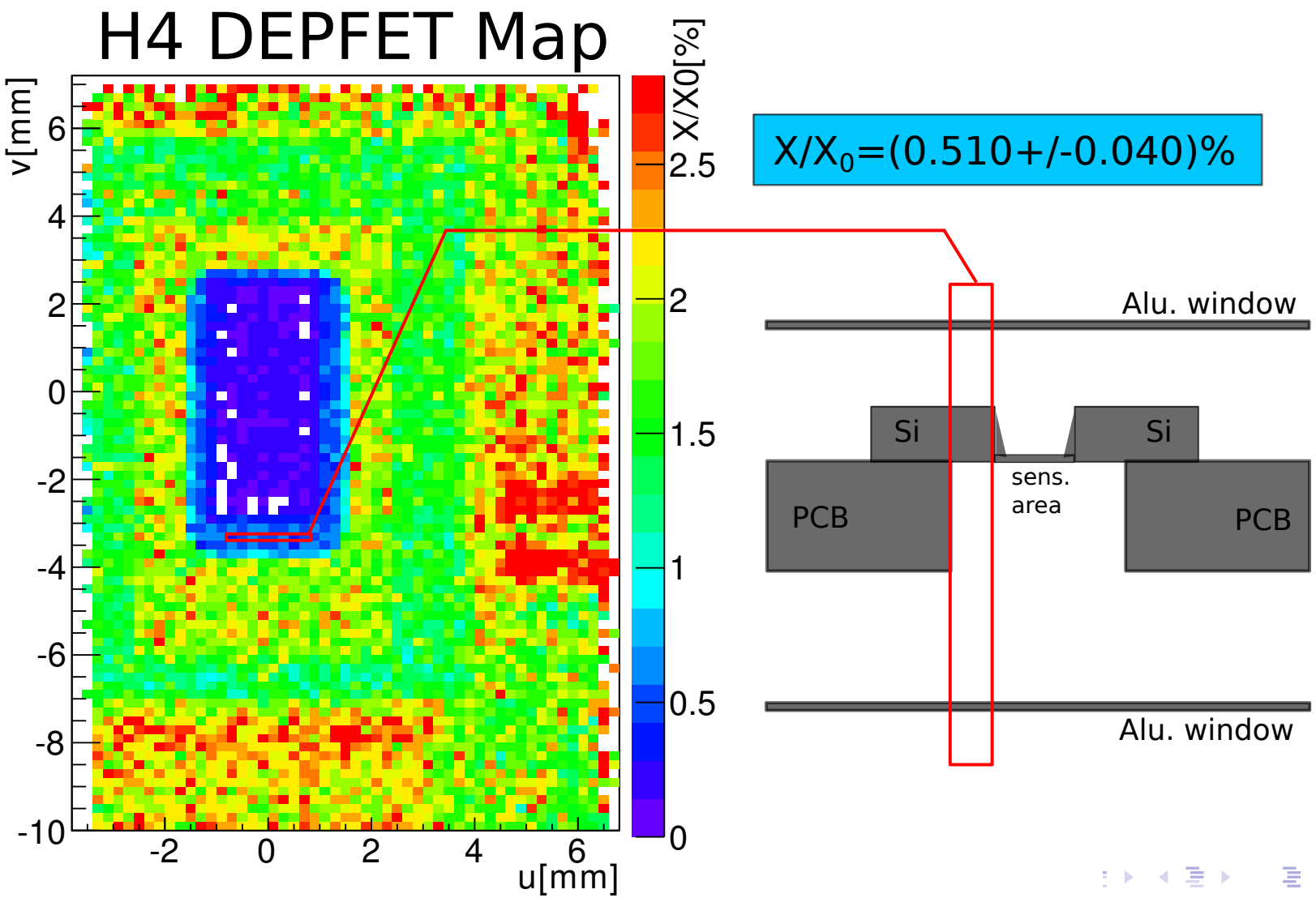
DEPFET Hybrid 4 Map



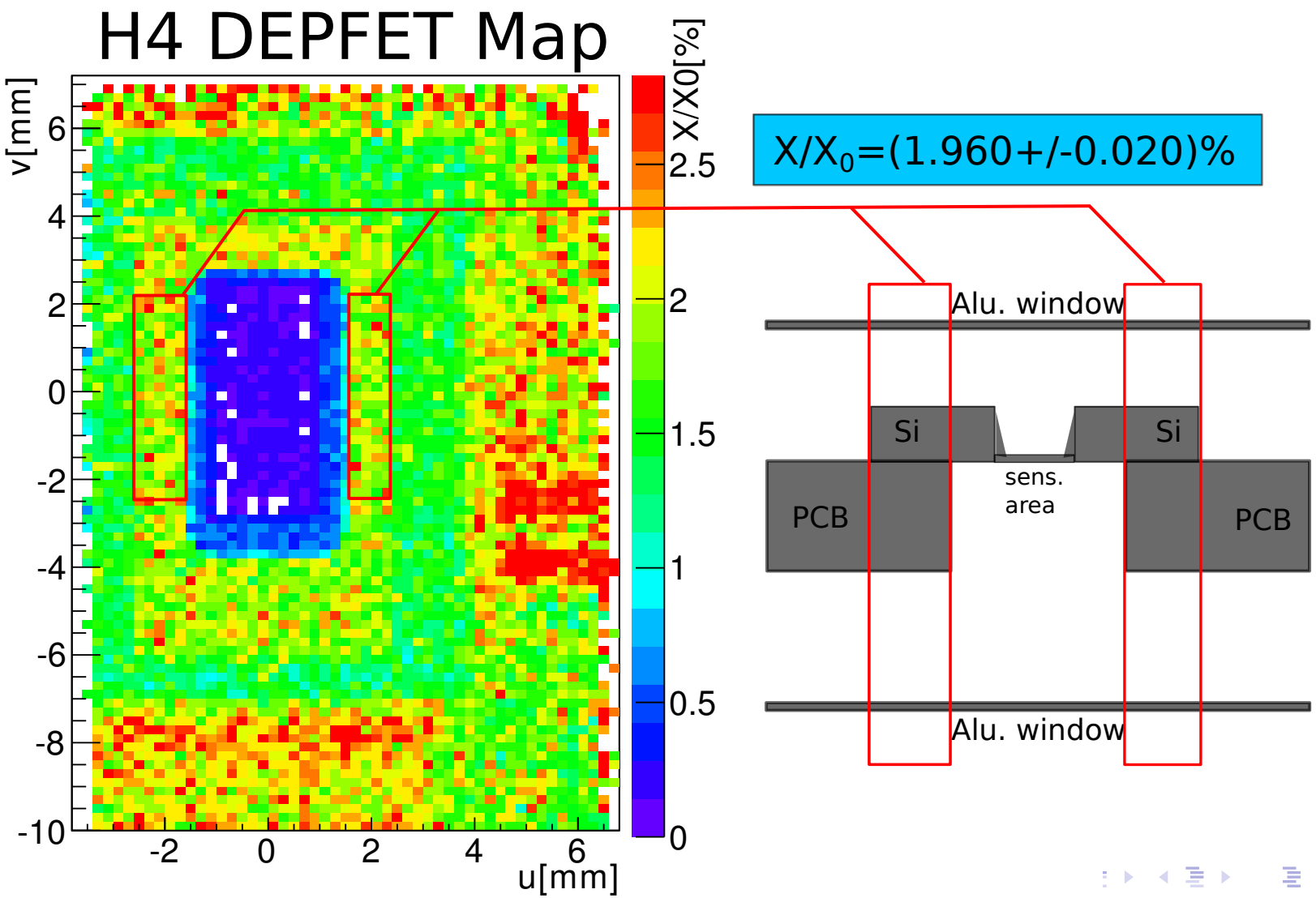
DEPFET Hybrid 4 Map



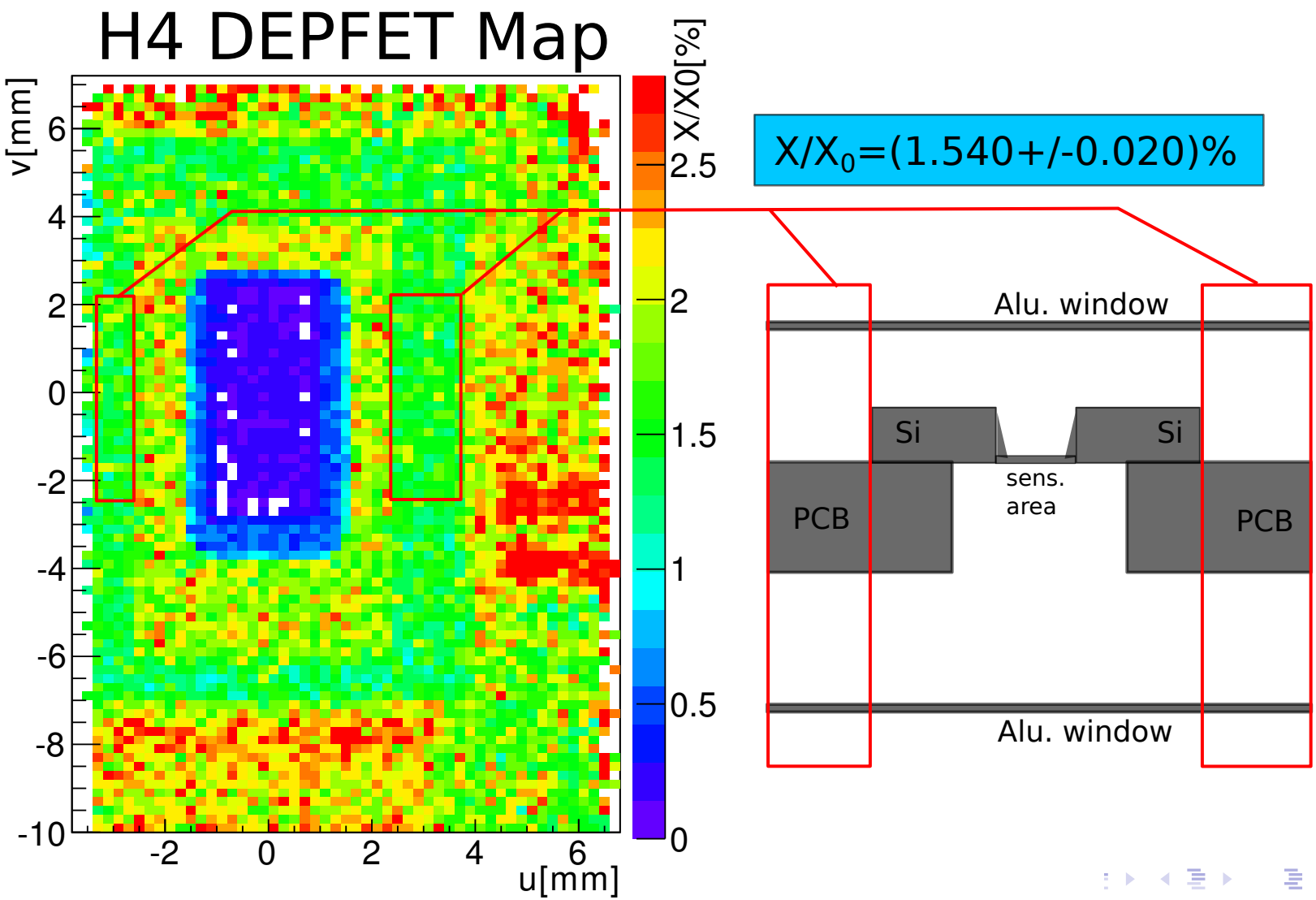
DEPFET Hybrid 4 Map



DEPFET Hybrid 4 Map



DEPFET Hybrid 4 Map



Results

Measurement results

$$
\begin{aligned}
(X/X_0)_{\mathrm{PCB+Si+Al}} &= (1.96 \pm 0.02)\,\% \\
(X/X_0)_{\mathrm{PCB+Al}} &= (1.54 \pm 0.02)\,\% \\
(X/X_0)_{\mathrm{Si+Al}} &= (0.510 \pm 0.04)\,\% \\
(X/X_0)_{\mathrm{ThinSi+Al}} &= (0.16 \pm 0.003)\,\%
\end{aligned}
$$

Radiation length and thickness of the silicon chip

$$
\begin{aligned}
(X/X_0)_{\mathrm{SiChip}} &= (X/X_0)_{\mathrm{PCB+Si+Al}} - (X/X_0)_{\mathrm{PCB+Al}} \\
&= (0.42 \pm 0.03)\,\% \\
\rightarrow d_{\mathrm{SiChip}} &= (390 \pm 30)\,\mu\mathrm{m} \quad (\mathrm{expected}: 420\,\mu\mathrm{m})
\end{aligned}
$$

Results

Measurement results

$$
\begin{aligned}
(X/X_0)_{\mathrm{PCB+Si+Al}} &= (1.96 \pm 0.02)\,\% \\
(X/X_0)_{\mathrm{PCB+Al}} &= (1.54 \pm 0.02)\,\% \\
(X/X_0)_{\mathrm{Si+Al}} &= (0.510 \pm 0.04)\,\% \\
(X/X_0)_{\mathrm{ThinSi+Al}} &= (0.16 \pm 0.003)\,\%
\end{aligned}
$$

Thickness difference between silicon chip and thin silicon layer

$$
\begin{aligned}
(X/X_0)_{\mathrm{SiGap}} &= (X/X_0)_{\mathrm{Si+Al}} - (X/X_0)_{\mathrm{ThinSi+Al}} \\
&= (0.35 \pm 0.04)\,\% \\
\rightarrow d_{\mathrm{SiGap}} &= (330 \pm 40)\,\mu\mathrm{m} \quad (\mathrm{expected}: 370\,\mu\mathrm{m})
\end{aligned}
$$

Results

Measurement results

$$
\begin{aligned}
(X/X_0)_{\mathrm{PCB+Si+Al}} &= (1.96 \pm 0.02)\,\% \\
(X/X_0)_{\mathrm{PCB+Al}} &= (1.54 \pm 0.02)\,\% \\
(X/X_0)_{\mathrm{Si+Al}} &= (0.510 \pm 0.04)\,\% \\
(X/X_0)_{\mathrm{ThinSi+Al}} &= (0.16 \pm 0.003)\,\%
\end{aligned}
$$

Radiation length and thickness of the sensitive area

$$
\begin{aligned}
(X/X_0)_{\mathrm{ThinSi}} &= (X/X_0)_{\mathrm{SiChip}} - (X/X_0)_{\mathrm{SiGap}} \\
&= (0.07 \pm 0.05)\,\% \\
\rightarrow d_{\mathrm{ThinSi}} &= (70 \pm 50)\,\mu\mathrm{m} \quad (\mathrm{expected}: 50\,\mu\mathrm{m})
\end{aligned}
$$

Results

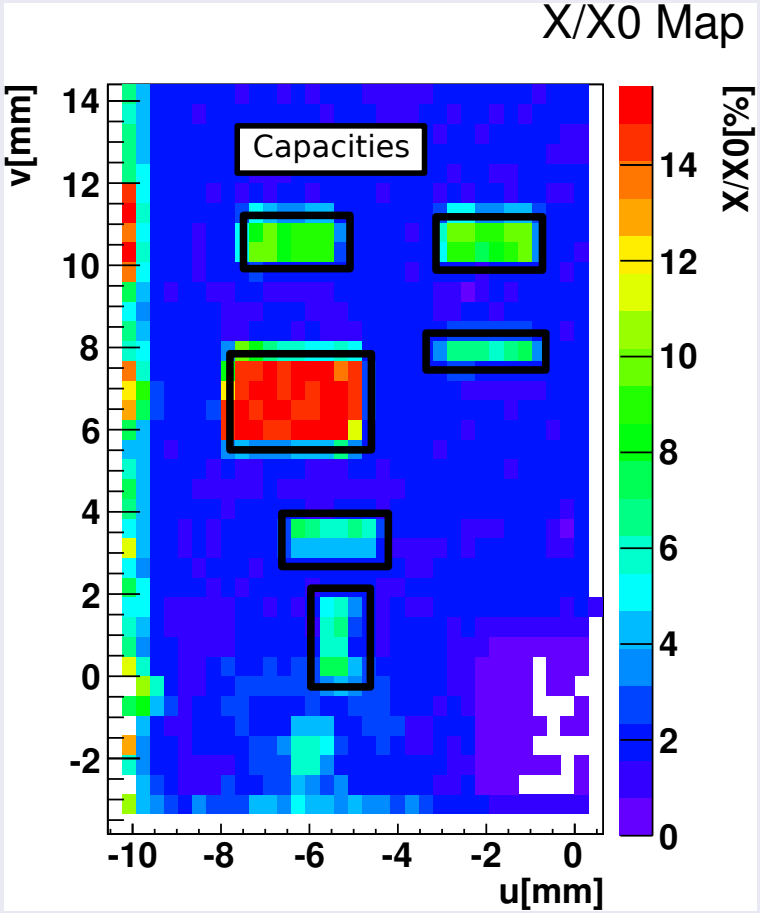
Measurement results

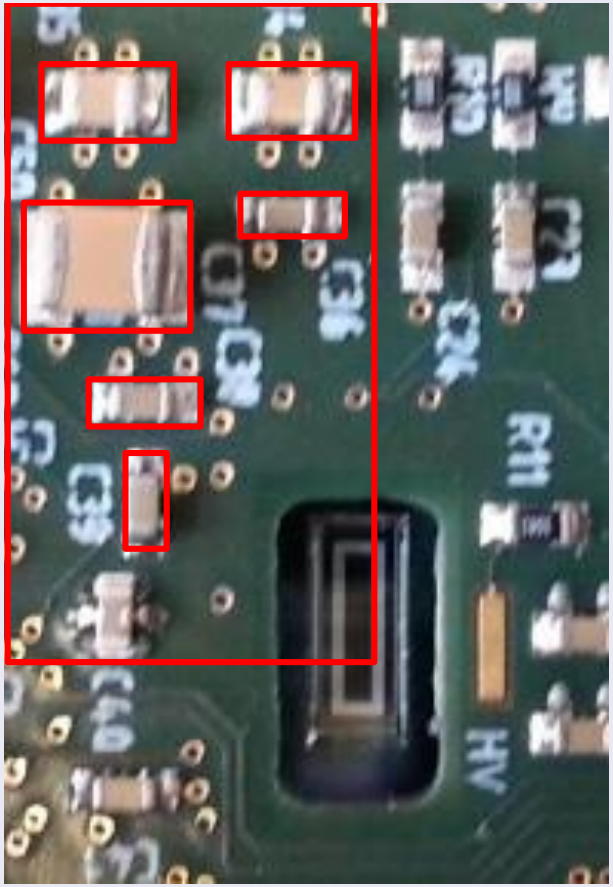
$$
\begin{aligned}
(X/X_0)_{\mathrm{PCB+Si+Al}} &= (1.96 \pm 0.02)\,\% \\
(X/X_0)_{\mathrm{PCB+Al}} &= (1.54 \pm 0.02)\,\% \\
(X/X_0)_{\mathrm{Si+Al}} &= (0.510 \pm 0.04)\,\% \\
(X/X_0)_{\mathrm{ThinSi+Al}} &= (0.16 \pm 0.003)\,\%
\end{aligned}
$$

Radiation length of the PCB

$$
\begin{aligned}
(X/X_0)_{\mathrm{PCB}} &= (X/X_0)_{\mathrm{PCB+Si+Al}} - (X/X_0)_{\mathrm{ThinSi+Al}} \\
&- (X/X_0)_{\mathrm{SiGap}} = (1.48 \pm 0.06)\,\% \\
(\mathrm{expected}: X/X_0 &= 1.50\,\%)
\end{aligned}
$$

DEPFET Hybrid 5 Map





Conclusion and future plans

Conclusion

- A method to estimate the spatial resolved radiation length has been developed and tested on TB data
- Relative and absolute values of X/X_0 can be determined
- A calibration of the reconstruction errors is needed for a precise measurement of the radiation length of thin materials (100 μm Si)

Future plans

- What are the effects of other systematicals like energy loss of the particle beam in the telescope?
- Larger track samples/merging track samples: Improve spatial resolution in the map and reduce statistical errors of the pixels

Many Thanks for your attention!

Back Up Slides

Kolmogorov Smirnov Test

- Used for testing, if two data sets are based on the same PDF
- Test parameter: maximum distance between the two cumulative distributions
- Used here: 5 % significance level
- Critical value for the test parameter: $D_{\max} = \frac{1.36}{\sqrt{N}}$,
  N: Number of data points

Forward and Backward KF

Forward KF

- gives In-State
- prediction of track state on sensor i+1 based on tracks state i

$$\begin{aligned} \eta_{i+1} &= F_{i\to i+1}\eta_i \\ V_{i+1} &= F_{i\to i+1}V_i F^{\mathrm{T}}_{i\to i+1} \\ &+ Q_{\mathrm{F;i}} \end{aligned}$$

Backward KF

- gives Out-State
- prediction of track state on sensor i based on tracks state i+1

$$\begin{aligned} \eta_i &= F_{i+1\to i}\eta_{i+1} \\ V_i &= F_{i+1\to i}V_{i+1}F^{\mathrm{T}}{}_{i+1\to i} \\ &+ Q_{\mathrm{B};i+1} \end{aligned}$$

- F: Extrapolation matrix
- Q: MSC effects