STANDARD MODEL

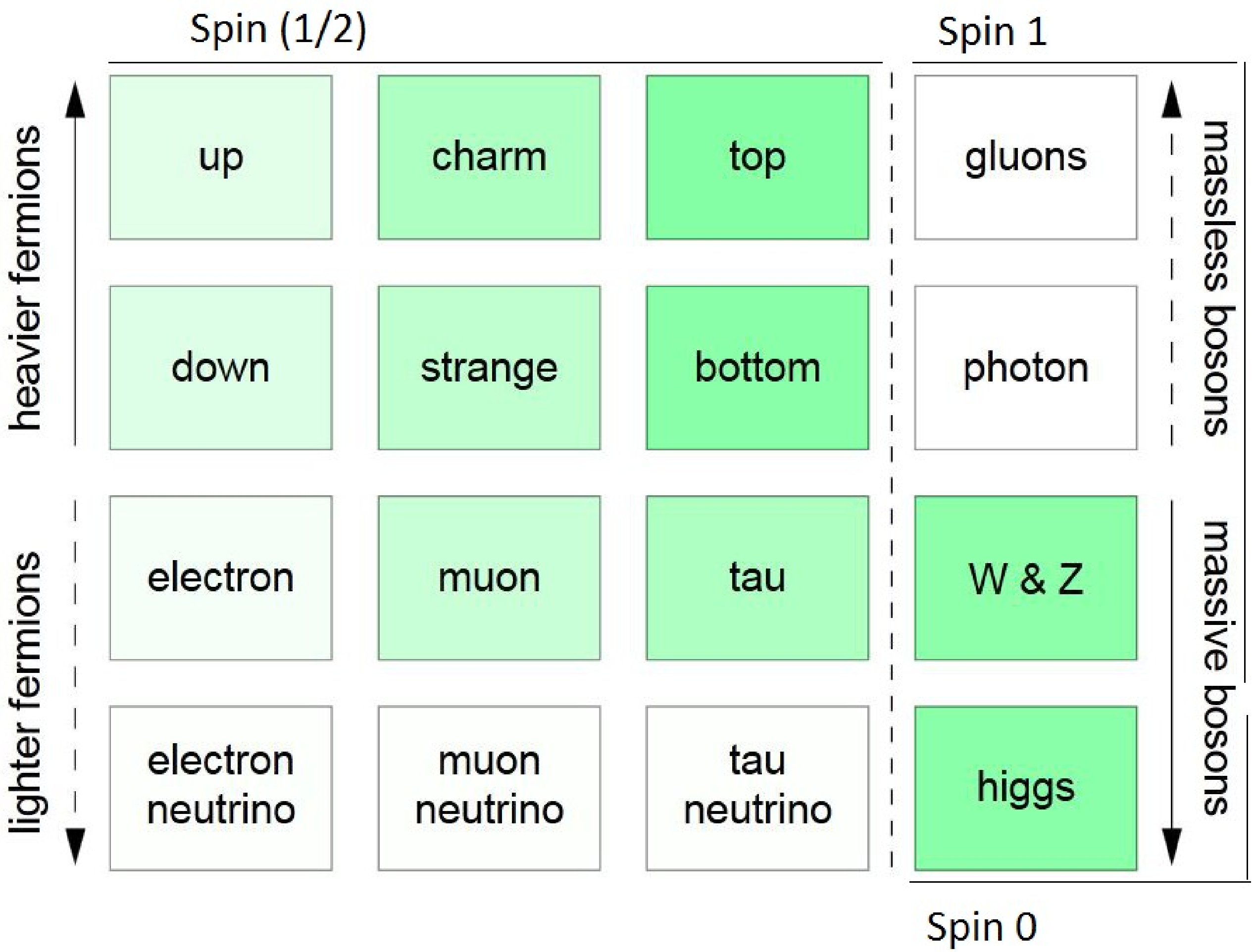
By Abhishek Khanna

A Brief history

- 1934 Fermi 4-point theory
- 1967/68 Standard Model (Glashow, Salam, Weinberg)
- 1971 Renormalizibility of non-abelian theories

  ('t Hooft Veltmann)
- 1973 Assymptotic Freedom of QCD(Gross, Wilzcek)
- 1974 Discovery of Neutral Currents (CERN)
- 1979 Discovery of gluon (PETRA,DESY)
- 1983 Observation of W and Z (CERN)
- 1995 Discovery of Top (Tevatron, Fermilab)
- 2012 Higgs Particle

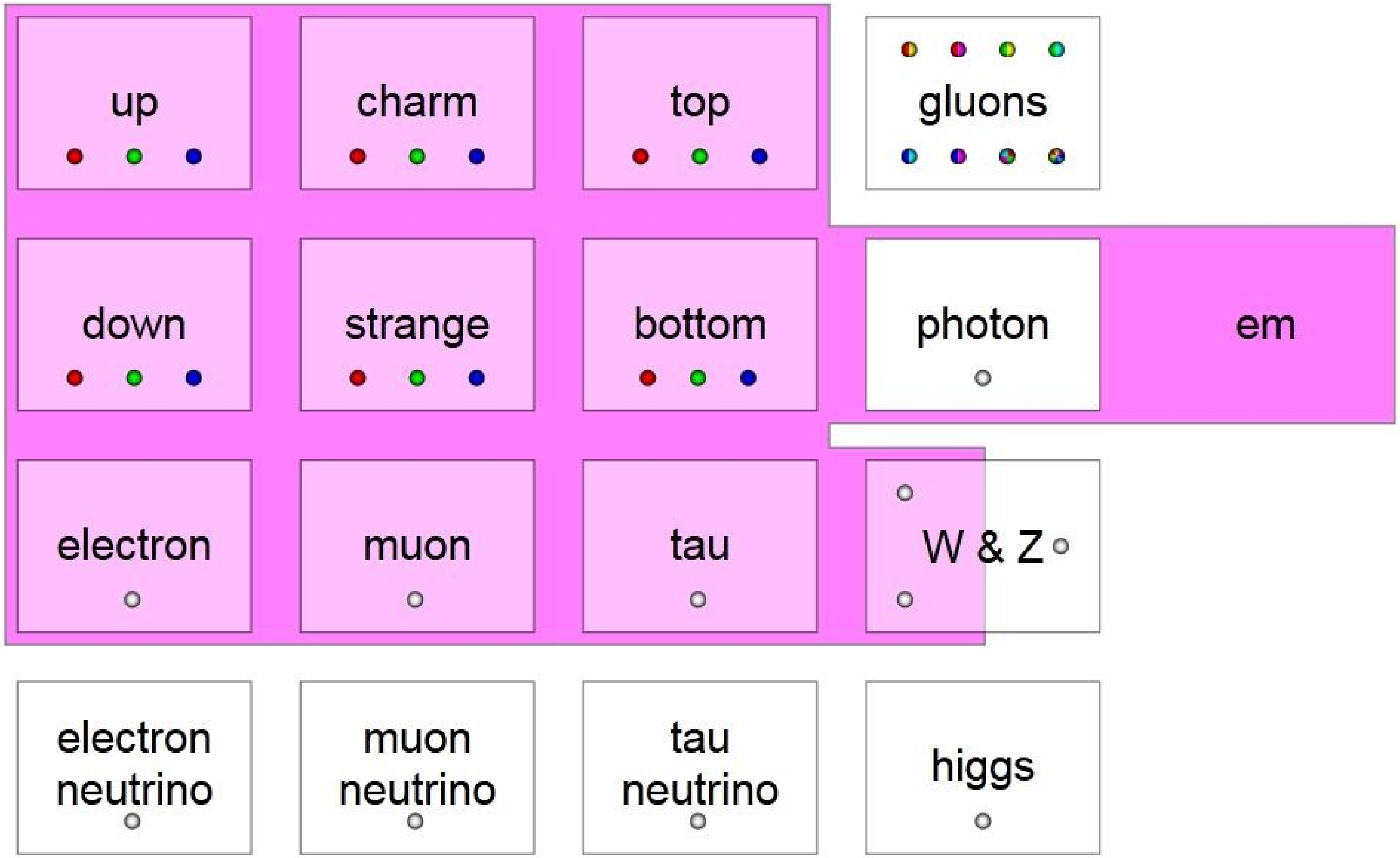
Overview


Spin (1/2)
Spin 1
heavier fermions
lighter fermions
up
charm
top
gluons
down
strange
bottom
photon
electron
muon
tau
W & Z
electron neutrino
muon neutrino
tau neutrino
higgs
massless bosons
massive bosons
Spin 0

Overview

	I	II	III		
+2/3	up	charm	top	gluons	0
−1/3	down	strange	bottom	photon	0
−1	electron	muon	tau	W & Z	+1 / 0 / −1
0	electron neutrino	muon neutrino	tau neutrino	higgs	0

Overview: EM


up
charm
top
gluons
down
strange
bottom
photon
em
electron
muon
tau
W & Z
electron neutrino
muon neutrino
tau neutrino
higgs

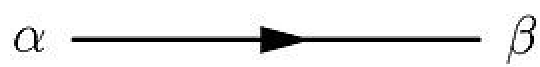
Overview: Strong

Overview: Weak

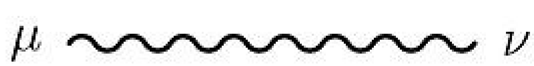
up	charm	top	gluons	
down	strange	bottom	photon	
electron	muon	tau	W & Z	weak
electron neutrino	muon neutrino	tau neutrino	higgs	

When Higgs particle is also considered, we may call the interaction as "Higgs"

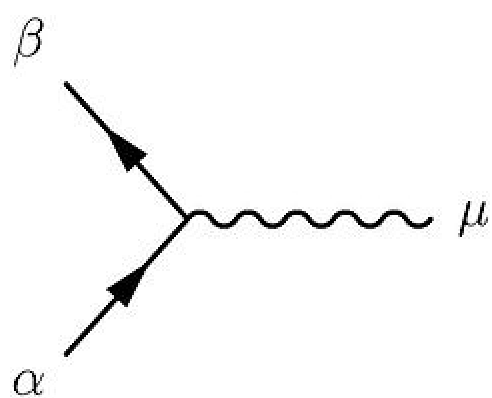
Quantum Electrodynamics



$$\rightarrow \quad \left(\frac{i}{\not{p} - m + i\varepsilon}\right)_{\beta\alpha}$$



$$\rightarrow \quad \frac{-i\eta_{\mu\nu}}{p^2 + i\varepsilon}$$

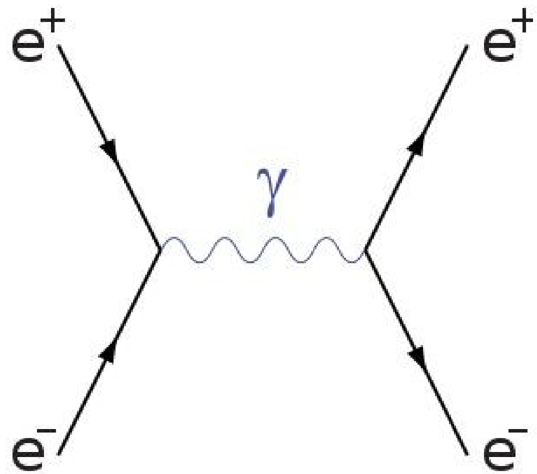


$$\rightarrow \quad -ie\gamma^{\mu}_{\beta\alpha}(2\pi)^4\delta^{(4)}(p_1 + p_2 + p_3).$$

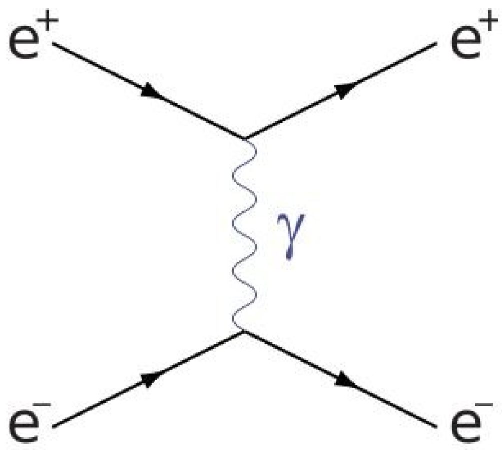
Quantum Electrodynamics

Incoming fermion:	α 	$\rightarrow$	$u_\alpha(\vec{p}, s)$
Incoming antifermion:	α 	$\rightarrow$	$\overline{v}_\alpha(\vec{p}, s)$
Outgoing fermion:	 α	$\rightarrow$	$\overline{u}_\alpha(\vec{p}, s)$
Outgoing antifermion:	 α	$\rightarrow$	$v_\alpha(p, s)$
Incoming photon:	μ 	$\rightarrow$	$\epsilon_\mu(\vec{k}, \lambda)$
Outgoing photon:	 μ	$\rightarrow$	$\epsilon_\mu(\vec{k}, \lambda)^*$

Quantum Electrodynamics

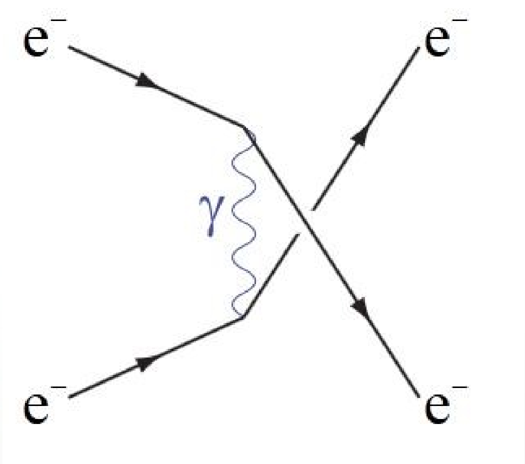


S - Channel



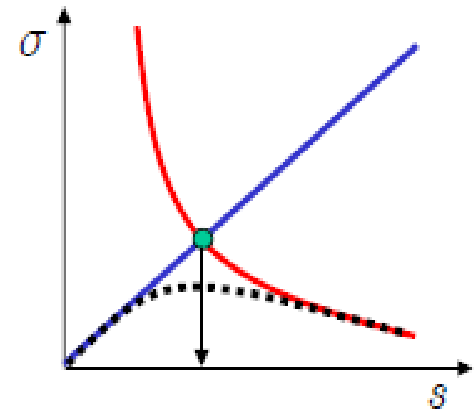
T – Channel

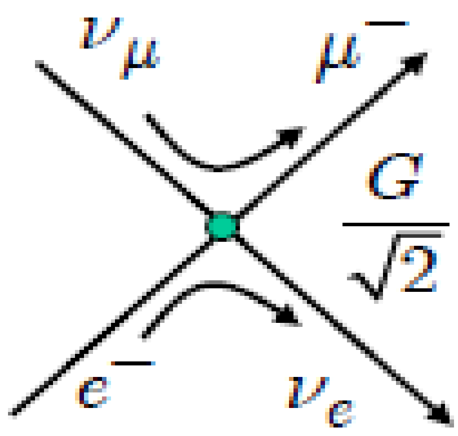
U – Channel



Weak Interaction

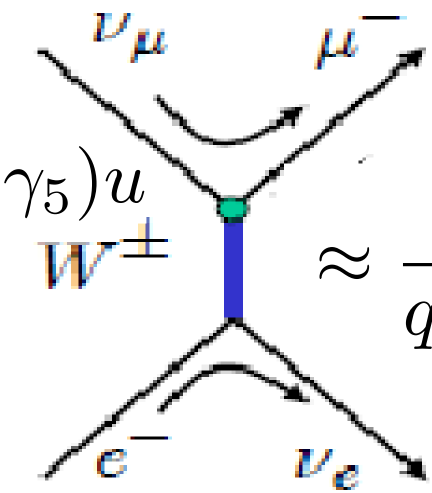
- Fermi's 4-point theory
- First theory: 1934





$$\frac{G}{\sqrt{2}}$$

$$-ig\bar{u}\gamma_\mu(1-\gamma_5)u$$



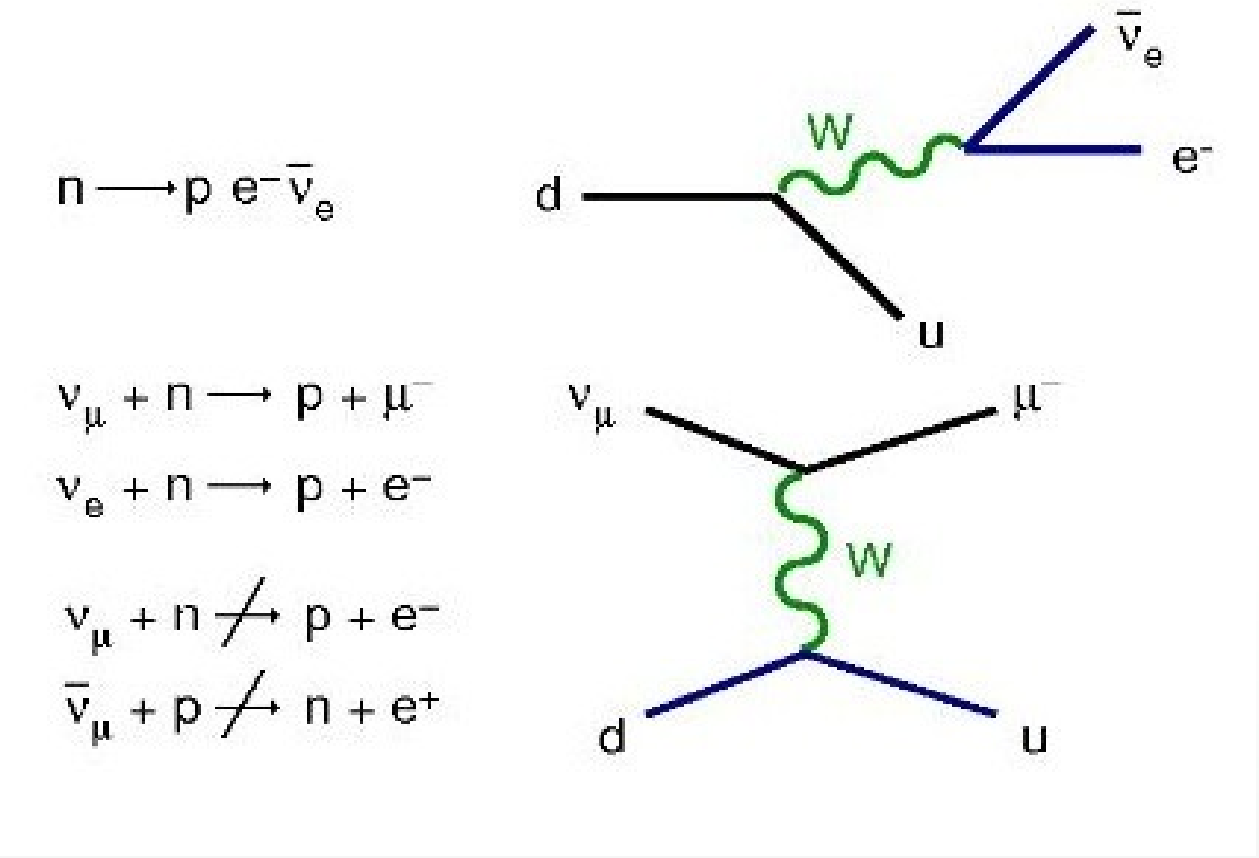
$$\approx \frac{ig^{\mu\nu}}{q^2 - M_W^2}$$

$$\sigma \approx G^2 s$$

$$\sigma \approx \frac{1}{s};\, s \to \infty$$

Weak Interaction

- Typical Interaction of W and Z bosons
- Lepton Number preserved



Weak Interaction by QED

$$L_{globalU(1),massive} = \bar{\psi}(i\partial\!\!\!/ + m)\psi$$

- Not Local U(1)

$$\partial_\mu \rightarrow D_\mu = \partial_\mu + ig\partial_\mu(x)$$

- Parity Violation under SU(2) transformation

$$\psi = \psi_L + \psi_R$$
$$\psi_L = P_L\psi$$
$$\psi_R = P_R\psi$$

  - Left Handed Particles: Doublets
  - Right Handed Particles: Singles

Weak Interaction by QED

- Projections:

$$P_{R,L} = \frac{1 \pm \gamma^5}{2}$$

$$P_L^2 = P_L, P_R^2 = P_R$$

$$P_R P_L = P_L P_R = 0$$

- SU(2) Invariant

$$\bar{\psi} m \psi = \bar{\psi}_L m \psi_R + \bar{\psi}_R m \psi_L$$

$$SU(2) \rightarrow \psi_L^\dagger U^\dagger \gamma^o m \psi_R + \psi_R \gamma^0 m U \psi_L$$

Common Description?

- Common Description of QED and Weak Interactions?

$$QED : U(1)_{QED} : (1 - iQ\beta)$$

$$(1 - iQ\beta) = \begin{pmatrix} 1 - iq_\nu\beta \\ 1 - iq_e\beta \end{pmatrix} \qquad \psi = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

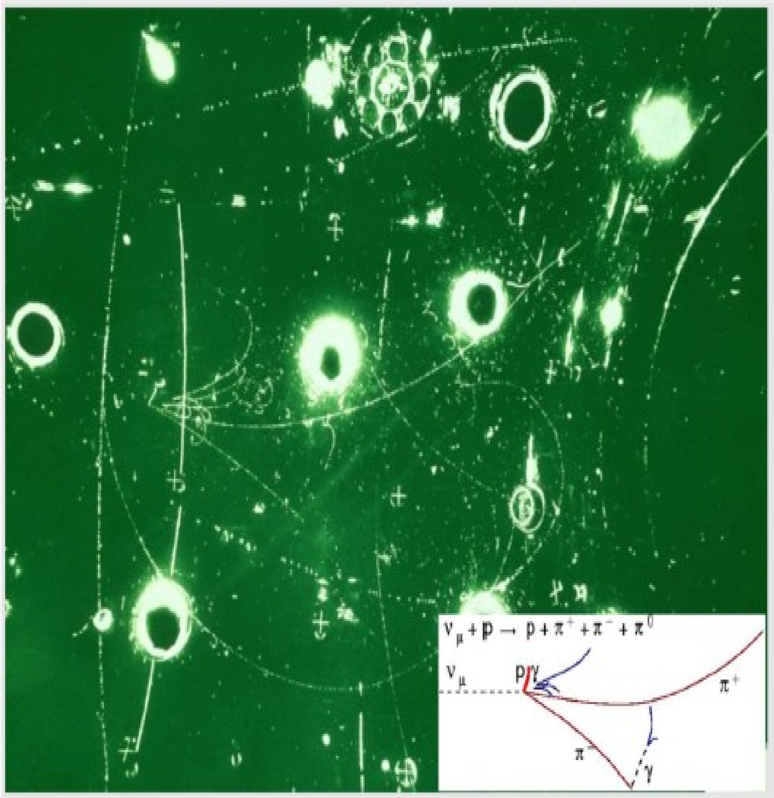
$$WeakInteraction : SU(2)_L : (1 - \frac{i}{2}\vec{\alpha}.\vec{\tau})$$

$$Combination : SU(2)_L \otimes U(1)_{QED}$$

$$Commutation :\Rightarrow q_\nu = q_e$$

Electroweak Interactions

- 1973: Neutral currents predicted by Salam, Glashow and Wienberg
- Many others competitive theories: Heavy Lepton Theory: most popular
- 1983: Neutral currents of Q and Z observed in UA(1), CERN experiments



Electroweak Interactions

- Gauge Group: $SU(2)_L \otimes U(1)_Y$
- Gauge Bosons: $U(1)_Y : B_\mu$

$$SU(2)_L : \vec{w}_\mu = (w_1, w_2, w_3)$$

- Interactions: $R : \partial_\mu \rightarrow D_\mu = \partial_\mu + \frac{1}{2} i g' Y_R B_\mu(x)$

$$L : \partial_\mu \rightarrow D_\mu = \partial_\mu + \frac{1}{2} i g' Y_L B_\mu(x) + \frac{1}{2} i g \vec{Z}.\vec{W}_\mu$$

- Gauge Transformations:

$$U(1)_Y : L' = exp(-\frac{i}{2} Y_L \beta(x))L; e'_R = exp(-\frac{i}{2} Y_R \beta(x)] e_R$$

$$SU(2) : L' = exp(-\frac{i}{2} \vec{\alpha}(x) \vec{\tau})L; e'_R = e_R$$

Electroweak Interactions

- Gauge Transformations:

$$B'_\mu(x) = B_\mu(x) + \frac{1}{g'}\partial_\mu \beta(x)$$

$$\vec{W}'_\mu(x) = \vec{W}_\mu(x) + \frac{1}{g}\partial_\mu \vec{\alpha}(x) - \vec{W}_\mu \times \vec{\alpha}(x)$$

- Gauge Bosons mix to form mass eigenstates with θ_ω known as Weinberg Angle

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2) : W^\pm$$

$$A_\mu = B_\mu cos\theta_\omega + W_\mu^3 sin\theta_\omega : \gamma$$

$$Z_\mu = -B_\mu sin\theta_\omega + W_\mu^3 cos\theta_\omega : Z^o$$

$$gSin\theta_\omega = g'Cos\theta_\omega = e$$

Electroweak Interactions

$$L_{Dirac} = \bar{\psi} i \not{\partial} \psi$$

$$L = \underbrace{\bar{L} i \gamma^{\mu} D_{\mu}^{L} L + \bar{e}_R i \gamma^{\mu} D_{\mu}^{R} e_R}_{L_F + L_{FB}} - \underbrace{\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} \vec{W}_{\mu\nu} \vec{W}^{\mu\nu}}_{L_B}$$

$$L_F = \bar{L} i \gamma^{\mu} \gamma_{\mu} L + \bar{e}_R i \gamma^{\mu} \partial_{\mu} e_R$$

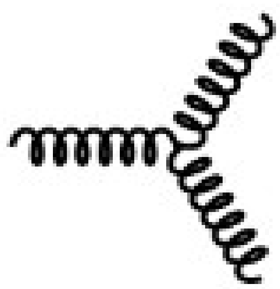
$$L_{FB} = -\bar{L}(\frac{1}{2} g' Y_L \gamma^{\mu} B_{\mu}) L - \bar{L}[\frac{1}{2} g \gamma^{\mu} \bar{\tau} \bar{W}_{\mu}) L - \bar{e}_R (\frac{1}{2} g' Y_R \gamma^{\mu} B_{\mu}) e_R$$

$$L_B : B_{\mu\nu} = \partial_{\mu} B_{\nu} - \partial_{\nu} B \mu$$

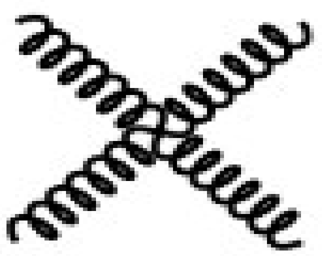
$$\bar{W}_{\mu\nu} = \partial_{\mu} \bar{W}_{\nu} - \partial_{\nu} \bar{W}_{\mu} - \underbrace{g \bar{W}_{\mu} \times \bar{W}_{\nu}}_{\text{Self-Interactions}}$$

Quantum Chromodynamics

- Interactions of quarks+gluons
- Gauge Theory
- Symmetry Group : SU(3)*SU(2)*U(1)
- Quarks carry color charge
  - Colours: R, G, B Analogous: u, d, s
  - Anti-colours: $\overline{R}$, $\overline{G}$, $\overline{B}$,   Analogous: $\overline{u}$, $\overline{d}$, $\overline{s}$
- Glouns are bicoloured
- Glouns have self-couplings
- Hadrons: Colourless
- Glouns and photons are massless

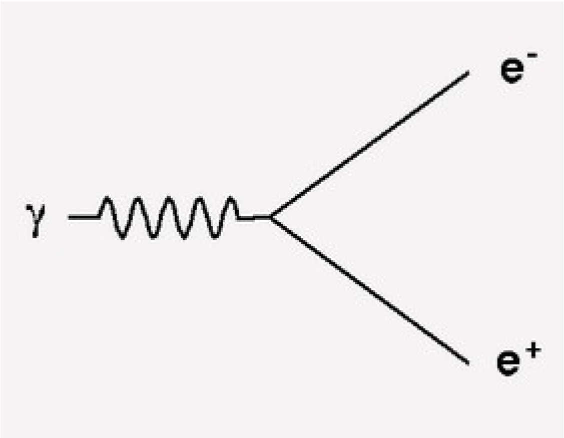


Gluon
Self-Couplings



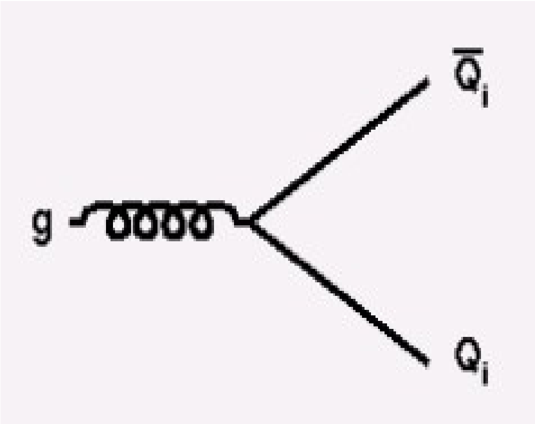
Quantum Chromodynamics

QED Vertex



$$\alpha_s = \frac{g_s^2}{4\pi}$$

QCD Vertex



$$\alpha = \frac{e^2}{4\pi}$$

- SU(3) is group with 3*3 matrix elements
- With Unitarity , we get 8 generators, T_a : 8 glouns

$$[T^a, T^b] = i f_{abc} T^c$$

- F_{abc} are called structure constants

Quantum Chromodynamics

- Gauge Transformations:

$$q_i(x) \rightarrow q_i' = \Omega(x)_i^j q_j(x); \Omega(x) = exp(i\alpha_a(x)T^a)$$

$$T_a A_\mu^a(x) \rightarrow T_a A_\mu'^a = \Omega(x) T_a A^a \Omega^{-1}(x) + \frac{i}{g}(\partial_\mu \Omega(x))\Omega^{-1}(x)$$

- Quark Fields: q transform as spin(1/2) Dirac spinors under Lorentz Transformation and as color triplet in fundamental representation under SU(3)
- Guage Bosons: Gluon Fields, A transform as vectors under Lorentz transformations and as color octets under SU(3)

$$Triplets : (D_\mu)_{ij} = \delta_{ij}\partial_\mu + igT_{aij}A_\mu^a$$

$$Octets : (D_\mu)_{bc} = \delta_{bc}\partial_\mu + ig(T_{Adj}^a)_{bc}A_\mu^a$$

Quantum Chromodynamics

- Field Strength Tensor of QCD:

$$F_{\mu\nu} := F^a_{\mu\nu} T^a \equiv \frac{1}{ig}[D_\mu, D_\nu]$$

$$F_{\mu\nu} = \frac{i}{ig}[\partial_\mu + igA_\mu, \partial_\nu + igA_\nu]$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f_{abc} A^b_\mu A^c_\nu$$

$$F_{\mu\nu} \to F'_{\mu\nu} =_{\mu\nu} \Omega^{-1}$$

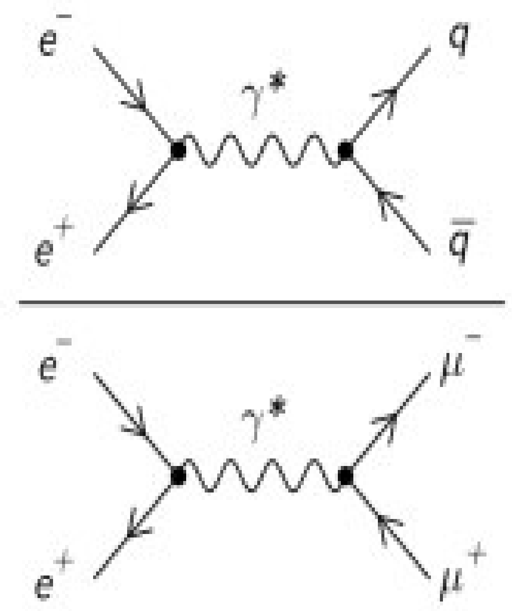
- QCD Langrangian:

$$L_{QCD} = \bar{\psi}_i (i \not{D}_{\mu ij} - m\delta_{ij})\psi_j - \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a$$

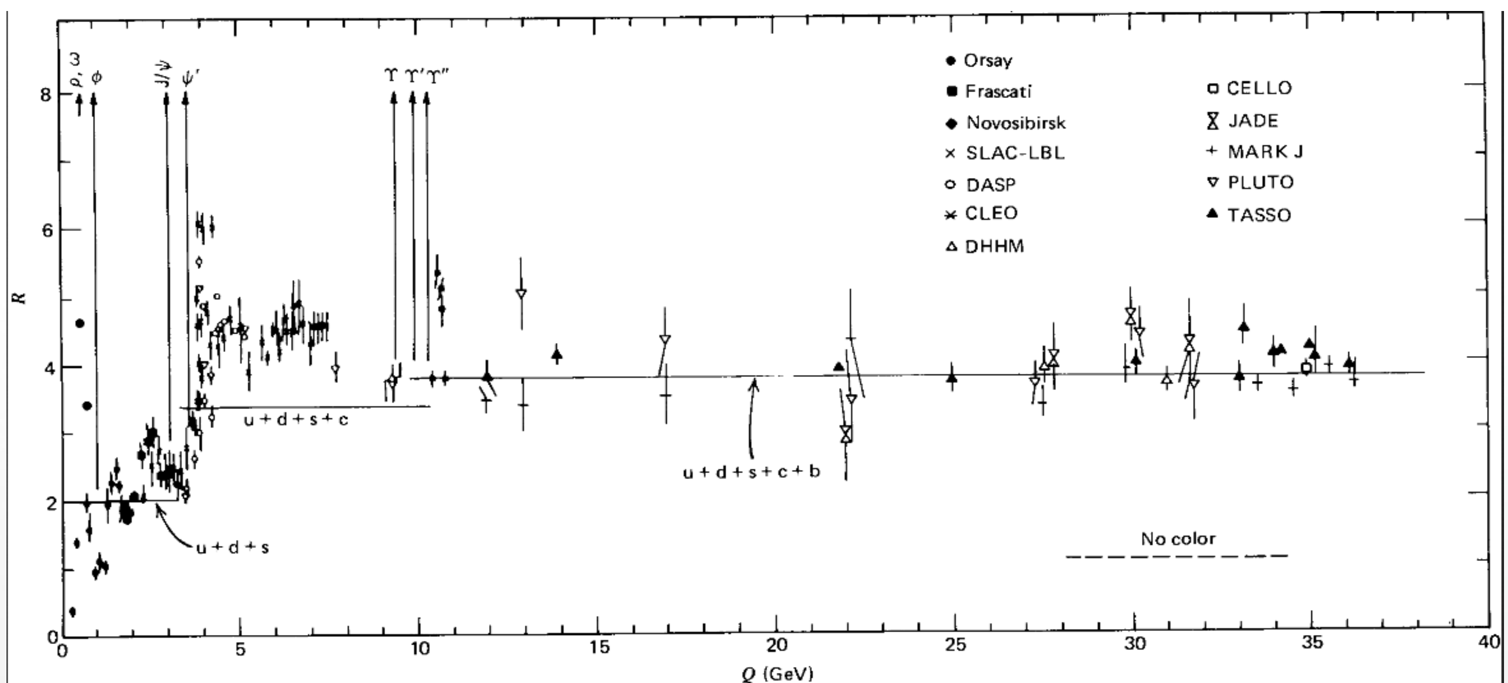
Quantum Chromodynamics

- Number of Colors?

$$R = \frac{\sigma(e^-e^+ \to \text{hadrons})}{\sigma(e^-e^+ \to \mu^-\mu^+)} \approx$$

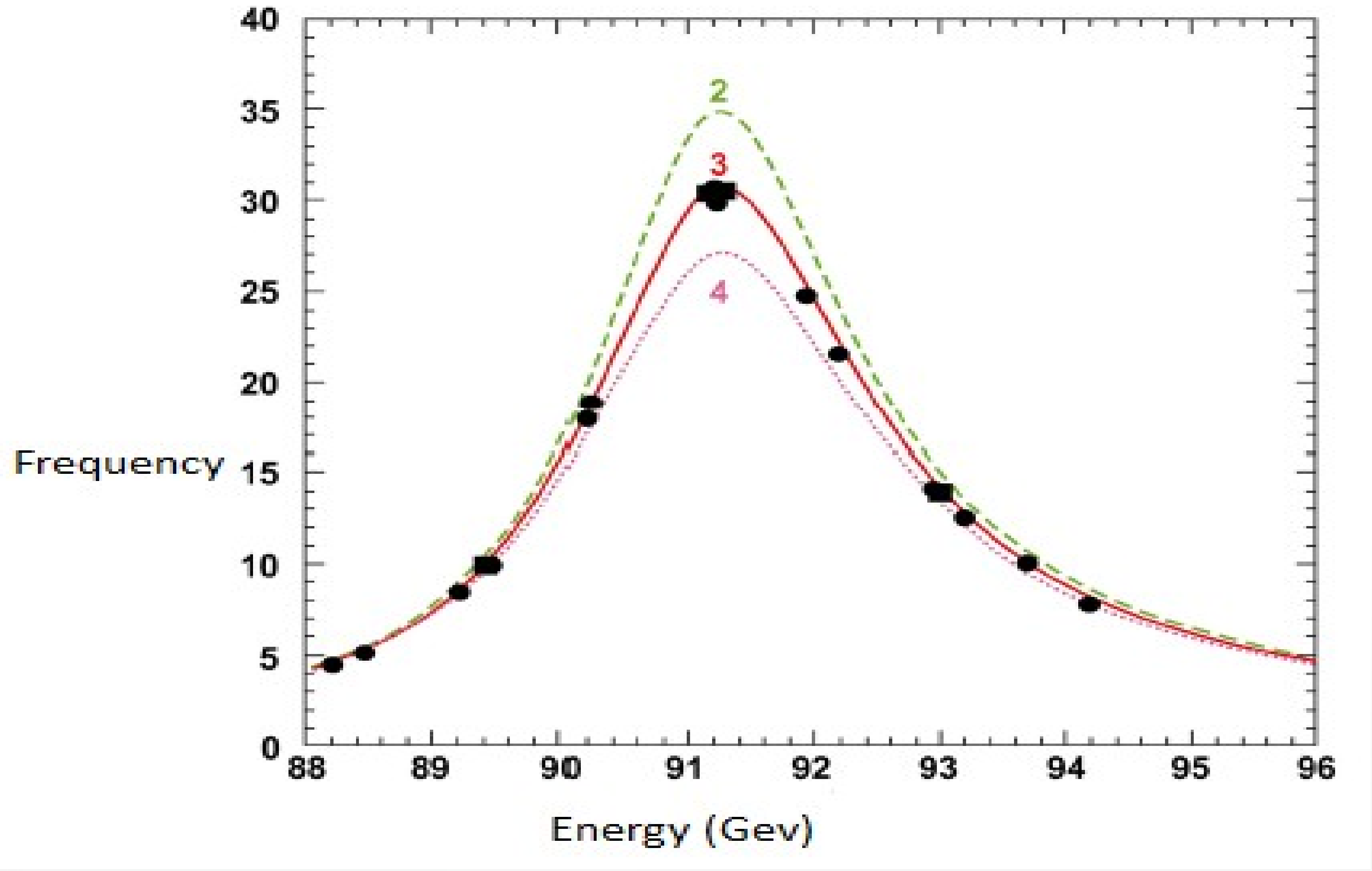


$$R = N_C \Sigma q\bar{q}$$



Quantum Chromodynamics

- LEP
- Number of Neutrino Generation



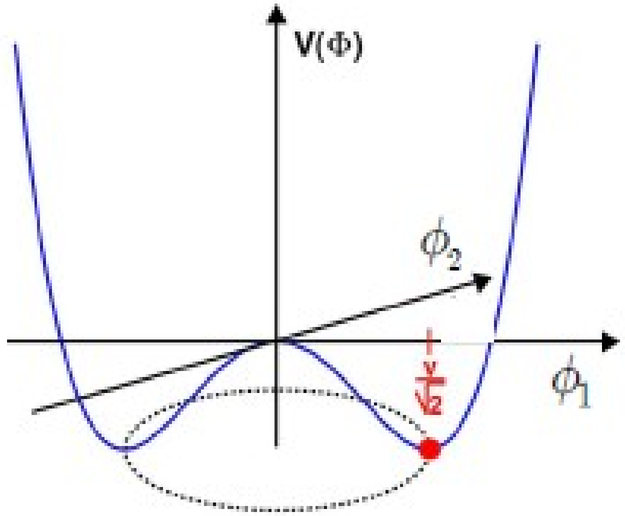
Higgs

- Scalar U(1) invariant Lagrangian

$$L = (D_\mu \phi D^\mu \phi) - V(\phi) - \frac{1}{4} F_{\mu\nu} F_{\mu\nu}$$

- Cowboy Hat Potential

$$V(\phi) = -\mu|\phi|^2 + \lambda^2|\phi|^2$$



- Fixing particular direction breaks symmetry. For eg:

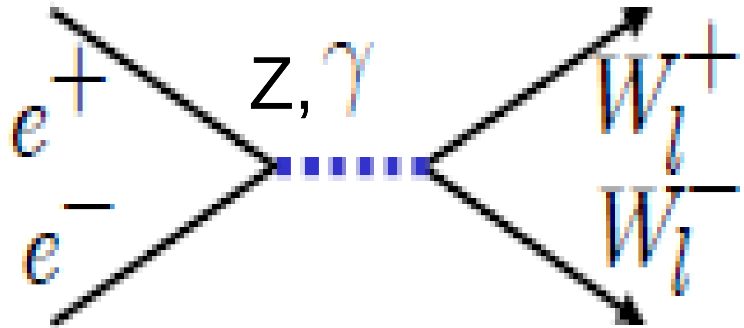
$$\phi(x) = \frac{1}{\sqrt{2}}(\nu + \eta(x) + iG(x))$$

$$M - Term : \frac{1}{2} q^2 \nu^2 A_\nu A^\nu$$

- Symmtery Breaks: Massive higgs boson: Parity violated

Conclusion

- Masses?
- Unitarity? Higgs?
- Stay Tuned!



Hopefully, we go beyond SM!

Thank You!

References

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- Advanced Particles Physics Lecture, LMU
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