# Diffusion Constants in Electroweak Phase Transition: Leading-Log Results

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# Electroweak Baryogenesis

## baryon asymmetric universe (BAU)

- observations: **baryon abundance** over anti-baryons $\left(Y_B \equiv \frac{n_b - n_{\bar{b}}}{s}\right)$

$$6.7 \times 10^{-11} < Y_B^{BBN} < 9.2 \times 10^{-11} \text{ (95\% C.L.)[1]}$$
$$8.36 \times 10^{-11} < Y_B^{CMB} < 9.32 \times 10^{-11} \text{ (95\% C.L.)[1, 2]}$$

- correspondence confirms **experimental evidence**
- **baryogenesis** transforms a baryon symmetric universe into an asymetric one

## Shakarov conditions

1. B-number violation
2. C- and CP-violation
3. non-equilibrium

## electroweak baryogenesis

- most attractive and promising scenario[3]
- during electroweak phase transition at $T \lesssim 100$ GeV

# Electroweak Baryogenesis

## strong enough first order electroweak phase transition (EWPT)

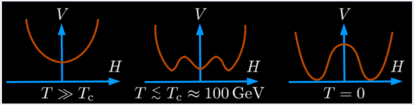



Figure 1: Higgs potential over Higgs field[4].

- **sphaleron transition** for $T \lesssim T_C \approx 100$ GeV
- Higgs acquires non vanishing **VEV** $\langle\phi\rangle \neq 0$ (**SSB**)
- requires $m_H \lesssim 70$ GeV $\Rightarrow$ **not** in SM [5]

## bubble nucleation

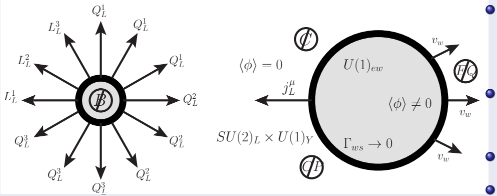



Figure 2: Sphaleron process and bubble nucleation during EWPT.

- particle scatterings ($\not{C}$, $\not{CP}$) with the wall lead to **left-handed** charges $n_{\text{left}}$
- $n_{\text{left}}$ **diffuse** into symmetric phase: $n_{\text{left}} \propto e^{\frac{v_W}{D} z}$
- sphalerons: $n_{\text{left}} \to n_B$
- $n_B$ gets swept up by expanding bubble $\Rightarrow$ **freezes in**

# Diffusion Constants in Electroweak Phase Transition

## importance of diffusion constants

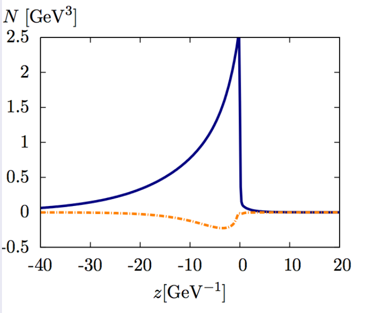



Figure 3: Charge densities over $z$. Higgs (blue,solid), quarks (orange,dashed)[6].

sphaleron process converts left-handed charges $n_{\text{left}} \propto e^{\frac{v_w}{D}z}$ to **net baryon charge** $n_B$[7]:

$$n_B = -3\frac{\Gamma_{\text{ws}}}{v_w}\int_{-\infty}^{0} dz\; n_{\text{left}}(z) e^{\frac{15}{4}\frac{\Gamma_{\text{ws}}}{v_w}z}.$$

$\Rightarrow$ compute **diffusion constant** $D$ to **probe** electroweak barygenesis

# Outline

## goal

calculate diffusion constants to **leading-log** accuracy in $SU(2)_L \times U(1)_Y$ gauge-theory

$$J_i = -D\partial_i n,$$

by making use of the **closed-time-path formalism** and **compare** the results to previous estimates.

## Closed-Time-Path (CTP) formalism[8]

derive **transport equation** in CTP as analogon to Boltzmann equation

## collision term

- consider all contributing **diagrams**
- use **resummed propagators**
- **Debye screening** implicit

## variational approach[9]

**set of functions** can be used to extract diffusion constants

## numerical calculation

**3-dim integrals** has to be solved for the different couplings numerically

## expected result

$$D^{-1} = \left(ag^4 + bg^4 \log 1/g\right) T$$

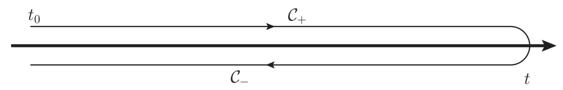
# In-Out v.s Closed-Time-Path Formalism

## in-out theory

- **asymptotic free** in- and out-states
- **interaction regime** can be switched on and off
- doing well in computing $S$-matrix elements in **scattering theory**

## in-in theory

- **situation**: hot universe $\rightarrow$ interactions with the plasma
- **problem**: How to construct field theory that accounts for **finite density** problems?
- Schwinger(1961) and Keldysh (1965): **Closed-Time-Path Formalism**




- general **physical** states
- adaptable to both **equilibrium** and **non-equilibrium** problems at **finite temperatures**

# Green-Functions in CTP

## CTP indices

- CTP-index $a = \{+, -\}$ is referred to each path

## four two-point functions

$$S^{++}(u,v) \equiv S^{t}(u,v) = \langle T[\psi(u)\bar{\psi}(v)]\rangle$$
$$S^{--}(u,v) \equiv S^{\bar{t}}(u,v) = \langle \bar{T}[\psi(u)\bar{\psi}(v)]\rangle$$
$$S^{+-}(u,v) \equiv S^{<}(u,v) = -\langle \bar{\psi}(v)\psi(u)\rangle$$
$$S^{-+}(u,v) \equiv S^{>}(u,v) = \langle \psi(u)\bar{\psi}(v)\rangle$$

## useful combinations

$$G^{\mathcal{A}} \equiv \frac{1}{2}(G^{>} - G^{<})$$
$$G^{H} \equiv \frac{1}{2}(G^{t} - G^{\bar{t}})$$
$$G^{r} \equiv G^{t} - G^{<}$$
$$G^{a} \equiv G^{t} - G^{>}$$

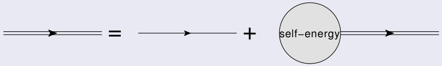
## example of tree-level Green functions

$$iS^{++}(p) = +\frac{i(\not{p}+m)}{p^2 - m^2 + i\epsilon} - 2\pi(\not{p}+m)f(|\mathbf{p}|)\delta(p^2 - m^2)$$
$$iS^{+-}(k) = -2\pi(\not{p}+m)\delta(p^2 - m^2)\mathrm{sign}(p^0)f(p^0)$$

# Kinetic equations in CTP

## Schwinger-Dyson equation (SDE)

- 1PI resummation yields **full propagator**




$$S(u,v) = S^0(u,v) + \underbrace{\int d^4w \Sigma(u,w) \cdot S(w,v)}_{\equiv \Sigma(u,w) \odot S(w,v)}$$

## Kadanoff-Baym-equation (KBE)

$$i\partial\!\!\!/ iS^{<,>} - \Sigma^H \odot S^{<,>} - \Sigma^{<,>} \odot S^H = \frac{1}{2}(\Sigma^> \odot S^< - \Sigma^< \odot S^>)$$

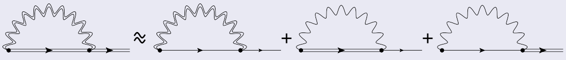
## transport equation for fermions

$$\left[\partial_t + \frac{|\vec{k}| \cdot \vec{\nabla}}{|k|}\right] f(|k|) = \frac{1}{2\pi} \int_0^\infty dk_0 \frac{1}{4} \mathrm{Tr}\left[\mathcal{C}_\Psi(k) + \mathcal{C}_\Psi^\dagger(k)\right]$$

- textbfcollision term $\mathcal{C}_\Psi(k) \equiv i\Sigma^>(k) iS^<(k) - i\Sigma^<(k) iS^>(k)$

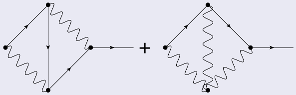
# Collision Term

## wave function contributions



- **resummed propagators** (double lined) necessary
- expansion in the coupling
- contribute at leading-log due to **logarithmic enhancement** for small $k^2 \lesssim g^2 T^2$ near mass-pole

## vertex type contributions



- free propagators sufficient
- do **not** contribute at leading-log

## resummed propagators

$$\Delta^{\mathcal{A}}_{\mu\nu} = \sum_{T,L} P^{T,L}_{\mu\nu} \frac{\Pi^{\mathcal{A}}_{T,L}}{[k^2 + \Pi^{H}_{T,L}]^2 + [\Pi^{\mathcal{A}}_{T,L}]^2}$$

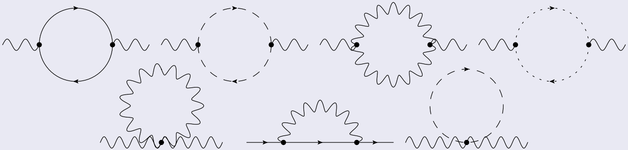
$$S^{\mathcal{A}}(k) = P_X \frac{2\left(\not{k} - \not{\Sigma}^H\right)\Sigma^A(k - \Sigma^H) - \not{\Sigma}^A\left(\not{k} - \not{\Sigma}^H\right)^2 + \not{\Sigma}^{A3}}{\left[\left(\not{k} - \not{\Sigma}^H\right)^2 - \not{\Sigma}^{A2}\right]^2 + 4\left[\Sigma^A(k - \Sigma^H)\right]^2}$$

- $G^H$ $\Rightarrow$ **mass shift**
- $G^{\mathcal{A}}$ $\Rightarrow$ **finite width**

$\Rightarrow$ **cures** ***t*****-channel divergence (screening)**

# Self-Energies

## self-energy diagrams via functional derivative of 2PI vacuum bubbles



## spectral self-energy

- tadpole (seagull) do **not** contribute
- computed **without** any approximation **analytically**

## hermitian self-energy

- all diagrams contribute
- extract **leading term** in temperature $\propto g^2 T^2$
- provides particles **thermal masses** (Debye masses)

$\Rightarrow$ **cures** ***t*****-channel divergence (screening)**

## $G^H \propto$ thermal masses

$$m_B^2 = \frac{g_1^2 T^2}{6} \sum_{i=\text{f, H}} Y_i$$

$$m_W^2 = \frac{g_2^2 T^2}{12} \left[N_f + 4 + N_H\right]$$

$$m_f^2 = \frac{g_1^2 Y_f^2 + g_2^2 C_2^F}{8} T^2$$

# Variational Approach

## diffusion equation

$J_i = -D\partial_i n \propto \partial_i \mu$

## Transport equation

$\hat{k} \cdot \vec{\nabla} f(|k|) \propto \int_0^\infty dk_0 \mathrm{Tr}\left[\mathcal{C}_\Psi(k)\right]$

## linearisation

- $f(p^0) = f_0(p^0) + \underbrace{f_0(p^0)[1 \pm f_0(p^0)]f_1(p^0)}_{\equiv \delta f \propto \nabla \mu}$
- $f_0(p^0) = [e^{\beta(p^0-\mu)} \mp 1]^{-1}$ a local equilibrium distribution function
- $\mathcal{C}[f] = \underbrace{\mathcal{C}[f_0]}_{\to 0} + \underbrace{\mathcal{C}[f_1]}_{\text{linear in perturbation}} + \mathcal{O}(f_1^2)$

## extraction of diffusion constant

**variational approach** to transport equation yields

$$D^{-1} \propto \underbrace{\int d^4k}_{\text{outer momentum}} \; \mathcal{C}[\phi](k) \cdot \phi(k)$$

with function $\phi(k) = \frac{|\mathbf{k}|}{T}$

# Numerics

## numerical calculation

- still **ongoing**
- compute diffusion constant for different couplings $g \in \{0.1, 1\}$
- find **fit** with fit-function $D^{-1} = \left(ag^4 + bg^4 \log 1/g\right) T$
- **leading-log** term corresponds to parameter $b$

## snippet of ongoing computation

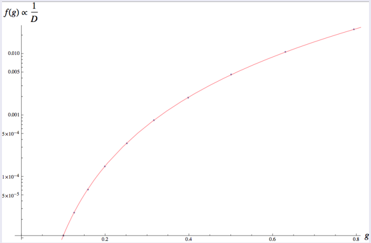



Figure 4: A function proportional to the inverse diffusion constant over the coupling g.

# Conclusion

## conclusion

- diffusion constants can be used to **probe electroweak baryogenesis**
- we derived **Boltzmann equation** in **CTP**
- screening due to **thermal masses** is implicit and cures $t$-channel divergence
- vertex-type diagrams do not contribute
- use **resummed propagators** in wave function contributions
- **variational approach** applied to compute diffusion constant out of the **linearised transport equation**
- fit numerical results on order to obtain **leading-log contributions** of diffusion constant
- **compare** with previous estimates

# References


1 Particle Data Group collaboration, W.M. Yao et al., Review of particle physics, J. Phys. G 33 (2006) 1

2 WMAP collaboration, J. Dunkley et al., Five-year Wilkinson Microwave Anisotropy Probe (WMAP) observations: likelihoods and parameters from the WMAP data, Astrophys. J. Suppl. 180 (2009) 306 [arXiv:0803.0586]

3 V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B 155, 36 (1985)

4 "How Diffusion Works for Electroweak Baryogenesis-and what we Learn for Leptogenesis" Talk by Bjoern Garbrecht

5 A. I. Bochkarev and M. E. Shaposhnikov, Mod. Phys. Lett. A 2, 417 (1987)

6 D. J. H. Chung, B. Garbrecht, M. J. Ramsey-Musolf, and S. Tulin, Supergauge interactions and electroweak baryogenesis, Journal of High Energy Physics, vol. 12, p. 067, 2009

7 P. Huet and A.E. Nelson, Electroweak baryogenesis in supersymmetric models, Phys. Rev. D 53 (1996) 4578 [hep-ph/9506477]

8 T. Prokopec, M. G. Schmidt and S. Weinstock, Transport equations for chiral fermions to order h-bar and electroweak baryogenesis, Annals Phys. 314 (2004) 208 [arXiv:hep-ph/0312110]

9 P. Arnold, G. D. Moore and L. G. Yaffe, JHEP 0011, 001 (2000) [hep-ph/0010177]