Supersymmetry and D-brane calibrations in G_2-compactifications of type II string theory

Alexander Seifert

Leibniz Universität Hannover

07.07.2014

Outline

Introduction

Introduction

- The study of supergravity of type II backgrounds (with fluxes) has received much attention in last 10-15 years.

- The study of supergravity of type II backgrounds (with fluxes) has received much attention in last 10-15 years.
- The supersymmetry conditions for the type II background of the form $\mathbb{R}^{1,3} \times M_6$ can be written in terms of the pure spinors associated to $SU(3) \times SU(3)$-structure on $TM_6 \oplus T^*M_6$

- The study of supergravity of type II backgrounds (with fluxes) has received much attention in last 10-15 years.
- The supersymmetry conditions for the type II background of the form $\mathbb{R}^{1,3} \times M_6$ can be written in terms of the pure spinors associated to $SU(3) \times SU(3)$-structure on $TM_6 \oplus T^*M_6$
- One-to-one-correspondence between supersymmetry conditions and generalised D-brane calibration.

- The study of supergravity of type II backgrounds (with fluxes) has received much attention in last 10-15 years.
- The supersymmetry conditions for the type II background of the form $\mathbb{R}^{1,3} \times M_6$ can be written in terms of the pure spinors associated to $SU(3) \times SU(3)$-structure on $TM_6 \oplus T^*M_6$
- One-to-one-correspondence between supersymmetry conditions and generalised D-brane calibration.
- The idea is to verify that this correspondence is also valid for the type II backgrounds of the form $\mathbb{R}^{1,2} \times M_7$

$\mathcal{N}=1$ SUSY (IIA type)

$\mathcal{N} = 1$ SUSY (IIA type)

- Interest in type IIA backgrounds (with fluxes) preserving

$\mathcal{N} = 1$ SUSY (IIA type)

- Interest in type IIA backgrounds (with fluxes) preserving
  - three-dimensional Poincaré invariance,

$\mathcal{N} = 1$ SUSY (IIA type)

- Interest in type IIA backgrounds (with fluxes) preserving
  - three-dimensional Poincaré invariance,
  - $\mathcal{N} = 1$ supersymmetry.

$\mathcal{N} = 1$ SUSY (IIA type)

- Interest in type IIA backgrounds (with fluxes) preserving
  - three-dimensional Poincaré invariance,
  - $\mathcal{N} = 1$ supersymmetry.
- The ansatz for ten-dimensional metric G_{MN}

$$ds^2 = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{2B(y)}\tilde{g}_{mn}(y)dy^m dy^n.$$

$\mathcal{N} = 1$ SUSY (IIA type)

- Interest in type IIA backgrounds (with fluxes) preserving
  - three-dimensional Poincaré invariance,
  - $\mathcal{N} = 1$ supersymmetry.
- The ansatz for ten-dimensional metric G_{MN}

$$ds^2 = e^{2A(y)}\eta_{\mu\nu}dx^{\mu}dx^{\nu} + e^{2B(y)}\tilde{g}_{mn}(y)dy^m dy^n.$$

- The RR-fields decomposition is

$\mathcal{N} = 1$ SUSY (IIA type)

- Interest in type IIA backgrounds (with fluxes) preserving
  - three-dimensional Poincaré invariance,
  - $\mathcal{N} = 1$ supersymmetry.
- The ansatz for ten-dimensional metric G_{MN}

$$ds^2 = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{2B(y)}\tilde{g}_{mn}(y)dy^m dy^n.$$

- The RR-fields decomposition is

$$F_n = \hat{F}_n + \mathrm{Vol}_3 \wedge \tilde{F}_{n-3}, \quad n \text{ even}$$

$\mathcal{N} = 1$ SUSY (IIA type)

- Interest in type IIA backgrounds (with fluxes) preserving
  - three-dimensional Poincaré invariance,
  - $\mathcal{N} = 1$ supersymmetry.
- The ansatz for ten-dimensional metric G_{MN}

$$ds^2 = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{2B(y)}\tilde{g}_{mn}(y)dy^m dy^n.$$

- The RR-fields decomposition is

$$F_n = \hat{F}_n + \mathrm{Vol}_3 \wedge \tilde{F}_{n-3}, \quad n \text{ even}$$

with the relation between the internal components

$$\tilde{F}_{7-n} = (-1)^{\frac{n(n-1)}{2}} e^{3A} e^{(7-2n)B} *_7 \hat{F}_n$$

- Furthermore we suppose the susy-generators $\varepsilon^{1,2}$ have the same norm and decompose them

$\mathcal{N} = 1$ SUSY (IIA type)

- Furthermore we suppose the susy-generators $\varepsilon^{1,2}$ have the same norm and decompose them

$$\varepsilon^1 = \zeta \otimes \xi^1(y) \otimes (0,\ 1)^t, \quad \varepsilon^2 = \zeta \otimes \xi^2(y) \otimes (1,\ 0)^t$$

$\mathcal{N} = 1$ SUSY (IIA type)

- Furthermore we suppose the susy-generators $\varepsilon^{1,2}$ have the same norm and decompose them

$$\varepsilon^1 = \zeta \otimes \xi^1(y) \otimes (0,\ 1)^t, \quad \varepsilon^2 = \zeta \otimes \xi^2(y) \otimes (1,\ 0)^t$$

where ζ is a constant spinor on Minkowski space, ξ^i are the spinors on the internal manifold

$\mathscr{N} = 1$ SUSY (IIA type)

- Furthermore we suppose the susy-generators $\varepsilon^{1,2}$ have the same norm and decompose them

$$\varepsilon^1 = \zeta \otimes \xi^1(y) \otimes (0,\ 1)^t, \quad \varepsilon^2 = \zeta \otimes \xi^2(y) \otimes (1,\ 0)^t$$

where ζ is a constant spinor on Minkowski space, ξ^i are the spinors on the internal manifold with the conditions

$$\zeta^* = \alpha^{-1}\zeta, \quad \xi^{1*} = \alpha\xi^1, \quad \xi^{2*} = -\alpha\xi^2,\ \alpha \in \mathbb{C} \text{ and } \|\alpha\| = 1$$

$\mathscr{N} = 1$ SUSY (IIA type)

- Furthermore we suppose the susy-generators $\varepsilon^{1,2}$ have the same norm and decompose them

$$\varepsilon^1 = \zeta \otimes \xi^1(y) \otimes (0,\ 1)^t, \quad \varepsilon^2 = \zeta \otimes \xi^2(y) \otimes (1,\ 0)^t$$

where ζ is a constant spinor on Minkowski space, ξ^i are the spinors on the internal manifold with the conditions

$$\zeta^* = \alpha^{-1}\zeta, \quad \xi^{1*} = \alpha\xi^1, \quad \xi^{2*} = -\alpha\xi^2,\ \alpha \in \mathbb{C} \text{ and } \|\alpha\| = 1$$

- The internal spinors $\xi^{1,2}$ are fixed and characterize the reduction of the structure group of $TM_7 \oplus T^*M_7$ from $SO(7,7)$ to $G_2 \times G_2$ and can be associated to an even polyform Ψ^+ using the Clifford map.

- Furthermore we suppose the susy-generators $\varepsilon^{1,2}$ have the same norm and decompose them

$$\varepsilon^1 = \zeta \otimes \xi^1(y) \otimes (0,\ 1)^t, \quad \varepsilon^2 = \zeta \otimes \xi^2(y) \otimes (1,\ 0)^t$$

where ζ is a constant spinor on Minkowski space, ξ^i are the spinors on the internal manifold with the conditions

$$\zeta^* = \alpha^{-1}\zeta, \quad \xi^{1*} = \alpha\xi^1, \quad \xi^{2*} = -\alpha\xi^2, \ \alpha \in \mathbb{C} \text{ and } \|\alpha\| = 1$$

- The internal spinors $\xi^{1,2}$ are fixed and characterize the reduction of the structure group of $TM_7 \oplus T^*M_7$ from $SO(7,7)$ to $G_2 \times G_2$ and can be associated to an even polyform Ψ^+ using the Clifford map.
- The $G_2 \times G_2$-structure $\rightarrow \begin{cases} G_2, & \text{if } \xi^{1,2} \text{ are parallel,} \\ & \end{cases}$

$\mathcal{N} = 1$ SUSY (IIA type)

- Furthermore we suppose the susy-generators $\varepsilon^{1,2}$ have the same norm and decompose them

$$\varepsilon^1 = \zeta \otimes \xi^1(y) \otimes (0,\ 1)^t, \quad \varepsilon^2 = \zeta \otimes \xi^2(y) \otimes (1,\ 0)^t$$

  where ζ is a constant spinor on Minkowski space, ξ^i are the spinors on the internal manifold with the conditions

$$\zeta^* = \alpha^{-1}\zeta, \quad \xi^{1*} = \alpha\xi^1, \quad \xi^{2*} = -\alpha\xi^2,\ \alpha \in \mathbb{C} \text{ and } \|\alpha\| = 1$$

- The internal spinors $\xi^{1,2}$ are fixed and characterize the reduction of the structure group of $TM_7 \oplus T^*M_7$ from $SO(7,7)$ to $G_2 \times G_2$ and can be associated to an even polyform Ψ^+ using the Clifford map.
- The $G_2 \times G_2$-structure $\rightarrow \begin{cases} G_2, & \text{if } \xi^{1,2} \text{ are parallel,} \\ SU(3), & \text{if } \xi^{1,2} \text{ are orthogonal} \end{cases}$

- The necessary and sufficient conditions to have the supersymmetric type IIA background of form $\mathbb{R}^{1,2} \times M_7$

$$\mathrm{d}_H(e^{3A-\Phi}\Psi^+) = \tilde{F}_A, \qquad (1a)$$

$$\mathrm{d}_H(e^{2A-\Phi} *_7 \sigma(\Psi^+)) = 0, \qquad (1b)$$

$$\tilde{F}_A \wedge \sigma(\Psi^+)_{|7} = 0, \qquad (1c)$$

$$\|\xi^1\|^2 = \|\xi^2\|^2 = e^A \qquad (1d)$$

where $\tilde{F}_A = \tilde{F}_1 + \tilde{F}_3 + \tilde{F}_5 + \tilde{F}_7$ and $d_H := d + H\wedge$

$\mathscr{N} = 1$ SUSY (IIA type)

- The supersymmetry equations for type IIB can be obtained by exchange

$$\Psi^{+} \leftrightarrow *_7\sigma(\Psi^{+}) \quad \text{and} \quad \tilde{F}_A \leftrightarrow \tilde{F}_B$$

where $\tilde{F}_B = \tilde{F}_0 + \tilde{F}_2 + \tilde{F}_4 + \tilde{F}_6$

$\mathcal{N} = 1$ SUSY (IIA type)

- The supersymmetry equations for type IIB can be obtained by exchange

$$\Psi^+ \leftrightarrow *_7\sigma(\Psi^+) \quad \text{and} \quad \tilde{F}_A \leftrightarrow \tilde{F}_B$$

  where $\tilde{F}_B = \tilde{F}_0 + \tilde{F}_2 + \tilde{F}_4 + \tilde{F}_6$
- This relation can be seen as a generalised mirror symmetry for type II backgrounds with $G_2 \times G_2$-structure

Calibrations

- Furthermore we consider the static, magnetic D-branes $(\Sigma, \mathscr{F})$,

- Furthermore we consider the static, magnetic D-branes $(\Sigma, \mathscr{F})$, filling $1+q$ ($q = 1, 2$) spacetime dimensions and wrapping a cycle Σ in the internal space,

- Furthermore we consider the static, magnetic D-branes $(\Sigma, \mathscr{F})$, filling $1 + q$ $(q = 1, 2)$ spacetime dimensions and wrapping a cycle Σ in the internal space, with the worldvolume field strength $\mathscr{F}$ $(d\mathscr{F} = P_\Sigma[H])$

- Furthermore we consider the static, magnetic D-branes $(\Sigma, \mathscr{F})$, filling $1 + q$ ($q = 1, 2$) spacetime dimensions and wrapping a cycle Σ in the internal space, with the worldvolume field strength $\mathscr{F}$ ($d\mathscr{F} = P_\Sigma[H]$)
- Also we restrict the case $\|\xi^1\|^2 = \|\xi^2\|^2 = e^A$ and

Calibrations

- Furthermore we consider the static, magnetic D-branes $(\Sigma, \mathscr{F})$, filling $1+q$ $(q=1,2)$ spacetime dimensions and wrapping a cycle Σ in the internal space, with the worldvolume field strength $\mathscr{F}$ $(d\mathscr{F} = P_\Sigma[H])$
- Also we restrict the case $\|\xi^1\|^2 = \|\xi^2\|^2 = e^A$ and $\tilde{F}_A \wedge \sigma(\Psi^+)_{|7} = 0$

Calibrations

- Furthermore we consider the static, magnetic D-branes $(\Sigma, \mathscr{F})$, filling $1 + q$ $(q = 1, 2)$ spacetime dimensions and wrapping a cycle Σ in the internal space, with the worldvolume field strength $\mathscr{F}$ $(d\mathscr{F} = P_\Sigma[H])$
- Also we restrict the case $\|\xi^1\|^2 = \|\xi^2\|^2 = e^A$ and $\tilde{F}_A \wedge \sigma(\Psi^+)_{|7} = 0$
- *A generalised calibration* is a polyform ω such that

- Furthermore we consider the static, magnetic D-branes $(\Sigma, \mathscr{F})$, filling $1+q$ $(q=1,2)$ spacetime dimensions and wrapping a cycle Σ in the internal space, with the worldvolume field strength $\mathscr{F}$ $(d\mathscr{F} = P_\Sigma[H])$
- Also we restrict the case $\|\xi^1\|^2 = \|\xi^2\|^2 = e^A$ and $\tilde{F}_A \wedge \sigma(\Psi^+)_{|7} = 0$
- *A generalised calibration* is a polyform ω such that
  a) $d_H\omega = 0$

Calibrations

- Furthermore we consider the static, magnetic D-branes $(\Sigma, \mathscr{F})$, filling $1+q$ $(q=1,2)$ spacetime dimensions and wrapping a cycle Σ in the internal space, with the worldvolume field strength $\mathscr{F}$ $(d\mathscr{F} = P_\Sigma[H])$
- Also we restrict the case $\|\xi^1\|^2 = \|\xi^2\|^2 = e^A$ and $\tilde{F}_A \wedge \sigma(\Psi^+)_{|7} = 0$
- *A generalised calibration* is a polyform ω such that

  a) $d_H\omega = 0$

  b) $P_\Sigma[\omega] \wedge e^{\mathscr{F}} \leq \mathscr{E}(\Sigma, \mathscr{F})$ for the energy density $\mathscr{E}$ and any D-brane $(\Sigma, \mathscr{F})$

Calibrations

- Furthermore we consider the static, magnetic D-branes $(\Sigma, \mathscr{F})$, filling $1+q$ $(q=1,2)$ spacetime dimensions and wrapping a cycle Σ in the internal space, with the worldvolume field strength $\mathscr{F}$ $(d\mathscr{F} = P_\Sigma[H])$
- Also we restrict the case $\|\xi^1\|^2 = \|\xi^2\|^2 = e^A$ and $\tilde{F}_A \wedge \sigma(\Psi^+)_{|7} = 0$
- *A generalised calibration* is a polyform ω such that
  - a) $d_H\omega = 0$
  - b) $P_\Sigma[\omega] \wedge e^{\mathscr{F}} \leq \mathscr{E}(\Sigma, \mathscr{F})$ for the energy density $\mathscr{E}$ and any D-brane $(\Sigma, \mathscr{F})$
- A D-brane is *calibrated* iff the inequality above is saturated

- We define the following polyforms

$$\begin{aligned} \omega^{(q=1)} &:= e^{2A-\Phi} *_7 \sigma(\Psi^+) \\ \omega^{(q=2)} &:= e^{3A-\Phi}\Psi^+ - \sum_k \tilde{C}_{(2k)} \end{aligned} \tag{2}$$

Calibrations

- We define the following polyforms

$$\begin{aligned} \omega^{(q=1)} &:= e^{2A-\Phi} *_7 \sigma(\Psi^+) \\ \omega^{(q=2)} &:= e^{3A-\Phi}\Psi^+ - \sum_k \tilde{C}_{(2k)} \end{aligned} \tag{2}$$

- With the κ-symmetry and worldvolume chiral operator we can show that (2) satisfies the bound condition (*b*).

Calibrations

- Using the operator d_H and the condition $\tilde{F}_A = d_H \sum_k \tilde{C}_{(2k)}$ we obtain

$$d_H\omega^{(q=2)} = d_H(e^{3A-\Phi}\Psi^+ - \tilde{F}_A)$$
$$d_H\omega^{(q=1)} = d_H(e^{2A-\Phi} *_7 \sigma(\Psi^+))$$

Calibrations

- Using the operator d_H and the condition $\tilde{F}_A = d_H \sum_k \tilde{C}_{(2k)}$ we obtain

$$d_H\omega^{(q=2)} = d_H(e^{3A-\Phi}\Psi^+ - \tilde{F}_A)$$
$$d_H\omega^{(q=1)} = d_H(e^{2A-\Phi} *_7 \sigma(\Psi^+))$$

- They are equivalent to the first two susy-equations

$$\mathrm{d}_H(e^{3A-\Phi}\Psi^+) = \tilde{F}_A, \quad (1a)$$
$$\mathrm{d}_H(e^{2A-\Phi} *_7 \sigma(\Psi^+)) = 0, \quad (1b)$$
$$\tilde{F}_A \wedge \sigma(\Psi^+)_{|7} = 0, \quad (1c)$$
$$\|\xi^1\|^2 = \|\xi^2\|^2 = e^A \quad (1d)$$

Calibrations

- We define the following polyforms

$$\begin{aligned} \omega^{(q=1)} &:= e^{2A-\Phi} * \sigma(\Psi^{+}) \\ \omega^{(q=2)} &:= e^{3A-\Phi}\Psi^{+} - \sum_k \tilde{C}_{(2k)} \end{aligned} \qquad (2)$$

- With the κ-symmetry and worldvolume chiral operator we can show that (2) satisfies the bound condition (b).

Calibrations

- We define the following polyforms

$$\begin{aligned} \omega^{(q=1)} &:= e^{2A-\Phi} * \sigma(\Psi^{+}) \\ \omega^{(q=2)} &:= e^{3A-\Phi}\Psi^{+} - \sum_k \tilde{C}_{(2k)} \end{aligned} \quad (2)$$

- With the κ-symmetry and worldvolume chiral operator we can show that (2) satisfies the bound condition (b).
- The condition (a) is to the susy conditions (1) equivalent.

Calibrations

- We define the following polyforms

$$\begin{aligned} \omega^{(q=1)} &:= e^{2A-\Phi} * \sigma(\Psi^{+}) \\ \omega^{(q=2)} &:= e^{3A-\Phi}\Psi^{+} - \sum_k \tilde{C}_{(2k)} \end{aligned} \qquad (2)$$

- With the κ-symmetry and worldvolume chiral operator we can show that (2) satisfies the bound condition (b).
- The condition (a) is to the susy conditions (1) equivalent.
- $\Rightarrow$ the one-to-one-correspondence between the calibrations and susy equations

Calibrations

- We define the following polyforms

$$\begin{aligned} \omega^{(q=1)} &:= e^{2A-\Phi} * \sigma(\Psi^+) \\ \omega^{(q=2)} &:= e^{3A-\Phi}\Psi^+ - \sum_k \tilde{C}_{(2k)} \end{aligned} \quad (2)$$

- With the κ-symmetry and worldvolume chiral operator we can show that (2) satisfies the bound condition (*b*).
- The condition (a) is to the susy conditions (1) equivalent.
- $\Rightarrow$ the one-to-one-correspondence between the calibrations and susy equations
- The D-brane is calibrated iff it is supersymmetric.

Conclusions

Conclusions

- I studied $\mathscr{N} = 1$ supersymmetric type II backgrounds of the form $\mathbb{R}^{1,2} \times M_7$ during my master thesis.

Conclusions

- I studied $\mathcal{N} = 1$ supersymmetric type II backgrounds of the form $\mathbb{R}^{1,2} \times M_7$ during my master thesis.
- I showed that the SUSY conditions can be expressed in terms of polyform that defines the $G_2 \times G_2$-structure on the internal manifold M_7,

Conclusions

- I studied $\mathcal{N} = 1$ supersymmetric type II backgrounds of the form $\mathbb{R}^{1,2} \times M_7$ during my master thesis.
- I showed that the SUSY conditions can be expressed in terms of polyform that defines the $G_2 \times G_2$-structure on the internal manifold M_7,
- and there is an one-to-one correspondence between background susy equations and generalised D-brane calibrations. (These results were already explored in [hep-th/0905.1582])

Conclusions

- I studied $\mathcal{N} = 1$ supersymmetric type II backgrounds of the form $\mathbb{R}^{1,2} \times M_7$ during my master thesis.
- I showed that the SUSY conditions can be expressed in terms of polyform that defines the $G_2 \times G_2$-structure on the internal manifold M_7,
- and there is an one-to-one correspondence between background susy equations and generalised D-brane calibrations. (These results were already explored in [hep-th/0905.1582])
- The D-branes are calibrated iff they are supersymmetric.

Conclusions

- I studied $\mathcal{N} = 1$ supersymmetric type II backgrounds of the form $\mathbb{R}^{1,2} \times M_7$ during my master thesis.
- I showed that the SUSY conditions can be expressed in terms of polyform that defines the $G_2 \times G_2$-structure on the internal manifold M_7,
- and there is an one-to-one correspondence between background susy equations and generalised D-brane calibrations. (These results were already explored in [hep-th/0905.1582])
- The D-branes are calibrated iff they are supersymmetric.
- It would be interesting to pursue this analysis and to calculate susy conditions for $AdS_3 \times M_7$-background

Conclusions

- I studied $\mathcal{N} = 1$ supersymmetric type II backgrounds of the form $\mathbb{R}^{1,2} \times M_7$ during my master thesis.
- I showed that the SUSY conditions can be expressed in terms of polyform that defines the $G_2 \times G_2$-structure on the internal manifold M_7,
- and there is an one-to-one correspondence between background susy equations and generalised D-brane calibrations. (These results were already explored in [hep-th/0905.1582])
- The D-branes are calibrated iff they are supersymmetric.
- It would be interesting to pursue this analysis and to calculate susy conditions for $AdS_3 \times M_7$-background
- and find out the conditions for susy-breaking