Non-associative Deformations of Geometry in Double Field Theory

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based on JHEP 04(2014)141 by R. Blumenhagen, MF, F. Haßler, D. Lüst, R. Sun

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Outline

- Deformation Quantization
- T-Duality and non-associativity in String Theory
- Non-associative Deformations of Geometry in DFT

Canonical Quantization

Replace Poisson-bracket by commutator:

$$\{x, p\}_{PB} = 1 \qquad \rightarrow \qquad [x, p] = i\hbar$$

Fulfilled for instance by operators: $\hat{p} = -i\hbar\frac{\partial}{\partial x}$

Deformation Quantization:

No operators, instead change multiplication law: Replace $f \cdot g$ by

$$f \star g := f \cdot g + \frac{i\hbar}{2} \begin{pmatrix} \partial_x f \\ \partial_p f \end{pmatrix}^T \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \partial_x g \\ \partial_p g \end{pmatrix}$$

Insert coordinate and momentum:

$$\left.\begin{aligned} x \star p &= x \cdot p + \tfrac{i\hbar}{2} \\ p \star x &= p \cdot x - \tfrac{i\hbar}{2} \end{aligned}\right\} [x, p] = x \star p - p \star x = i\hbar$$

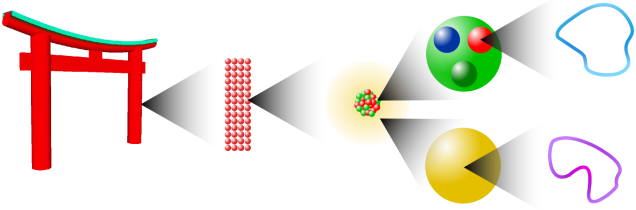
Take home message:

Commutation relations realized by deformed product 9709040

$$f \star g := f \cdot g + \frac{i\hbar}{2} \omega^{ij} \partial_i f \, \partial_j g + \mathcal{O}(\hbar^2)$$

String Theory

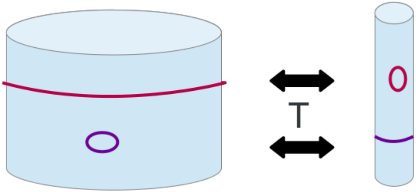
Fundamental objects not points, but strings



Strings must live in 10D → compactify!

T-Duality

Closed strings wind around compactified dimensions:



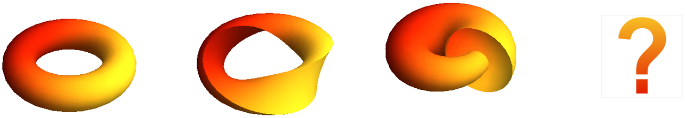
$$\text{momenta } p_i \quad \overset{\text{T-Duality}}{\longleftrightarrow} \quad \text{winding momenta } \tilde{p}^i$$

$$\updownarrow \qquad\qquad \updownarrow$$

$$\text{coordinate } x^i \quad \overset{\text{T-Duality}}{\longleftrightarrow} \quad \text{winding coordinate } \tilde{x}_i$$

Non-geometric Fluxes

T-Duality mixes G and B $\Rightarrow$ change of geometry

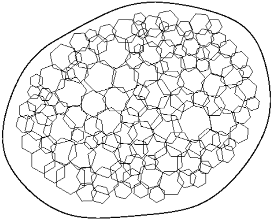


$$H_{ijk} \overset{T^k}{\longleftrightarrow} F_{ij}{}^k \overset{T^j}{\longleftrightarrow} Q_i{}^{jk} \overset{T^i}{\longleftrightarrow} R^{ijk}$$

Non-associative Geometry

Blumenhagen, Lüst, Plauschinn et alii:

$$[x^a, x^b] \cong R^{abc} p_c$$



“fuzzy” geometry due to Heisenberg uncertainty:

$$\Delta x^a \Delta x^b \cong \langle [x^a, x^b] \rangle \neq 0$$

non-vanishing Jacobi identity! $\hat{=}$ non-associative operators!

Not possible in ordinary quantum mechanics!

Deformed product vanishes for observables by momentum conservation! 1106.0316 by Blumenhagen, Deser, Lüst, Plauschinn, Rennecke

Our work: Investigate in double field theory how non-associativity vanishes!

Double Field Theory

Combine $\binom{\text{normal}}{\text{winding}}$ in 2D vector

$$P_M = \begin{pmatrix} p_i \\ \tilde{p}^i \end{pmatrix} \quad \partial_M = \begin{pmatrix} \partial_i \\ \tilde{\partial}^i \end{pmatrix} \quad X_M = \begin{pmatrix} x_i \\ \tilde{x}^i \end{pmatrix}$$

$\Rightarrow$ Coordinates and winding on equal footage!

BUT: Constraints needed for consistency!

Non-associative Deformations of Geometry in DFT

Translate deformed product into DFT:

$$f \vartriangle g \vartriangle h = f \cdot g \cdot h + \underbrace{\mathcal{F}_{ABC}}_{\text{contains H,f,Q,R}} \partial^A f\, \partial^B g\, \partial^C h$$

We found: Deformation vanishes by consistency constraints!

Deformation in physical situations (action) $\hat{=}$ integration:

$$\int_{DFT} \mathcal{F}^{ABC}\, \mathcal{D}_A f\, \mathcal{D}_B g\, \mathcal{D}_C h \overset{\text{PI}}{=} -\int_{DFT} \underbrace{\mathcal{Z}^{AB}}_{Bianchi\ \mathcal{Z}^{AB}=0!} f\, \mathcal{D}_A g\, \mathcal{D}_B h$$

Another deformation:

DFT allows for another deformation:

$$f \triangle g \triangle h = f \cdot g \cdot h + \check{\mathcal{F}}_{ABC}\, \partial^A f\, \partial^B g\, \partial^C h$$

$\Rightarrow$ Generalization of open strings in B-field background 9812219

No reason to vanish! Integral:

$$\int_{DFT} \check{\mathcal{F}}^{ABC}\, \mathcal{D}_A f\, \mathcal{D}_B g\, \mathcal{D}_C h \overset{\text{PI}}{=} - \int_{DFT} \underbrace{\mathcal{G}^{AB}}_{eom:\, \mathcal{G}^{AB}=0!}\, f\, \mathcal{D}_A g\, \mathcal{D}_B h$$

Conclusion

associativity of observables preserved by crucial ingredients of double field theory

$\check{\mathcal{F}}_{ABC}\,\partial^A f \partial^B g \partial^C h$	$\mathcal{F}_{ABC}\,\partial^A f \partial^B g \partial^C h$
equation of motion	Bianchi identity
continuity equation	closure of algebra

Outlook

Future research directions:

- Derive higher orders of the product (ongoing)
- Non-associativity in Hamiltionian formalism