The Higgs particle

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Table of contents

1. Introduction
   - Gauge theory
   - Derivation of the Higgs mechanism
2. The Higgs mechanism
   - Higgs-field in analogy to the ferro-magnet
   - The Higgs mechanism in a $U(1)$ gauge theory
   - The Higgs mechanism in the electroweak interaction
3. Detection of the Higgs particle
   - Higgs mass
   - Proportion of Higgs mass

Gauge theory

Gauge principle:

- equation of motion is invariant under local phase transformation
- there exists an external field (interaction) which receives a compensating transformation

$\rightarrow$ the gauge bosons have to be massless

Example: electro-magnetic interaction with massless photon

Dirac equation with interaction:

$$(i\,\gamma^{\mu}\,D_{\mu} - m)\,\psi\,(x) = 0$$

$$D_{\mu} = \partial_{\mu} + i\,q\,A_{\mu}\,(x)$$

Gauge theory

$$(i\,\gamma^{\mu}\,D_{\mu} - m)\,\psi\,(x) = 0$$

→ equation is invariant under local phase transformation

Coupled transformation:

$$\psi\,(x) \;\; \rightarrow \psi'\,(x) = e^{iq\rho(x)}\,\psi\,(x)$$

$$A_{\mu}\,(x) \rightarrow A'_{\mu}\,(x) = A_{\mu}\,(x) - \partial_{\mu}\,\rho\,(x)$$

Lagrangian for massive spin-1 field:

$$\mathcal{L} = -\frac{1}{4}\,F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}\,M^2 A^{\mu}A_{\mu}$$

→ for $\mathcal{L}_{photon}$ to be invariant the mass M has to be zero

$$\mathcal{L}_{photon} = -\frac{1}{4}\,F^{\mu\nu}F_{\mu\nu}$$

Gauge theory

But: consider weak interaction

- very small range of interaction
- there for: heavy gauge bosons Z^0, $W^\pm$

$\rightarrow$ problem of gauge boson mass

Idea:

- treat weak interaction with infinite range
- introduce background field which screens range

$\rightarrow$ the gauge bosons receive a mass

Note: no loss of gauge invariance

Additional formalism

relativistic energy-momentum relationship: $E^2 = p^2 + m^2$

Note: $\Box := \partial_\mu \, \partial^\mu$

Klein-Gordon equation:

$$(\Box + m^2) \, \phi \, (x) = 0$$

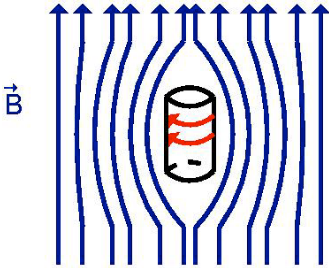
$\rightarrow$ equation of motion for spin-0 field with mass m

Klein-Gordon Lagrangian:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2} m^2 \phi^2$$

Superconductor

Consider: Meissner-Ochsenfeld effect



- static B field causes currents j_S in a superconductor
- below a critical temperature T_C screening currents generate counter-field
- counter-field compensates external static B field

Superconductor

→ allowed density of electrons in the same state cannot cause counter-fields

Explanation of M.-O. effect by postulated equation:

$$\text{London eq.: } \nabla \times j = -\frac{q^2\, n_c}{m_c} B$$

BCS theory: at low temperatures electrons can form a bound state due to the interaction with the lattice

→ Cooper-pairs: two electrons with opposite spin and momentum form a bosonic state (spin 0)

→ macroscopic occupancy of single state possible

n_c: density, $q = 2e$, $m_c = 2m_e$

Superconductor - insertion 1

Find simultaneously solution of:

I: $\Box A^\mu = j^\mu$ inhom. Maxwell eq. with $\partial_\mu A^\mu = 0$

II: $(\Box + m^2) A^\mu = 0$

Static version of equations:

I: $\nabla^2 A = -j$

II: $\nabla^2 A = +m^2 A$

$$\Rightarrow \quad j = -m^2 A \qquad | \cdot \nabla \times$$

$$\nabla \times j = -m^2 B \qquad (\star)$$

Superconductor - insertion 2

Look at 4. static Maxwell equation:

$$\nabla \times B = j \qquad | \cdot \nabla \times$$

$$\nabla \times (\nabla \times B) = \nabla(\nabla \cdot B) - \nabla^2 B = -\nabla^2 B$$
$$\nabla \times j = -m^2 B$$

$$\nabla^2 B = m^2 B$$

$\rightarrow$ equation looks like the B field has a mass

Superconductor

Compare the London eq. with eq. $(\star)$

London eq.: $\nabla \times j = -\frac{q^2 n_c}{m_c} B$

$(\star)$: $\nabla \times j = -m^2 B$

$\rightarrow$ it is possible to calculate the effective mass of the photon

n_c: density, $q = 2e$, $m_c = 2m_e$

Superconductor - result

Cooper pairs:

- below T_C all Cooper pairs are in the same ground state
- Cooper pairs mediate the "supra" current
- Cooper pairs form a scalar background-field
- photons get an effective mass in matter

Point:

If we were not aware of this background-field
$\rightarrow$ we conclude that photons are massive

$\Rightarrow$ Question: Is there a background-field which gives mass to Z^0, $W^\pm$

Mass generation in the vacuum

Vacuum in QFT:

Vacuum expectation value $\langle \phi_0 \rangle \approx$ ground of the system

Consider: Higgs (background)-field as free spin-0 particles $\phi(x)$

$$(\Box + m^2)\,\phi\,(x) = 0$$

construct the current:

$$j^\mu = iq[\phi^\star(\partial^\mu \phi) - (\partial^\mu \phi)^\star \phi]$$

Introduce interaction with photon field:

$$D^\mu = \partial^\mu + iqA^\mu$$
$$j^\mu = iq[\phi^\star(\partial^\mu \phi) - (\partial^\mu \phi)^\star \phi] - 2q^2 A^\mu I \phi I^2$$

Mass generation in the vacuum

2 cases for the vacuum:

$I: \quad \phi_0 = 0 \rightarrow$ vacuum current: $j_0^\mu = 0$

$II: \quad \phi_0 =$ const. $\rightarrow$ vacuum current: $j_0^\mu = -2q^2 A^\mu |\phi_0|^2$

Define the mass of A^μ :

$$m = q\sqrt{2}\,|\phi_0|$$

$$\Box A^\mu = j_0^\mu \quad \rightarrow \quad K.G.: (\Box + m^2) A^\mu = 0$$

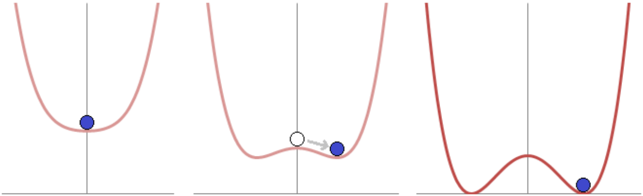
$\Rightarrow$ mass generation only possible for $\phi_0 \neq 0$

Spontaneous symmetry breaking

Spontaneous symmetry breaking:

- basic equation of a system have a certain symmetry
- symmetry does not exist in the ground state
- non vanishing ground state: $\phi_0 = \text{const.}$

Requirement for $\phi_0 = \text{const.}$: self-interacting Higgs field



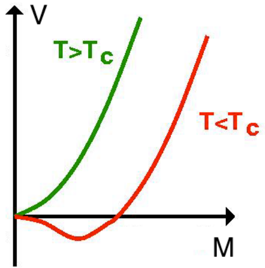
Spontaneous symmetry breaking

Example: superconductor

- all Cooper pairs are correlated (same phase, same c.o.m. momentum)
- current j^μ is invariant under phase transformation
- ground state is a specific choice of phase (not invariant)

Spontaneous symmetry breaking

Example: ferro-magnet



$V = \alpha M^2 + \beta M^4$,M: magnetisation

$$I: \quad T > T_C : \alpha > 0 \quad \text{groundstate: } M = 0$$
$$II: \quad T < T_C : \alpha < 0 \quad \text{groundstate: } M \neq 0$$

$M \neq 0$: preferred direction of spins (no rotational symmetry)

Higgs potential - wine bottle potential

Analog: potential with non vanishing ground state:

$$V(\phi) = -\mu^2 |\phi|^2 + \lambda^2 |\phi|^4 \qquad , \phi(x) = \frac{1}{\sqrt{2}}[\phi_1(x) + i\,\phi_2(x)]$$

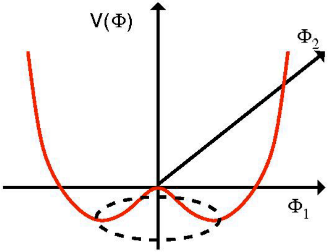
$\phi(x)$ scalar field

Minimum of V:

$$\frac{\partial V}{\partial \phi} \stackrel{!}{=} 0$$

$$\phi_1^2 + \phi_2^2 = \frac{\mu^2}{2\lambda^2}$$

$$\rightarrow \quad |\phi_0| = \frac{v}{\sqrt{2}} \qquad , \; v = \frac{\mu}{\lambda}$$



The Higgs mechanism

System has a U(1) symmetry: $\phi \rightarrow \phi' = e^{i\alpha}\phi$

Lagrangian is given by:

$$\mathcal{L} = (\partial_\mu \phi)(\partial^\mu \phi)^* - V(\phi)$$

$\rightarrow \mathcal{L}$ is invariant under phase transformations

Goldstone theorem:

- $\mathcal{L}$ has an exact continuous global symmetry
- this symmetry is not present in the ground state

$\rightarrow$ the theory contains massless particles
$\rightarrow$ massless particles = Goldstone bosons

The Higgs mechanism

- $\mathcal{L}$ has U(1) symmetry
- symmetry breaks down in ground state (no symmetry group)

$\rightarrow$ theorem predicts 1 Goldstone boson

Rewrite ϕ by oscillations around ground state:

$$\phi(x) = \frac{1}{\sqrt{2}}[v + \eta(x) + i\,\xi(x)]$$

$\eta(x)$: radial direction

$\xi(x)$: azimuthal direction

The Higgs mechanism

Insert in potential:

$$V(\phi) = \mu^2\eta^2 - \frac{1}{4}\mu^2 v^2 + \dots$$

Insert in Lagrangian:

$$\begin{aligned}\mathcal{L} &= (\partial_\mu\phi)(\partial^\mu\phi)^* - V(\phi) \\ &= \tfrac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) - \mu^2\eta^2 + \tfrac{1}{2}(\partial_\mu\xi)(\partial^\mu\xi) + \dots\end{aligned}$$

$\rightarrow$ one "real" massterm in $\mathcal{L}$

$$\mathcal{L}_{mass} = -\mu^2\eta^2 = -\frac{1}{2}m_\eta^2\,\eta^2$$

$$m_\eta = \sqrt{2}\mu \quad , m_\xi = 0$$

$\rightarrow$ ξ is Goldstone boson

The Higgs mechanism - gauge theory

Make the theory gauge invariant: $D_\mu = \partial_\mu + iqA_\mu$

$$\mathcal{L} = (D_\mu\phi)(D^\mu\phi)^* - V(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Use expansion: $\phi(x) = \frac{1}{\sqrt{2}}[v + \eta(x) + i\,\xi(x)]$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) - \mu^2\eta^2 + \frac{1}{2}(\partial_\mu\xi)(\partial^\mu\xi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$
$$+\frac{1}{2}q^2v^2\,A_\mu\,A^\mu + q\,v\,A_\mu(\partial^\mu\xi)$$

What about the last two terms?

The Higgs mechanism - gauge theory

$\mathcal{L}$ is gauge invariant:

$$\phi(x) \rightarrow \phi'(x) = e^{iq\rho(x)}\,\phi(x)$$

$$A_\mu(x) \rightarrow A'_\mu(x) = A_\mu(x) - \partial_\mu\,\rho(x)$$

Perfect choice:

$$\rho(x) = -\frac{1}{qv}\xi(x)$$

Insert ϕ', A'_μ in $\mathcal{L}$:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) - \mu^2\eta^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}q^2v^2\,A'_\mu\,A'^\mu + \ldots$$

gauge theory eliminated the Goldstone Boson $\Rightarrow \xi$ has vanished

A_μ became massive $m_A = qv$

The Higgs mechanism - result

Counting degrees of freedom:

$\rightarrow$ before interaction

- 1 dof: η field $m_\eta = \sqrt{2}\mu$
- 1 dof: ξ field $m_\xi = 0$ (Goldstone boson)
- 2 dof: A_μ field $m_A = 0$ (2 transverse dof.)

$\rightarrow$ after gauge

- 1 dof: η field $m_\eta = \sqrt{2}\mu$ ("Higgs")
- 3 dof: A_μ field $m_A = qv$ (+1 longitudinal dof.)

massive photon field A_μ, $m_A = qv$ given by Higgs vacuum expectation value

The Higgs mechanism in $U(1)_Y \otimes SU(2)_L$

$U(1)_Y \otimes SU(2)_L$: $\quad \mathcal{L} = \mathcal{L}_F + \mathcal{L}_{FB} + \mathcal{L}_B + \mathcal{L}_H + \mathcal{L}_{HB} + \mathcal{L}_{HF}$

$$\mathcal{L}_H + \mathcal{L}_{HB} = (D^\mu \Phi)^\dagger (D_\mu \Phi) - V(\Phi^\dagger \Phi)$$
$$D_\mu = \partial_\mu + \tfrac{1}{2} i g' Y B_\mu(x) + \tfrac{1}{2} i g \vec{\tau} \vec{W}_\mu(x)$$

- $\mathcal{L}$ is gauge invariant (elimination of GB)
- coupling constant: $g \approx e$

simplest Higgs structure to give mass to $Z^0, W^\pm$ is a doublet:

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

ϕ is complex scalar $SU(2)_L$ doublet: 4 dof. (after gauge 1 dof.)

weak isospin $I(\Phi) = \frac{1}{2}$: $\quad I_3 = \pm \frac{1}{2}$

The Higgs mechanism in $U(1)_Y \otimes SU(2)_L$

groundstate: $\langle\Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\v\end{pmatrix}$, v real

gauge bosons $\vec{W}_\mu, B_\mu$ can be mapped to exchange bosons

$$W^\pm \rightarrow W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp i\,W^2_\mu)$$

$$Z^0 \rightarrow Z_\mu = -B_\mu \sin\theta_w + W^3_\mu \cos\theta_w$$

$$\text{Photon} \rightarrow A_\mu = B_\mu \cos\theta_w + W^3_\mu \sin\theta_w$$

- $W^\pm$ mass eigenstates with $m_W = \frac{gv}{2}$
- Z^0 with $m_Z = \frac{m_W}{cos\theta_W}$
- A_μ with $m_A = 0$

scalar Higgs boson: $m_{Higgs} = \sqrt{2}\mu = \sqrt{2}\lambda v$

Limits on the Higgs boson mass

the vacuum expectation value is already determined:

$m_W = \frac{gv}{2}$ and $\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2}$

$G_F = 1,16639 \cdot 10^{-5} GeV^{-2}$

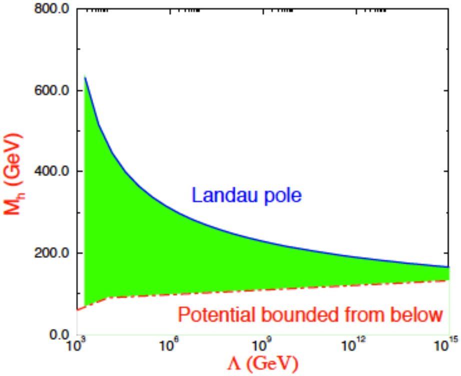
$$\frac{G_F}{\sqrt{2}} = \frac{1}{2v^2} \quad \rightarrow v = 246\, GeV$$

$$m_{Higgs} = \sqrt{2}\lambda v$$

- lower bound: v $\rightarrow$ reference scale
- upper bound: self-coupling of Higgs: $V(\phi) \propto \lambda^2 \phi^4$
  require finite coupling: $\frac{1}{\lambda(\Lambda)} > 0$

$$\lambda^2 = \frac{m_H^2}{2v^2} \quad \xrightarrow{\text{large scale } \Lambda} \quad m_H < \frac{8\pi^2 v^2}{3log(\Lambda^2/v^2)}$$

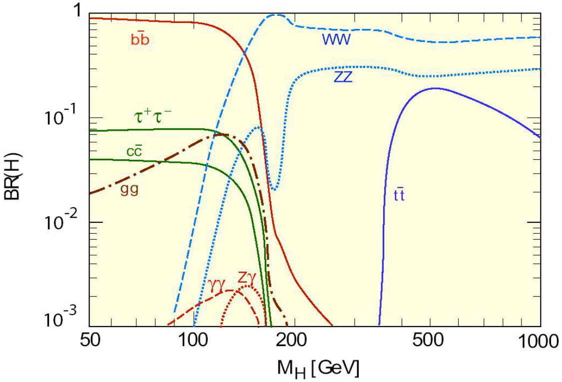
Limits on the Higgs boson mass



- no new physics before 10^{16}GeV: $m_H < 160$GeV
- Landau pole: forbidden region (coupling is finte)
- for 3TeV: $m_H < 600$GeV

Branching Ratios

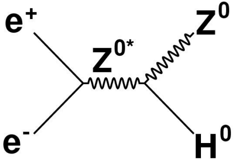
branching ratios of allowed Higgs decays as a function of m_H



Higgs tends to decay into the heaviest particle
→ constraints on m_H : no decay into $t\bar{t}$

Higgs search at LEP ($e^+e^- - collider$)

dominant Higgs production process at LEP2: radiation off a Z^0

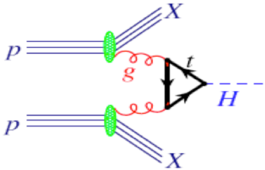


- $\sqrt{s} > m_H + m_Z$
- dominant decay: $H \to b\bar{b}$
- additional kinematic constraint for fermion pair from Z^0: $m_{inv}(f\bar{f}) = m_Z$ (background suppression)
- background: $e^+e^- \to W^+W^-, Z^0Z^0, f\bar{f}f\bar{f}$

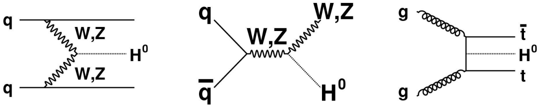
mass range exclusion LEP2: $m_H > 114.4\text{GeV}$ (95%)

Higgs production at LHC

Atlas detector: proton-proton collisions, $\sqrt{s} = 8\text{TeV}$ (2012)



- dominant production processes for the Higgs boson: gg fusion
- Higgs does not couple to massless gluons (quark loop)



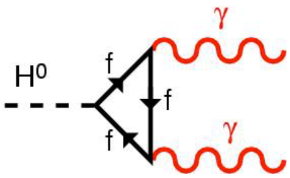
Direct Higgs searches at LHC

important experimentally mass range:

$$114.4 Gev \leq m_H \leq 130 Gev$$

- lower bound from LEP 2 (95 % CL)
- kinematically forbidden: $H \to ZZ, H \to WW$

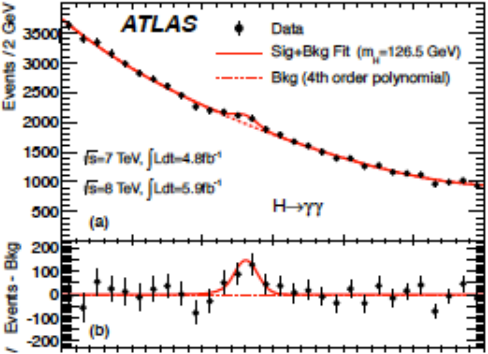
$H \to \gamma\gamma$ Higgs decays via fermion loop



other decays: $H \to ZZ^* \to 4l$, $H \to WW^* \to e\nu\mu\nu$,

Direct Higgs searches at LHC

distribution for invariant mass of $\gamma\gamma$



$$m_H = 126.0 \pm 0.4(stat) \pm 0.4(sys) Gev$$

Matter and energy of the universe

- mass of atoms 4%
- dark matter 23%
- dark energy 73%

Proportion of Higgs mass

estimate the proportion of the "Higgs mass" of the universe

$\rightarrow$ use proton or neutron mass

Quark content of proton

- $p = |uud\rangle$
- $m_p \approx 1000$ MeV
- bare quark mass ≈ 13 MeV

$\rightarrow$ Higgs field causes 1.3 % mass of the universe

Note: 99 % are caused by the strong interaction between quarks

Thank you for your attention

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