From curved spacetimes to the Kondo effect

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Overview

- Qualitative explanation of AdS/CFT
- Entanglement Entropy
- A holographic model of the Kondo effect
- Entanglement Entropy in the Kondo effect

What is AdS/CFT?

Simply speaking:

The conjecture that certain conformal quantum field theories (CFTs) have a dual description in terms of gravity with anti de-Sitter (AdS) backgrounds.

Prospects:

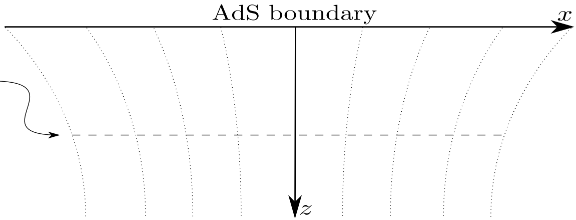
- Hard computations in the QFT may be simple in the dual gravity picture.
- Set up a *holographic dictionary* relating QFT and gravity objects.

What is AdS/CFT?

AdS metric:

$$ds^2 = \frac{1}{z^2}\left(-dt^2 + dx_i dx^i + dz^2\right)$$

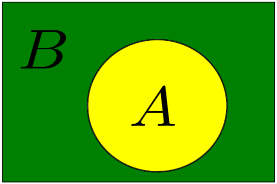
$$ds^2\Big|_{z\equiv const.} = \frac{1}{z^2}\left(\eta_{mn}dx^m dx^n\right)$$

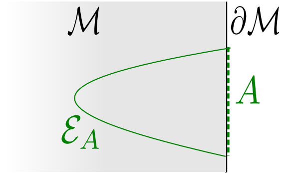


- Timelike asymptotic infinity (*boundary*) at $z = 0$ with induced Minkowsky metric.
- CFT is understood to live at boundary.

The holographic dictionary

One entry to the holographic dictionary is the entanglement entropy. It is a measure of entanglement between two subystems A and B of the CFT.





$\mathcal{S}_{EE}(A) = -\mathrm{Tr}_A[\rho_A \log(\rho_A)]$ with reduced density matrix $\rho_A \equiv \mathrm{Tr}_B[\rho_{A\cup B}]$ $\Leftrightarrow$ $\mathcal{S}_{EE}(A) = \mathrm{Area}\,(\mathcal{E}_A)\,/4G_N$ where $\mathcal{E}_A$ is a spacelike extremal surface

→ Generalisation of Bekenstein-Hawking entropy formula!

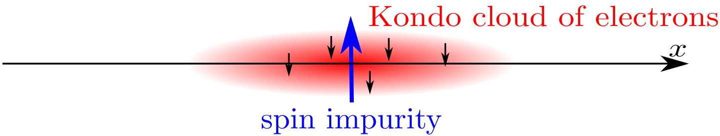
The holographic Kondo model

- Field theory side:
  - Spin-spin interaction of electrons with a localised magnetic impurity.
  - Can be mapped to $1+1$ dimensional conformal system [Affleck et. al. 1991].
  - Below a temperature T_K, electrons form a bound state around impurity: *Kondo cloud*.

- Holographic gravity side: [Erdmenger et. al.: 1310.3271]
  - Dual gravity model has $2+1$ (bulk-) dimensions.
  - Localised spin impurity is represented by co-dimension one brane extending from boundary into the bulk.
  - Finite T is implemented by black hole background with finite Hawking-temperature.

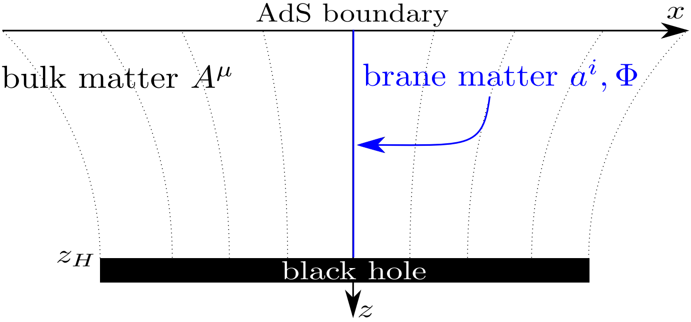
The holographic Kondo model

Field theory picture:



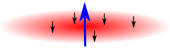
$\Updownarrow$

Gravity picture:



The holographic Kondo model

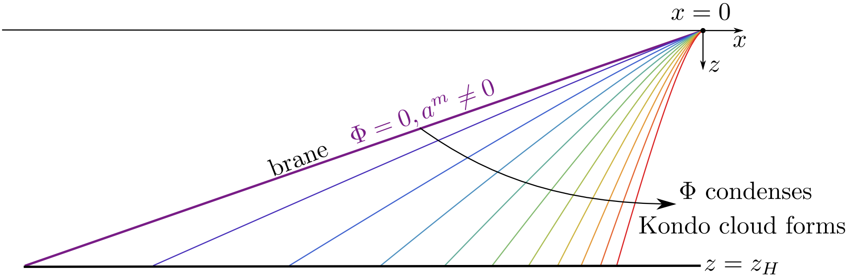
How can we obtain information about the Kondo cloud from our model?

- Kondo cloud is formed by anti-aligned spins 
- $\Rightarrow$ expect imprint on entanglement entropy $\mathcal{S}_{EE}$, e.g. entanglement of state $|\Psi\rangle = \frac{1}{\mathcal{N}}\left(|\uparrow\downarrow\downarrow\ldots\rangle - |\downarrow\uparrow\uparrow\ldots\rangle\right)$ does not vanish.
- $\mathcal{S}_{EE}$ is defined by spacelike geodesics $\Rightarrow$ to calculate it, we need to take backreaction on geometry into account.
- What is the backreaction of an infinitely thin hypersurface carrying energy-momentum? *Israel junction conditions!*

Kondo model: backreaction

$$S_{brane}[a^m, \Phi] = -\int dV_{brane}\left(\frac{1}{4}f^{mn}f_{mn} + \gamma^{mn}(D_m\Phi)^\dagger D_n\Phi + V(\Phi^\dagger\Phi)\right)$$

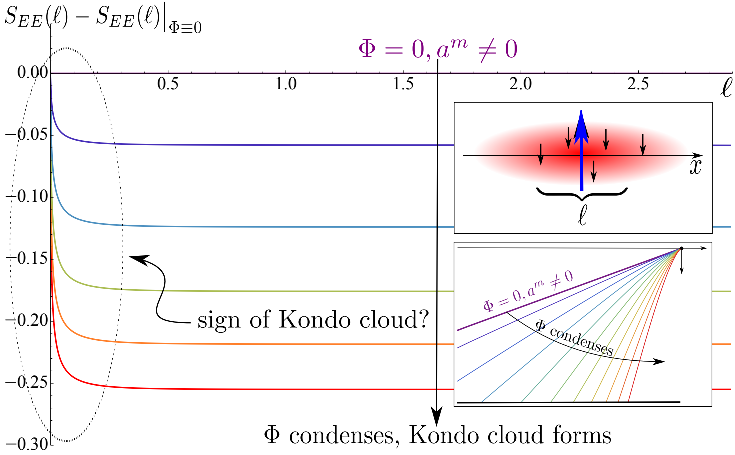
- Brane starts at boundary and falls into black hole.
- As Φ condenses, Kondo cloud forms at boundary.
- Qualitative bending of brane constrained by energy conditions.



[Erdmenger, M.F., Newrzella: 1410.7811]

Kondo model: entanglement entropy

Preliminary results on entanglement entropy: Difference of $S_{EE}(\ell)$ relative to solutions with $\Phi = 0$.



[Erdmenger, M.F., Hoyos, Newrzella, O'Bannon, Wu: work in progress]

Summary and Outlook

Holography allows us to investigate problems in quantum field theory by studying a dual gravitational system.

Example: The holographic Kondo model

- Gravity dual involves thin brane carrying energy-momentum.
- We obtained general results constraining possible geometries of the brane by energy conditions [1410.7811].
- Specific Kondo model will be solved numerically.

Thank you for your attention

Our goal: Study Israel junction conditions

In electromagnetism: To describe field around an infinitely thin charged surface Σ, integrate Maxwells equations in a box around Σ:

$$\Rightarrow \vec{E}_{||} \text{ continuous, } \; \vec{E}_{\perp} \text{ discontinuous on } \Sigma$$

In gravity: To describe backreaction of an infinitely thin surface, integrate Einsteins equations in a box $\Rightarrow$ Israel junction conditions:

$$(K^+_{ij} - \gamma_{ij} K^+) - (K^-_{ij} - \gamma_{ij} K^-) = -\kappa\, S_{ij}$$

S_{ij}: energy momentum tensor on the brane, γ_{ij}: induced metric,
$K^{\pm}$: extrinsic curvatures depending on embedding.

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S_{ij}: energy momentum tensor on the brane, γ_{ij}: induced metric,
$K^{\pm}$: extrinsic curvatures depending on embedding.

$\Rightarrow$ Embedding (location of the brane) will not be $x \equiv 0$ anymore, it becomes a dynamical function $x(z)$ with $(*)$ its own equations of motion.

[Israel, 1966]